The Semantics of Plurals, Part 2: Cumulativity, Distributivity, and Quantification

1. Our Current Picture of Plurals

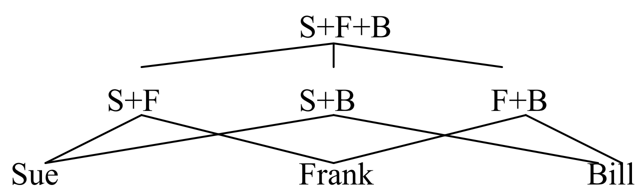
At the conclusion of 'Part 1' of our discussion of plurals, we had built a semantics for interpreting plural NPs/DPs that had the following key ingredients.

(1) **Ontology of Plural Entities**

In addition to the domain D of 'singular entities', we have the extended domain *D, which contains all the 'singular entities' *and* all the possible 'plural entities' (groups) that one can construct from the set of singular entities.

a. Definition of **D* The smallest set such that
(i) $D \subseteq *D$
(ii) If x, y ∈ *D, then x+y ∈ *D

b. Picture of **D*
(i) *The Domain of Individuals, D:* Sue, Frank, Bill
(ii) *The Domain of Plural Entities, *D:*



(2) **The Interpretation of Plural NPs**

A plural NP is interpreted as the set of all the possible groups that could be formed from the extension of the singular NP.

a. Semantics of Plural NP: [[NP pl]] = *[[NP]]

b. Picture of Plural NP Semantics
(i) [[boy]] = {Frank, Bill, Dave}
(ii) [[boys]] = *[[boy]] = { Frank, Bill, Dave, Frank+Bill, Frank+Dave, Dave+Bill, Frank+Dave+Bill }

(3) **The Interpretation of Conjoined DPs**

If 'DP1' is type e and 'DP2' is type e, then [DP1 and DP2] is of type e, and

[[DP1 and DP2]] = [[DP1]]+[[DP2]].

(4) **The Interpretation of Plural Definites**

A plural definite denotes the 'maximal group' from the extension of the plural NP. This is the group that contains *all* the entities in the extension of the singular NP

a. Semantics of the Definite Article

[[the]] = $\lambda P_{<et>}$. MAX(P)

b. The Semantics in a Picture

(i) [[boy]] = { Frank , Bill, Dave }

(ii) [[boys]] = { Frank, Bill, Dave, Frank+Bill, Frank+Dave, Dave+Bill, **Frank+Dave+Bill** }

(iii) [[the boys]] = MAX([[boys]])

= MAX({ Frank, Bill, Dave, Frank+Bill, Frank+Dave, Dave+Bill, **Frank+Dave+Bill** }

= Frank+Bill+Dave

(5) **The Ability for Vs to Directly Take Plurals as Arguments**

- In our semantic system, all plural entities are of type 'e'.
- Consequently, all plural definites are of type 'e'.
- Consequently, plural definites can easily function as the arguments of expressions of type <et> or <e<et>>.
- Thus, in this system, verbs directly take plural DPs as arguments, and their extensions easily contain plural entities.

a. Possible Extensions in Our System

(i) An Expression of Type <et> (*e.g.* "Win")
{ Sue, Bill, **Dave+Frank** }

(ii) An Expression of Type <e<et>> (*e.g.* "See")
{ <Sue, Lauren> , <Dave, Rusty>, <Joe, **Frank+Bill+Dave** > }

Pretty much all approaches to plurals shares essentially this set of core assumptions.

In this set of notes, we will consider various facts that have lead to some of the key controversies and debates in the literature on plurals…

2. Cumulativity, A First Pass

We've seen that the inference in (6) is valid, and that the basic system above captures its validity.

(6) **Cumulative Inference (on NPs)**

a. Bill is a boy.
b. Frank is a boy.
c. Bill and Frank **are boys**.

For many VPs, this same form of inference also seems to be valid.

(7) **Cumulative Inference (on VPs)**

a. Bill is sleeping.
b. Frank is sleeping.
c. Bill and Frank **are sleeping.**

Issue:
The inference in (7), where the main predicate of the sentence is a *VP* (rather than an NP) is not valid in our current semantics.

However, this is arguably the right result, since there are other VPs for which the inference in (7) is much more questionable.

(8) **Cumulative Inference (on VPs)**

a. Bill owns a house.
b. Frank owns a house.
c. Bill and Frank **own a house.**

Key Observation:

- There is a reading of sentence (8c) under which the inference in (8) is *invalid.*
- But, there also *does* seem to be a reading of (8c) where the inference is *valid!*

(9) **Interim Goal**
Let us try to capture this apparent ambiguity of sentence (8c).

- Our system should generate T-conditions for (8c) where the inference in (8) is valid.
- (It already does generate a set of T-conditions where the inference in (8) is invalid.)

2.1 The Cumulativity Operator '*'

In our previous system, we've been making use of the 'meta-language' operator/notation '*', which we've been using to denote the closure of a set under the plural-forming operator '+'.

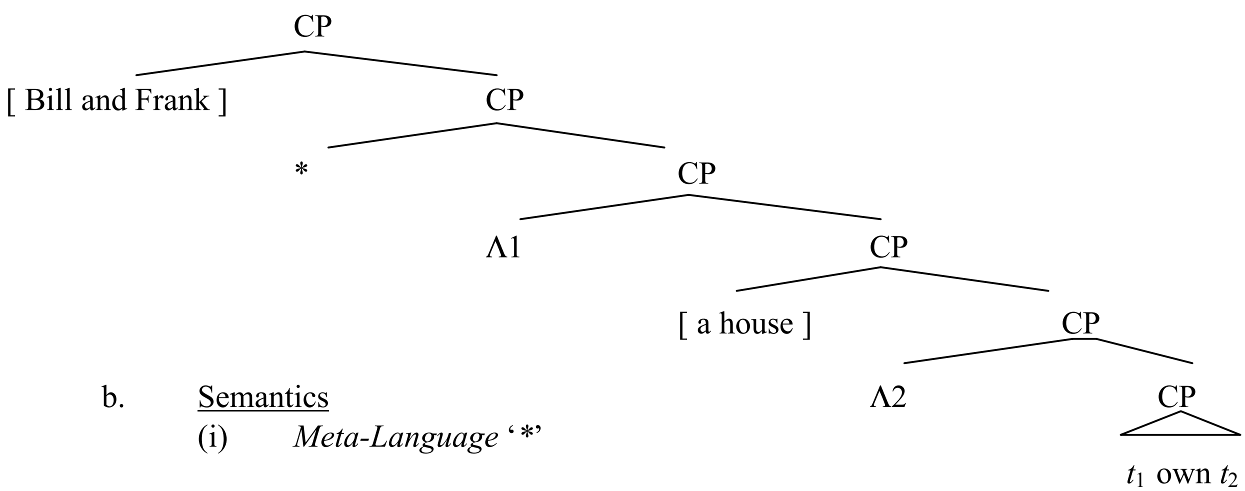
<u>The Big Leap:</u>

- Let's add this operator to our object-language representation.
- That is, let's assume that the lexicon of English also includes this (phonologically empty) operator '*', that has the following characteristic syntax and semantics.

(10) **The Cumulativity Operator**

a. <u>Syntax</u>
The cumulativity operator '*' can take as sister any type <e,t> phrase.

Example LF



b. <u>Semantics</u>

(i) *Meta-Language* '*'

$*S_{<et>}$ = The smallest set such that
i. $S \subseteq *S$
ii. If $x, y \in *S$, then $x+y \in *S$

(ii) *Meaning of Object-Language* '*'

[[* XP]] = * [[XP]]

(11) **Illustration**

a. [[[Λ1 [[a house] [Λ2 [t_1 own t_2] …]]] = {Bill, Frank}

b. [[[* [Λ1 [[a house] [Λ2 [t_1 own t_2] …]]] =

* [[[Λ1 [[a house] [Λ2 [t_1 own t_2] …]]] = {Bill, Frank, Bill+Frank}

(12) **Important General Consequence**

Given the semantics of the object-language 'cumulative operator' (*), as stated in (10b), the following inference pattern is valid.

a. Valid Inference in Proposed System

(i) [[[DP1 [* XP]]]] = T

(ii) [[[DP2 [* XP]]]] = T

(iii) Therefore, [[[[DP1 and DP2] [* XP]]]] = T

b. Quick Proof

(i) If (12ai) is T, then [[DP1]] is in the extension of *[[XP]]

(ii) If (12aii) is T, then [[DP2]] is in the extension of *[[XP]]

(iii) Given the definition of '*' (10b), it follows that [[DP1]]+[[DP2]] is in the extension of *[[XP]]

(iv) Thus, given (3), [[DP1 and DP2]] is in the extension of *[[XP]]

(v) Thus, (12aiii) is true.

(13) **Important Specific Consequence**

The following inference pattern is valid in our system.

a. [Bill [* [Λ1 [[a house] [Λ2 [t_1 own t_2] …]
Possible LF of 'Bill owns a house'

b. [Frank [* [Λ1 [[a house] [Λ2 [t_1 own t_2] …]
Possible LF of 'Frank owns a house'

c. Therefore, [Bill and Frank [* [Λ1 [[a house] [Λ2 [t_1 own t_2] …]
Possible LF of 'Bill and Frank own a house'.

Thus, by adding the (phonologically null) cumulativity operator to our theory of English, we predict that the sentences in (8) all have possible readings (LFs) under which the inference in (8) is *valid*…

2.2 Cumulativity on Non-Subject Position

In the system proposed above, we permit the cumulative operator to take as its sister *any* one-place predicate… *whether that predicate is lexical or derived in the syntax (see LFs above).*

This has the advantage that we predict that 'cumulativity' may hold on *any* argument place of the predicate. To illustrate, consider the inference in (14) below.

(14) **Cumulative Inference on a Prepositional Object**

a. Dave owns a house with Mary.

b. Dave owns a house with Sue.

c. Dave owns a house with Mary and Sue.

<u>Key Observation:</u>

- There is a reading of sentence (14c) under which the inference in (14) is *invalid.*
- But, there also *does* seem to be a reading of (14c) where the inference is *valid!*

(15) **Key Result**

Given the general consequence in (12), we predict that there are possible LFs for the sentences in (14) whereby the inference in (14) is *valid*.

a. [Mary [* [Λ1 [[a house] [Λ2 [Dave owns t_2 with t_1] …]

Possible LF of 'Dave owns a house with Mary.'

b. [Sue [* [Λ1 [[a house] [Λ2 [Dave owns t_2 with t_1] …]

Possible LF of 'Dave owns a house with Sue.'

c. Therefore, [Mary and Sue [* [Λ1 [[a house] [Λ2 [Dave owns t_2 with t_1] …]

Possible LF of 'Dave owns a house with Mary and Sue.'

<u>One Final Note</u>

- In the system developed above, the reading of (8c) where the inference is *valid* is one in which the subject is sister to a predicate 'marked' by the cumulative operator * (which has the indefinite 'a house' in its scope).

- The reading of (8c) where the inference is *invalid* could just be one where no such cumulative operator exists in the LF syntax.

- BUT WHAT ABOUT VPs LIKE 'SLEEP', FOR WHICH INFERENCES OF THE TYPE IN (8) ALWAYS SEEMS VALID (*cf.* (7))?

- **IDEA:** For these predicates, *cumulativity* is simply part of their lexical meaning. **To formally implement this, we might appeal to a 'meaning postulate' of the following form:**

  "If x, y ∈ [[sleep]], then x+y ∈ [[sleep]]"

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3. Distributivity

We've seen that the inference in (16) is valid, and that the basic system from 'Part 1' captures its validity.

(16) **Distributive Inference (on NPs)**

a. Bill and Frank **are boys**.
b. Bill **is a boy** and Frank **is a boy**.

For many VPs, this same form of inference also seems to be valid.

(17) **Distributive Inference (on VPs)**

a. Bill and Frank **are sleeping.**
b. Bill **is sleeping**, and Frank **is sleeping**.

<u>Issue:</u>
The inference in (17), where the main predicate of the sentence is a *VP* (rather than an NP) is not valid in our current semantics.

However, this is arguably the right result, since there are other VPs for which the inference in (17) is much more questionable.

(18) **Distributive Inference (on VPs)**

a. Bill and Frank **lifted a piano.**
b. Bill **lifted a piano**, and Frank **lifted a piano.**

<u>Key Observation:</u>

- There is a reading of sentence (18a) under which the inference in (18) is *invalid.*
- But, there also *does* seem to be a reading of (18a) where the inference is *valid!*

(19) **Interim Goal**

Let us try to capture this apparent ambiguity of sentence (18a).

- Our system should generate T-conditions for (18a) where the inference in (18) is valid.
- (It already does generate a set of T-conditions where the inference in (18) is *invalid*.)

(20) **Immediate Question**

Given the obvious similarity of this problem to the one we faced in Section 2, *could our solution to the problem in Section 2 solve this problem as well?*

That is, could the cumulative operator '*' be what's behind the 'valid' reading of (18)?

(21) **Immediate Answer: NO!**

Even if the LF in (21a) below were true, it wouldn't follow that the LF in (21b) is true.

a. [Bill and Frank [* [Λ1 [[a piano] [Λ2 [t_1 lifted t_2] …]

b. [Bill [* [Λ1 [[a piano] [Λ2 [t_1 lifted t_2] …]

The reason is that '*' only guarantees that ***if*** **x and y are in the extension of [* XP], then x+y is too**. *It doesn't necessarily guarantee the inverse…*

<u>Illustration:</u>

(i) [[[Λ1 [[a piano] [Λ2 [t_1 lifted t_2] …]]] = { Bill+Frank, Dave }

(ii) [[[* [Λ1 [[a piano] [Λ2 [t_1 lifted t_2] …]]] = { Bill+Frank, Dave, Bill+Frank+Dave }

3.1 The Distributivity Operator ‘ᴰ’

Let’s begin by noticing that the inference in (18) is unquestionably valid when the element *each* appears following the subject.

(22) **Unquestionably Valid Distributive Inference**

a. Bill and Frank <u>each</u> **lifted a piano.**
b. Bill **lifted a piano**, and Frank **lifted a piano.**

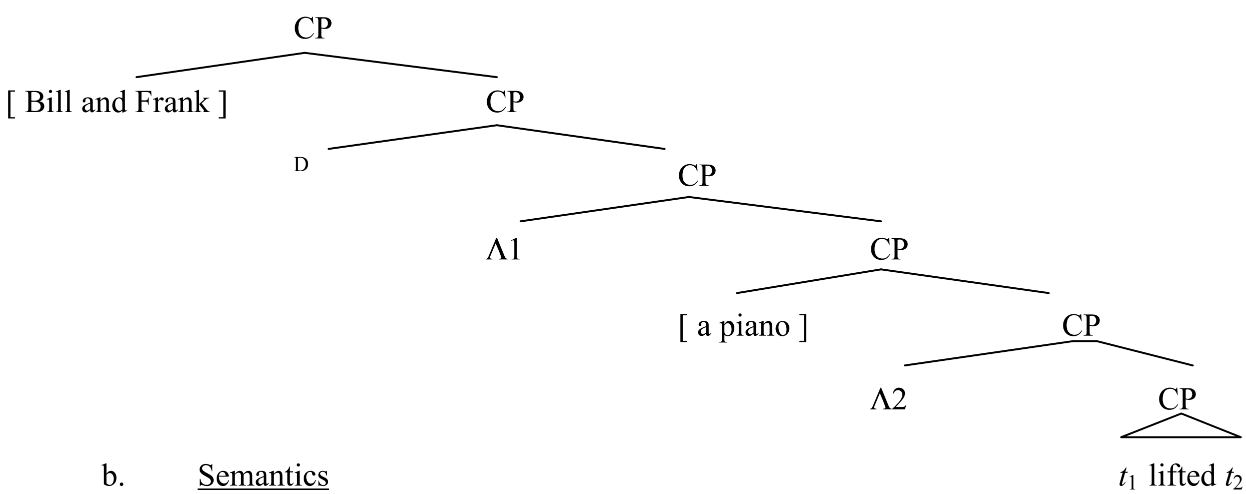
<u>The Big Leap:</u>

- Let’s suppose that, under its ‘validating reading’, the sentence in (18a) actually contains an unpronounced (phonologically empty) version of *each*
- That is, let’s assume that the lexicon of English also includes a (phonologically empty) operator ‘ᴰ’, that has the following characteristic syntax and semantics.

(23) **The Distributive Operator**

a. <u>Syntax</u>
The distributive operator ‘ᴰ’ can take as sister any type <et> phrase.

Example LF



b. <u>Semantics</u>

$[[\ ^{D}\ ]] = \lambda P_{<et>}.\ \lambda x\ .\ \forall y.\ (y \leq x\ \&\ AT(y)) \rightarrow P(y)$

Function which takes an <et> predicate P, and then an entity x, and returns T ***iff***
<u>Every atom y in x is such that P(y).</u>

(24) **Illustration**

a. [[[Bill and Frank [^{D} [Λ1 [[a piano] [Λ2 [t_1 lifted t_2] …]]] = T *iff*

b. ∀y. (y ≤ **Bill+Frank** & AT(y)) → ∃z. z is a piano & **y** lifted z *iff*

c. ∃z. z is a piano & **Bill** lifted z and ∃z. z is a piano & **Frank** lifted z *iff*

d. [[[Bill [^{D} [Λ1 [[a piano] [Λ2 [t_1 lifted t_2] …]]] = T *and*

[[[Frank [^{D} [Λ1 [[a piano] [Λ2 [t_1 lifted t_2] …]]] = T

(25) **Important General Consequence**

Given the semantics of the object-language 'distributive operator' (^{D}), as stated in (23b), the following inference pattern is valid. (Proof is clear from reasoning in (24))

a. <u>Valid Inference in Proposed System</u>

(i) [[[[DP1 and DP2] [^{D} XP]]]] = T

(ii) *Therefore,*

[[[DP1 [^{D} XP]]]] = T **<u>and</u>** [[[DP2 [^{D} XP]]]] = T

(26) **Important Specific Consequence**

The following inference pattern is valid in our system (as already made clear by (24))

a. [Bill and Frank [^{D} [Λ1 [[a piano] [Λ2 [t_1 lifted t_2] …]

Possible LF of 'Bill and Frank lifted a piano'.

b. Therefore, [[Bill [^{D} [Λ1 [[a piano] [Λ2 [t_1 lifted t_2] …] **and**
[Frank [^{D} [Λ1 [[a piano] [Λ2 [t_1 lifted t_2] …]]

Possible LF of 'Bill lifted a piano and Frank lifted a piano'

Thus, by adding the (phonologically null) distributivity operator to our theory of English, we predict that the sentences in (18) all have possible readings (LFs) under which the inference in (18) is *valid*…

3.2 Distributivity on Non-Subject Position

In the system proposed above, we permit the distributive operator to take as its sister *any* one-place predicate… *whether that predicate is lexical or derived in the syntax (see LFs above).*

This has the advantage that we predict that 'distributivity' may hold on *any* argument place of the predicate. To illustrate, consider the inference in (27) below.

(27) **Distributive Inference on a Prepositional Object**

a. Dave owns a house with Mary and Sue.
b. Therefore, Dave owns a house with Mary, and Dave owns a house with Sue.

<u>Key Observation:</u>

- There is a reading of sentence (27a) under which the inference in (27) is *invalid.*
- But, there also *does* seem to be a reading of (27a) where the inference is *valid!*

(28) **Key Result**

Given the general consequence in (25), we predict that there are possible LFs for the sentences in (27) whereby the inference in (27) is *valid*.

a. [Mary and Sue [^{D} [Λ1 [[a house] [Λ2 [Dave owns t_2 with t_1] …]
Possible LF of 'Dave owns a house with Mary and Sue.'

b. [[Mary [^{D} [Λ1 [[a house] [Λ2 [Dave owns t_2 with t_1] …] **and**
[Sue [^{D} [Λ1 [[a house] [Λ2 [Dave owns t_2 with t_1] …]]
Possible LF of 'Dave owns a house with Mary and Dave owns a house with Sue'

<u>An Additional Note</u>

- In the system developed above, the reading of (18a) where the inference is *valid* is one in which the subject is sister to a predicate 'marked' by the distributive operator '^{D}'
- The reading of (18a) where the inference is *invalid* could just be one where no such distritutive operator exists in the LF syntax.
- BUT WHAT ABOUT VPs LIKE 'SLEEP', FOR WHICH INFERENCES OF THE TYPE IN (18) ALWAYS SEEMS VALID (*cf.* (17))?
- **IDEA:** For these predicates, *distribtivity* is simply part of their lexical meaning. **To formally implement this, we might appeal to a 'meaning postulate' of the following form:**

  "If x+y ∈ [[sleep]], then x, y ∈ [[sleep]]"

3.3 A Point of Controversy

According to the semantics proposed in (23d), a 'distributively-marked' predicate [^{D} XP] universally quantifies over the *atoms* constituting its plural argument.

However, as has long been noted, this isn't completely accurate. Consider a sentence like (29a). *It seems to have a distributive reading that licenses the inference to (29b).*

(29) **Distribution over Pluralities**

a. The boys and the girls (each) met.

b. The boys met and the girls met.

<u>The Crucial Point:</u>

- As noted in 'Part 1', however, the predicate *met* in English cannot hold of singular individuals, but instead holds only of plural entities.

- Therefore, the distributive reading of (29) cannot be the following:

  $\forall$y. (y $\leq$ [[the boys and the girls]] & **<u>AT(y)</u>**) → y met.
  Every atomic member of 'the boys and the girls' met.

…So, what is the right fix?...

(30) **One Possibility (Schwarzschild 1996)**

- The context makes salient a particular way of 'dividing up' plural entities (a 'cover').

- The distributive operator quantifies over the divisions of the plurality provided by this contextually salient 'cover'.

…but there are some potential problems with this proposal as well..

For more information, see the works cited by Lasersohn (2008)…

4. Cumulativity, A Second Pass

Thus far, we've noticed that there are readings of the sentences in (31c) and (32a) under which the inferences in (31) and (32) are valid.

(31) **Cumulative Inference**

- a. Bill owns a house.
- b. Frank owns a house.
- c. Bill and Frank **own a house.**

(32) **Distributive Inference**

- a. Bill and Frank **lifted a piano.**
- b. Bill **lifted a piano**, and Frank **lifted a piano.**

- To capture the reading under which (31) is valid, we introduced the operator '*' in (10).
- To capture the reading under which (32) is valid, we introduced the operator '^{D}' in (23).

QUESTION: Given that we need the 'distributive operator' to analyze (32), *can we not use it to analyze (31) also?*

Notice that – following the proof in (24) – the following is a valid inference in our semantic system as well!

(33) **Valid Inference in Proposed System**

- a. [DP1 [^{D} XP]]
- b. [DP2 [^{D} XP]]
- c. [[DP1 and DP2] [^{D} XP]]

Thus, once we've introduced the distributive operator to capture the inference in (32), ***is there any longer any motivation for the 'cumulative operator' in our theory?***

ANSWER: Yes, there is a role to be played by the cumulative operator beyond simply validating the inference in (31)…

4.1 Cumulativity in Transitive Sentences

Suppose that there are three police officers: Officer1, Officer2, Officer3
Suppose that there are three targets: Target1, Target2, Target3

(34) **Observation 1**
There is a reading of sentence (34d) under which the inference below if valid.

a. Officer1 shot Target1
b. Officer2 shot Target2
c. Officer3 shot Target3
d. **The officers shot the targets**

(35) **Equivalent Observation**
Sentence (35a) is true in the situation described by the picture in (35b).

a. **The officers shot the targets.**

b.

Officers	(shooting)	*Targets*
Officer1	⟶	Target1
Officer2	⟶	Target2
Officer3	⟶	Target3

> *How do we capture the observations in (34) and (35)??*

(36) **The 'Distributivity Operator' is <u>NOT</u> the Answer**

No matter where we put the distributivity operator in the sentence, the reading we derive is *not* one that (i) validates the inference in (34) or (ii) is true in situation (35b).

a. *LF*: [The officers [^{D} [Λ1 [t_1 shot the targets] …]

T-Conditions: ∀y. (y ≤ [[the officers]] & AT(y)) → y shot the targets
Each of the officers shot the targets (together as a group)

b. *LF*: [The targets [^{D} [Λ1 [the officers shot t_1] …]

T-Conditions: ∀y. (y ≤ [[the targets]] & AT(y)) → the officers shot y
Each of the targets was shot by the officers (together as a group).

c. *LF:* [The officers [^{D} [Λ1 [the targets [^{D} [Λ2 [t_1 shot t_2] …]

T-Conditions: ∀y. (y ≤ [[the officers]] & AT(y)) →
∀z. (z ≤ [[the targets]] & AT(z) y shot z
Each of the officers shot each of the targets (individually).

<u>IDEA:</u> We can augment our theory of the cumulativity operator '*' so that it derives exactly the observations in (34) and (35).

…actually, what we will do is make an augmentation to our assumptions regarding the 'plurality-forming' operator "+".

(37) **Generalized Pluralization**

At the moment "+" is characterized as a function that takes *entities* as arguments.

Using this basic defintion for "+" (the joining of type 'e' entities), we can now extend our definition of "+" so that it can take arbitrarily long *tuples* as arguments.

a. <u>Definition of "+" for Tuples</u>

$<a_1, \dots a_n> + <b_1, \dots b_n> = <a_1 + b_1, \dots a_n + b_n>$

Consequently, we can now allow the cumulativity operator '*' to take as sister *any predicate of arbitrarity 'arity'* (using exactly the same semantics as in (10b))

b. <u>Illustration of '*' Taking Binary Predicate as Argument</u>

(i) [[shot]] = { <Officer1, Target1>, <Officer2, Target2>, <Officer3, Target3> }

(ii) [[* [shot]]] = * [[shot]] =

{ <Officer1, Target1>,
<Officer2, Target2>,
<Officer3, Target3>,
<Officer1+Officer2, Target1+Target2>
<Officer1+Officer3, Target1+Target3>
<Officer2+Officer3, Target2+Target3>
<Officer1+Officer2+Officer3, Target1+Target2+Target3> }

(38) **Key Consequence**

In situations where the individual 'shootings' are as represented in (37bi), it follows that the following LF will be true:

a. <u>True LF in Situation (37bi):</u> [[the officers] [[*shot] [the targets]]

Since this is a possible LF for sentences (34d)/(35a), we predict the observations in (34) and (35)!

4.2 Some Key Open Questions in the Literature

(39) **The Universal Cumulativity of Verbs**

- In the analysis above, we insert the 'cumulative operator' in the syntax.
- *But, could the 'cumulativity' of "shoot" be a property of the verb itself?*
- ***Could all Vs be inherently, universally cumulative on all their argument places?***

a. Potential Counter Evidence

There is a reading of (34d) where the inference in (34) is *invalid.*
Under this reading, the predicate "shoot" must *not* be cumulative…*right?*

b. Potential Alternative Account

Perhaps under its 'invalidating' reading, sentence (34d) has a covert "together".

(i) The officers (together) shot the targets (together).

Under this analysis, the verb "shoot" might still be inherently cumulative.
(*for an explicit semantics of 'together', see* Lasersohn 1990)

(40) **The Constraints on the Insertion of the Cumulative Operator**

- We know from earlier examples (*e.g.* (14)) that it should be possible for the cumulative operator '*' to be inserted in the syntax, on predicates derived by movement.
- *Should we allow this insertion to be completely free? Or is there evidence that the insertion is constrained in any way?*

For In-Depth Discussion of These Questions:

Kratzer, A. (2005) "On the Plurality of Verbs." In Doelling, J. & T. Heyde-Zybatow (eds) *Event Structures in Linguistic Form and Interpretation*. Mouton de Gruyter: Berlin

… and references therein…

5. Quantification

Thus far in our discussion, the only plural DPs that we've been considering are plural *definites.*

However, it's patently obvious that plural NPs can combine with all kinds of Ds besides "the".

(41) **Some Other Plural DPs**

a. Some dogs
b. Five dogs
c. Exactly five dogs
d. Many dogs
e. Most dogs
f. Few dogs
g. No dogs
(h. All dogs)

<u>Outstanding Question:</u>

How do we analyze each of these Ds so that they can combine with the meaning of their plural NP complements, and yield the correct truth-conditions for the sentences containing them?

5.1 The Semantics of Numerals with Plural NPs

Recall the following, classic 'GQ' semantics for numerals like *five*.

(42) **Semantics of Numerals in Classic Generalized Quantifier Theory**

[[five]] = $\lambda P_{<et>} . \lambda Q_{<et>} . | P \cap Q | \geq 5$
The function that takes a set P and a set Q, and returns T iff the cardinality of their intersection is at least five.

In a system where we 'ignore' the plurality of the NP, and treat all NPs as sets of atomic individuals, this semantics delivers the correct truth-conditions.

(43) **Truth-Conditions Predicted by (42), Ignoring Plurality**

[[Five boys smoke]] = T *iff* $| \{x: x \text{ is a boy}\} \cap \{x: x \text{ smokes}\} | \geq 5$
The set of boys that smoke is at least five.

<u>Issue:</u>

In a system where the plural number of the NP complement of "five" is interpreted, in the way that we do in the system developed here, *the semantics in (42) yields the wrong truth-conditions!*

(44) **Truth-Conditions Predicted by (42), *Interpreting* Plurality**

[[Five boys smoke]] = T *iff* | *{x: x is a boy} ∩ {x: x smokes} | ≥ 5
The set of ***groups*** *of boys that smoke is at least five.*
(*i.e.* Five **groups** of boys smoke.)

Besides just obviously *seeming* to be the wrong T-conditions (since they count *groups* of boys, rather than individual boys), we can actually show that the T-conditions in (44) make incorrect predictions.

(45) **The Inadequacy of the Truth-Conditions in (44)**

a. Suppose that Frank, Bill and Dave smoke.

b. Intuitively, then, all the following are groups of boys that smoke:

(i) Frank+Bill (*cf.* Frank and Bill smoke)
(ii) Frank+Dave (*cf.* Frank and Dave smoke)
(iii) Bill+Dave (*cf.* Bill and Dave smoke)
(iv) Frank+Bill+Dave (*cf.* Frank and Bill and Dave smoke)

c. Moreover recall that the extension of "boys" in our system will be the following:
{Frank, Bill, Dave, Frank+Bill, Frank+Dave, Bill+Dave, Frank+Bill+Dave}

d. Consequently, the following set will be equal to *{x: x is a boy} ∩ {x: x smokes}
S = {Frank, Bill, Dave, Frank+Bill, Frank+Dave, Bill+Dave, Frank+Bill+Dave}

e. Clearly | S | ≥ 5

f. Thus, the truth-conditions in (44) hold, and so the lexical entry in (42) wrongly predicts that "five boys smoke" should be true in a situation where (only) Frank, Bill and Dave smoke.

So… how do we fix the lexical entry in (42)?...

Let's start off by trying to characterize what we feel might be a more accurate set of T-conditions for "Five boys smoke".

(46) **More Accurate T-Conditions, First Pass**

[[Five boys smoke]] = T *iff* Some *group* of five boys smoke.
(*cf.* 'Five groups of boys smoke' in (44))

If we accept this informal statement of the T-conditions, we write it more formally as follows:

(47) **More Accurate T-Conditions, Formalized Statement**

[[Five boys smoke]] = T *iff*

$\exists$x. x $\in$ *{y : y is a boy} & | {z : z $\leq$ x & AT(z)} | $\geq$ 5 & x smokes.

There is some group of boys x, whose number of atoms is at least five, and x smokes

<u>Side-Note</u>

The truth-conditions in (47) would correctly predict that "five boys smoke" will be false in the scenario sketched in (45)… as there isn't any group of *five boys* in the extension of 'smokes'.

- The last step is to craft a lexical entry for "five" that will deliver the T-conditions in (47).
- The following will do the trick:

(48) **Semantics of *Five* in Our System for Plurals**

[[five]] = $\lambda P_{<et>} . \lambda Q_{<et>} . \exists x. P(x) = T \ \& \ | \{z : z \leq x \ \& \ AT(z)\} | \geq 5 \ \& \ Q(x) = T$

We can, of course, generalize this approach to all numerals:

(49) **Semantics of Numerals in Our System for Plurals**

a. [[one]] = $\lambda P_{<et>} . \lambda Q_{<et>} . \exists x. P(x) = T \ \& \ | \{z : z \leq x \ \& \ AT(z)\} | \geq \mathbf{1} \ \& \ Q(x) = T$

b. [[two]] = $\lambda P_{<et>} . \lambda Q_{<et>} . \exists x. P(x) = T \ \& \ | \{z : z \leq x \ \& \ AT(z)\} | \geq \mathbf{2} \ \& \ Q(x) = T$

c. [[three]] = $\lambda P_{<et>} . \lambda Q_{<et>} . \exists x. P(x) = T \ \& \ | \{z : z \leq x \ \& \ AT(z)\} | \geq \mathbf{3} \ \& \ Q(x) = T$

…

It's long been noted that this approach to the meaning of numerals makes an important (true) prediction, regarding the meaning of sentences that contain <u>multiple</u> numerically quantified DPs

(50) **Key Observation 1: 'Cumulative Readings' of Sentences with Multiple Numerals**

Sentence (50a) has a reading which is *true* in scenario (50b).

a. Three officers shot three targets.

b. <u>*Officers* (shooting) *Targets*</u>

Officer1 ⟶ Target1

Officer2 ⟶ Target2

Officer3 ⟶ Target3

(51) **Key Observation 2: 'Cumulative Readings' Can't be Derived in Classic GQ Theory**

If we assume a 'classic GQ' semantics for the numeral *three*, as in (51a) below, then we won't be able to predict any true reading of (50a) in situation (50a).

a. <u>Classic GQ Semantics of 'Three'</u>

[[three]] = $\lambda P_{<et>} . \lambda Q_{<et>} . | P \cap Q | \geq 3$

b. <u>Predicted Readings of Sentence (50a)</u>

(i) *Subject Scopes Over Object*

$| \{x: \text{x is an officer}\} \cap$
$\mathbf{\{ x : | \{ y: y \text{ is a target} \} \cap \{ z: x \text{ shot } z \} | \geq 3 \}} | \geq 3$

There are three officers such that they each shot three targets

(ii) *Object Scopes Over Subject*

$| \{x: \text{x is a target}\} \cap$
$\mathbf{\{ x : | \{ y: y \text{ is an officer} \} \cap \{ z: z \text{ shot } x \} | \geq 3 \}} | \geq 3$

There are three targets such that they each were shot by three officers.

(52) **Key Observation 3: 'Cumulative Readings' *Can* be Derived in Our System**

If we assume that sentence (50a) receives the LF in (52a), below, then the truth-conditions that our lexical entry in (49c) predicts are those in (52b).

a. [[three officers] [Λ1 [[three targets] [Λ2 [t_1 *shot t_2] …]

b. $\exists$x. x $\in$ *{y : y is an officer} & | {z : z $\leq$ x & AT(z)} | $\geq$ 3 &
$\exists$u. u $\in$ *{y : y is a target} & | {z : z $\leq$ u & AT(z)} | $\geq$ 3 & [[*shot]](u)(x)

There is some group of three officers x,
And some group of three targets u, such that x 'cumulatively shot' u.

Now, recall (from Section 4.1) that, in the scenario sketched in (50b), the extension of "[*shot]" is the set in (52c) below.

c. [[* [shot]]] = * [[shot]] =

{ <Officer1, Target1>,
<Officer2, Target2>,
<Officer3, Target3>,
<Officer1+Officer2, Target1+Target2>
<Officer1+Officer3, Target1+Target3>
<Officer2+Officer3, Target2+Target3>
<Officer1+Officer2+Officer3, Target1+Target2+Target3> }

Given that the pair '**<Officer1+Officer2+Officer3, Target1+Target2+Target3>**' is in the extension of "[*shot]" in scenario (50b), we find that the T-conditions in (52b) *do hold true* in that scenario.

Thus, our modified lexical entries in (49) – combined with our earlier theory of the cumulative operator '*' – uniquely predict the possibility of the such 'cumulative readings' of sentences like (50a)!!

(53) **Important Follow-Up Question**

The readings in (51b) – which are predicted by the classic 'GQ' theory of the meaning of numerals – *are* indeed possible readings of (50a).

So how do we capture the possibility of those readings in our system for plurals?...
… Amgibuity in the meaning of the ***numeral****?*

(54) **Capturing the Readings in (51)**

- We needn't appeal to the lexical entry in (51a) to capture the T-conditions in (51b).

- *Rather, we can simply appeal to the 'distributive operator' that our theory of plurals already independently needs!*

a. LFs Generated by Our Syntactic Theory of Plurals

(i) [[three officers] [$^{\mathbf{D}}$ [Λ1 [[three targets] [$^{\mathbf{D}}$ [Λ2 [t_1 shot t_2] …]

(ii) [[three targets] [$^{\mathbf{D}}$ [Λ2 [[three officers] [$^{\mathbf{D}}$ [Λ1 [t_1 shot t_2] …]

b. T-Conditions of the LFs in (54a)

(i) *T-Conditions of (54ai)*

∃x. x ∈ *{y : y is an officer} & | {z : z ≤ x & AT(z)} | ≥ 3 &
∀z. (z ≤ x & AT(z)) → ∃u. u ∈ *{y : y is a target} & | {s : s ≤ u & AT(s)} | ≥ 3 &
∀s. (s ≤ x & AT(s)) → z shot s

There is some group of three officers such that for each of the three officers z,
There is some group of three targets such that for each of the three targets s
z shot s

(*There are three officers such that they each shot three targets*)

(ii) *T-Conditions of (54aii)*

∃x. x ∈ *{y : y is a target} & | {z : z ≤ x & AT(z)} | ≥ 3 &
∀z. (z ≤ x & AT(z)) → ∃u. u ∈ *{y : y is a officer} & | {s : s ≤ u & AT(s)} | ≥ 3 &
∀s. (s ≤ x & AT(s)) → s shot z

There is some group of three targets such that for each of the three targets z,
There is some group of three officers such that for each of the three officers s
s shot z

(*There are three targets such that they were each shot by three officers*)

General Perspective on the 'Scopal Interaction' of (Upward Monotone) Quantifiers

- In a logical form where the only operators are existential quantifiers, permuting the scope of the existential quantifiers produces logically equivalent expressions.

  Illustration:
  ∃x. x is a man & ∃y. y is a dog & x walks y. **Logically Equivalent to:**
  ∃y. y is a dog & ∃x. x is a man & x walks y.

- Therefore, if we view numerals (and other upward monotone quantifiers) as simply **existential quantifiers** over pluralities (of a particular type), then we predict that *on their own*, they shouldn't scopally interact with each other…

  … that is, in the absence of any other operators, permuting the scope of these quantifiers won't result in any change of meaning.

- However, it's long been noted that there do seem to be ambiguities in sentences containing multiple instances of these quantifiers (*cf.* (50a), (51b)))… *ambiguities that seem to suggest that these quantifiers* ***do*** *scopally interact with each other.*

- In the account developed here, however, these ambiguities are not due to scopal interaction between the quantifiers themselves, *but rather between the quantifiers and the* ***distributivity operator '^{D}'***

- That is, insertion of '^{D}' introduces an additional operator into the logical form of the sentence, with the result that permutation of the scope of the existential quantifiers now might lead to logically distinct T-conditions.

5.2 Extending This Treatment to Other Quantifiers

In the previous section, we've seen how we might develop a semantics for plural DPs headed by numerals.

In this approach, numerals are treated as existential quantifiers over pluralities of a particular type (those whose number of atoms is at least as great as the amount specified by the numeral)

In this section, we'll see that this same general perspective can be extended to <u>some</u> other determiners, but not <u>all</u>.

The following quantifiers can also all be analyzed as existentials over a restricted set of pluralities. ***Note that they all share the property of being upward monotone (on their right argument).***

(55) **The Indefinite *Some***

Thus far, we've been treating the indefinite Ds, *some* and *a* in the following way:

a. [[some / a]] = $\lambda P_{<et>} . \lambda Q_{<et>} . |\ P \cap Q\ | \neq \varnothing$
$\lambda P_{<et>} . \lambda Q_{<et>} . \exists x. P(x) = 1 \ \&\ Q(x) = 1$

This lexical entry clearly captures the meaning of singular indefinites like (55b). **It also performs perfectly for plural indefinites like (55c).**

b. <u>Some dog</u> is barking.
c. <u>Some dogs</u> are barking.

The T-conditions that (55a) predicts for sentences like (55c) is as follows:

d. $|\ {}^*\{x\text{: x is a dog}\} \cap \{y\text{: y is barking}\}\ | \neq \varnothing$
$\exists$x. **x ∈ *{y : y is a dog}** & x is barking.

Some group of dogs is barking.

(56) **The Quantifier *No***

Under a classic GQ treatment, the D *no* receives an analysis under the following lines:

a. [[no]] = $\lambda P_{<et>} . \lambda Q_{<et>} . |\ P \cap Q\ | = \varnothing$
$\lambda P_{<et>} . \lambda Q_{<et>} . \neg\exists x. P(x) = 1 \ \&\ Q(x) = 1$

This lexical entry clearly captures the meaning of sentences like (56b). **It also performs perfectly for sentences like (56c).**

b. <u>No dog</u> is barking.
c. <u>No dogs</u> are barking.

The T-conditions that (56a) predicts for sentences like (56c) is as follows:

d. $|\ {}^*\{x\text{: x is a dog}\} \cap \{y\text{: y is barking}\}\ | = \varnothing$
¬∃x. **x ∈ *{y : y is a dog}** & x is barking.

No group or individual dog is barking.

(56) **The Quantifier *Many***

Under a classic GQ treatment, the D *many* receives an analysis along the following lines:

a. [[many]] = $\lambda P_{<et>} . \lambda Q_{<et>} . | P \cap Q | \geq n$
The intersection of [[NP]] *and* [[VP]] *is larger than some contextually salient amount 'n'.*

However, for reasons similar to what we saw above for numerals, this semantics will fail to deliver the right result in a system where the plurality of NPs is interpreted.

We can, though, easily alter our lexical entry for "many" along the lines that we did for numerals in (49).

b. [[many]] = $\lambda P . \lambda Q . \exists x. P(x) = T \ \& \ | \{z : z \leq x \ \& \ AT(z)\} | \geq \mathbf{n} \ \& \ Q(x) = T$

There is some 'large' group from *[[NP]] which is [[VP]].

(57) **The Quantifier *Most***

Under a classic GP treatment, the D *most* receives the following analysis:

a. [[most]] = $\lambda P_{<et>} . \lambda Q_{<et>} . | P \cap Q | \geq | P |/2$
The intersection of [[NP]] *and* [[VP]] *is greater than half the cardinality of* [[NP]].

For familiar reasons, this lexical entry won't do the right job in our semantics for plurals. We could, though, augment it to the following:

b. [[most]] = $\lambda P . \lambda Q . \exists x. P(x) = T \ \&$
$| \{z : z \leq x \ \& \ AT(z)\} | \geq | \{ y : y \leq MAX(P) \ \& \ AT(y) \} | / 2$
$\& \ Q(x) = T$

There is some group x from *[[NP]] *whose set of atoms is more than half the size of atoms in the <u>maximal</u> group from* *[[NP]]*, and x is* [[VP]]

(There is some group of over half the NPs that VP)

However, while these Ds could all be interpreted as existential quantifiers (over pluralities), *not all Ds admit of such an analysis…*

Note that that all the following quantifiers share the property of NOT being upward monotone (on their right argument)

(58) **Exactly *N***

Suppose we were to analyze the quantifier 'exactly 3' in the following fashion:

a. [[exactly three]] = $\lambda P.\ \lambda Q\ .\ \exists x.\ P(x)\ \&\ |\ \{z : z \leq x\ \&\ AT(z)\}\ | = \mathbf{3}\ \&\ Q(x)$
Some group of exactly three [[NP]] *are* [[QP]]

The T-conditions of the sentence 'Exactly 3 boys smoked' would be the following:

b. $\exists x.\ x \in {}^*\{y : y \text{ is a boy}\}\ \&\ |\ \{z : z \leq x\ \&\ AT(z)\}\ | = \mathbf{3}\ \&\ x$ smoked.
Some group of exactly three boys smoked.

However, this set of T-conditions would hold in a situation where *five* boys smoked – Bill, Dave, Frank, Tom and Lou – since there *is* a group of exactly three boys (Bill, Dave and Frank) who smoke.

(59) **Less than *N***

Suppose we were to analyze the quantifier 'less than 3' in the following fashion:

a. [[less than three]] = $\lambda P.\ \lambda Q\ .\ \exists x.\ P(x)\ \&\ |\ \{z : z \leq x\ \&\ AT(z)\}\ | \leq \mathbf{3}\ \&\ Q(x)$
Some group of less than three [[NP]] *are* [[QP]]

Then the T-conditions of the sentence 'Less than 3 boys smoked' would be the following:

b. $\exists x.\ x \in {}^*\{y : y \text{ is a boy}\}\ \&\ |\ \{z : z \leq x\ \&\ AT(z)\}\ | \leq \mathbf{3}\ \&\ x$ smoked.
Some group of less than three boys smoked.

However, this set of T-conditions would hold in a situation where *five* boys smoked – Bill, Dave, Frank, Tom and Lou – since there *is* a group of less than three boys (Dave and Frank) who smoke.

(60) **Few**

Suppose we were to analyze the quantifier 'few' in the following fashion:

a. [[few]] = $\lambda P.\ \lambda Q\ .\ \exists x.\ P(x)\ \&\ |\ \{z : z \leq x\ \&\ AT(z)\}\ | \leq \mathbf{n}\ \&\ Q(x)$
*Some group of fewer than **n** P (where **n** is a contextually given 'small' amount) are Q.*

Following the reasoning laid out above, these T-conditions would wrongly predict that a sentence like 'Few boys smoked' could be true in a situation where *many* boys smoked. (Since there will always be some small (sub-)group of boys who smoked.)

Issue: For quantifiers that are *not upward monotone* (on their right argument), an analysis where they are existential quantifiers over pluralities seems not to work...

… so, it seems that some *other* approach to their meaning must be adopted…

For more info on the meaning of plural DPs headed by 'exactly N', 'less than N' and 'few', see the following works:

Krifka, Manfred (1999) "At Least Some Determiners aren't Determiners." In Turner, K. (ed) *The Semantics/Pragmatics Interface from Different Points of View*. Elsevier: Amsterdam.

Winter, Yoad (2000) "Distributivity and Dependency" *Natural Language Semantics* 6: 303-337

Hackl, Martin (2000) *Comparative Quantifiers*. PhD Dissertation. MIT

Beck, Sigrid and Uli Sauerland (2001) "Cumulation is Needed: A Reply to Winter (2000)" *Natural Language Semantics 8:* 349-371.

5.3 One Quick Final Note on *All*

In the sections above, we discussed all the types of plural DPs under (41), *except for (41h)*:

(61) **Plural DPs with *All***

a. All dogs bark.
b. All the dogs are barking.

Issue: Despite the fact that natural language *all* is the expression logicians often use to 'translate' the logical operator '$\forall x$', its semantics extend far beyond that operator, and are a very difficult subject.

(62) **Fact 1: *All* Does Not Mean the Same Thing as *Every***

The DPs headed by *every*, which are *singular* in number, are inherently distributive. Thus, they cannot be the subject of 'plurality-seeking' predicate like *meet*:

a. * Every student met in the yard.

However, DPs modified by *all*, which are *plural* in number, can be collective.

b. All the students met in the yard.

(63) **The Core Question Regarding *All***

The sentences in (61) are *almost* identical in meaning the sentences below.

a. Dogs bark. (*cf.* All dogs bark.)
b. The dogs are barking. (*cf.* All the dogs are barking.)

There are some subtle differences in meaning, however…
…it is those differences in meaning that point the way to the proper treatment of 'all'

(64) **The Relevance of Exceptions**

One of the key ways in which the sentences in (63) differ from those in (61) containing *all* concerns the impact that 'a few exceptions' make on the truth of the sentence.

a. Exceptions and Bare Plurals
It's intuitively true that not all breeds of dogs have tails. There's *a few* that don't. Nevertheless, sentence (i) below seems true.

(i) Dogs have tails.

Interestingly, sentence (ii) seems false (given these circumstances)

(ii) All dogs have tails.

b. Exceptions and Definite Plurals
Suppose there was a faculty meeting, and *nearly* every faculty member attended. Sentence (i) seems to be true.

(i) The faculty met yesterday.

Interestingly, though, sentence (ii) seems false (given these circumstances)

(ii) All the faculty met yesterday.

(65) **Informal Generalization: *All* Means 'No Exceptions'**

- Much of the literature on *all* has focused on the contrasts noted in (64).
- As a result, the following picture has come into view on the meaning of *all*:

(i) Sentences containing bare plurals and (plain) definite plurals 'tolerate exceptions'
(ii) The meaning of *all* is essentially that it removes this 'tolerance of exceptions'

For more information, including a fully formalized treatment:
Brisson, Chistine (2003) "Plurals, *All*, and the Non-Uniformity of Collective Predication." *Linguistics and Philosophy* 26: 129-184