

The Semantics of Plurals, Part 1: The Basics

1. Introductory Questions and Guesses

(1) Blindingly Obvious Fact about Natural Language

There is number marking (singular, plural) on NPs, and this number seems to have an effect on the meaning of the sentence.

- a. Some **boy** just ran into the room.
- b. Some **boys** just ran into the room.

Thus far, our semantic system easily handles **singular** NPs like (1a).

Indeed, we've been ignoring number marking, and treating all NPs as if they were singular.

Clearly, though, *this isn't how natural languages actually operate...*

Plural number on NPs is semantically interpreted, in as much as it has an effect on the meaning of the sentence!

(2) Overarching Question

- What is the meaning of plural number marking?
- *What is the contribution of plural number marking to the truth-conditions of a sentence?*

(3) Some Very Basic Data to Get Us Started

The inference from (3a) and (3b) to (3c) seems valid.

- a. Bill is a boy.
- b. Frank is a boy.
- c. Bill and Frank are **boys**.

(4) A Very Basic Intuition to Get Us Started

If we think of “boy” as denoting a set of (individual) boys... perhaps the plural “boys” denotes **the set of all possible ‘groups’ of boys**.

Then the inference in (3) would go through!

If Bill is a boy and Frank is a boy, then Bill and Frank together are a possible ‘group’ of boys, and so should be in the extension of “boys”.

2. Plurals and Plural Entities

So, to start out, let's try to build enough into our semantics to capture the validity of (3) following the 'intuition' in (4).

(5) Fundamental Question

*What do we mean when we say "boys" denotes **groups** of boys?*

- a. Answer 1: Sets (Schwarzschild 1996)
 - "Boys" denotes all the possible sets of boys.
 - For a given plural NP "NP-pl", $[[\text{NP-pl}]] = P([[\text{NP}]])$ (power set of $[[\text{NP}]]$)
- b. Answer 2: Plural Entities (Link 1983, *et multia alia*)

A 'group of boys' is a kind of *complex entity*, a thing of type 'e' made out of other things of type 'e' (namely, individual boys)...

Given that – for better or worse – 'Answer 2' is the one most commonly employed in the literature on plurals, we'll develop a system that adopts this perspective...

Side-Note:

- As far as I am aware, there are no empirical arguments favoring (5b) over (5a).
- At one time, there were thought to be *purely conceptual / metaphysical* arguments for (5b) and against (5a), but the consensus now is that those arguments are not very compelling.
- Moreover, my own sense is that (5a) – particularly as it is developed by Schwarzschild (1996) – is conceptually simpler in a number of ways
- However, since (5a) is currently limited mostly to the work of Schwarzschild, for didactic reasons it's best to get everyone used to the way of doing things in (5b).

(5) The Background Philosophy / Metaphysics ('Mereology')

- a. Obvious, Common-Sense Observation

Clearly, objects can be made out of other objects.
- b. Huge Metaphysical Leap

For any two objects A and B, there is the abstract 'plural object' constructed from A and B.
- c. Notation 'A+B' = the plural object made from A and B.
- d. Examples Suzi+Clint, Seth+HulkHogan, (Suzi+Clint)+Seth
((Mike+Radek)+Nadine)+(Angelika+Kathy)

Let's start to clarify this idea some, by adding some additional concepts and formalisms to our 'meta-language'

(6) **The Group Forming Operator '+'**

- The (meta-language) operator '+' is a function of type $\langle e, \langle e, e \rangle \rangle$. It takes two entities of type 'e', and yields the (plural) entity of type 'e'.

- The operator '+' is idempotent, commutative and associative. Thus:

$$\begin{aligned} x+x &= x \\ x+y &= y+x \\ x+(y+z) &= (x+y)+z \end{aligned}$$

- Consequently, parentheses are not required in representing pluralities constructed from other pluralities.

$$x+(y+z) = (x+y)+z = \mathbf{x+y+z}$$

Side-Note:

- There is serious debate over whether certain natural language data show that the 'group-forming' operator should actually be *non*-associative (Schwarzchild 1998).
- For purposes of simplicity, we'll assume here that it is.

(7) **The Domain of Individual and Plural Entities**

Let D be our domain of 'individual entities'.

$*D =$ 'The domain of plural entities.'
The closure of D under the 'group forming' operator '+'

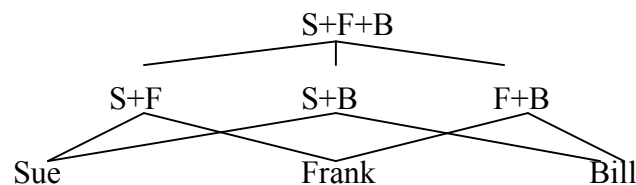
The smallest set such that

- $D \subseteq *D$
- If $x, y \in *D$, then $x+y \in *D$

(8) **A Picture of What the Domains Look Like**

a. The Domain of Individuals, D : Sue, Frank, Bill

b. The Domain of Plural Entities, $*D$:



Finally, before we start using this ontology of ‘plural entities’ to do natural language semantics, we’ll want to have some vocabulary for describing certain key properties and relations that the ‘denizens of *D’ can bear...

(9) **Two Key Concepts: ‘Part-of’ and Atomicity**

a. The Relation ‘Part of’

(i) *Notation*

$x \leq y$ ‘x is part of y’

(ii) *Definition*

$x \leq y$ iff $\exists z. x+z = y$

There is some z such that y is a group consisting of x and z.

Side-Note

- A key consequence of this definition is that **for all x, $x \leq x$**

b. The Property of ‘Atomicity’

(i) *Notation*

$AT(x)$ ‘x is an atom’

(ii) *Definition*

$AT(x)$ iff $\forall y. y \leq x \rightarrow y=x$

The only object that is a ‘part of x’ is x itself.

Side-Note

- Consequently, in the system as we have it here, $AT(x)$ iff $x \in D$

With the metaphysical machinery laid out here, we can now start to provide a semantics for plural NPs that captures the very basic data in (3)...

3. **A Basic Semantics for Plural NPs**

Remember our idea that “boys” denotes all the possible groups of (individual boys). Here’s a way to precisely state that idea in our formal meta-language.

(10) **Interpretation of Plural NPs**

$[[\text{NP pl}]]$ = $*[[\text{NP}]]$ = ‘the closure of $[[\text{NP}]]$ under +’

(11) **Pluralization in a Picture**

Suppose that the boys are Frank, Bill and Dave. Consequently:

$$a. \quad [[\text{boy}]] = \{ \text{Frank, Bill, Dave} \}$$

Moreover, given the rule in (10), it follows that:

$$b. \quad [[\text{boy pl}]] = *[[\text{boy}]] = \{ \text{Frank, Bill, Dave,} \\ \text{Frank+Bill, Frank+Dave, Dave+Bill,} \\ \text{Frank+Dave+Bill} \}$$

To complete the picture, we need a new rule for interpreting conjunctions of phrases of type ‘e’

(12) **Plural Conjunction**

If ‘DP1’ is type e and ‘DP2’ is type e, then [DP1 and DP2] is of type e, and

$$[[\text{DP1 and DP2}]] = [[\text{DP1}]]+[[\text{DP2}]].$$

The group formed from DP1 and DP2

We now have enough for our semantics to predict the inference in (3), repeated below:

(13) **Targeted Data**

The inference from (13a) and (13b) to (13c) seems valid.

- a. Bill is a boy.
- b. Frank is a boy.
- c. Bill and Frank are **boys**.

(14) **Proof of Validity of Inference**

- a. Suppose “Bill is a boy” and “Frank is a boy” are true.
- b. Clearly, our semantics predicts that $\text{Bill, Frank} \in \{ x : x \text{ is a boy} \}$
- c. Consequently, $\text{Bill+Frank} \in * \{ x : x \text{ is a boy} \}$
- d. Thus, $[[\text{Bill and Frank}]] \in *[[\text{NP pl}]]$
- e. Therefore, $[[\text{Bill and Frank are boys}]] = T$

HOORAY!! Plurals are solved!...right??

4. The Semantics of Definite Plurals

So far our system can handle simple sentences where the plural NP is a predicate.
But what about sentences where the plural is *the argument of a verb*?

(15) Definite Plurals as Arguments to Vs

- a. **The boys** are running.
- b. Sue saw **the boys**.

(16) Overarching Questions

- a. What is the meaning of such plural DPs like “the boys”?
- b. How can the meaning of the definite article “the” combine with the meaning of the plural NP “boys” to produce the observed meaning of “the boys”?

(17) Some Very Basic Data to Get Us Started

- a. The inference from (i) to (ii) seems valid.
 - (i) Frank, Bill and Dave are **the boys**.
 - (ii) Every boy is either Frank or Bill or Dave.
- b. The singular definite in (i) is undefined if there is *more than one boy*, while the plural definite in (ii) obviously isn't.
 - (i) The boy is nice. (Undefined if more than one boy)
 - (ii) The boys are nice. (Completely fine if there's more than one boy)

(18) A Very Basic Intuition to Get Us Started

- a. Like “Bill and Frank”, the plural DP “the boys” refers to a particular *plural* entity
- b. The plural entity that “the boys” refers to is *the group consisting of all the boys*

(19) Reintroduction of MAX

Remember ‘MAX’ from our theory of modals and conditionals?

Let S be a set of entities from *D.

MAX(S) = the unique x in S such that $\forall y. y \in S \rightarrow y \leq x$
That element in S which all other things in S are a part of
(otherwise undefined)

(20) **A New Interpretation of the Definite Article**

Thus far in the semester, we've carefully avoided having to give a semantics for the definite article. Now we can finally have one!

$$[[\text{the}]] = \lambda P_{\langle \text{et} \rangle} . \text{MAX}(P)$$

(21) **The Semantics of Definite Plurals**

Suppose that Frank and Bill and Dave are all the boys in the world. All the following holds in our system:

- a. $[[\text{boy}]] = \{ \text{Frank}, \text{Bill}, \text{Dave} \}$
- b. $[[\text{boys}]] = \{ \text{Frank}, \text{Bill}, \text{Dave}, \text{Frank+Bill}, \text{Frank+Dave}, \text{Dave+Bill}, \text{Frank+Dave+Bill} \}$
- c. $[[\text{the boys}]] = \text{MAX}([[\text{boys}]])$
 $= \text{MAX}(\{ \text{Frank}, \text{Bill}, \text{Dave}, \text{Frank+Bill}, \text{Frank+Dave}, \text{Dave+Bill}, \text{Frank+Dave+Bill} \})$
 $= \text{Frank+Bill+Dave}$

Thus, our system captures the 'intuition' that "the boys" refers to the group (plurality) consisting of *all the boys*.

Moreover, with a fairly simple assumption regarding DPs in predicate position, we accurately predict the validity of the inference in (17a).

(22) **Special Rule for Predicative DPs**

$$[[\text{is/are DP}]] = \lambda x . x = [[\text{DP}]]$$

(23) **Validity of Inference (17a)**

- a. Suppose that "Frank, Bill and Dave are the boys" is true.
- b. By our rule in (22), it follows that $[[\text{Frank and Bill and Dave}]] = [[\text{the boys}]]$
- c. Thus, $\text{Frank+Bill+Dave} = \text{MAX}(\{x: x \text{ is a boy}\})$
- d. Thus, Frank+Bill+Dave contain as parts *all* the possible groups of boys.
- e. Thus, if anything is a boy, then its either Frank or Bill or Dave

We also clearly predict that the definite plural “the boys” is fully defined if there is more than one boy.

And, as elaborated below, we also predict that the singular definite “the boy” is **undefined** if there is more than one boy.

(24) **The Semantics of Singular Definites**

a. When There is Only One Entity Satisfying the NP

- (i) Suppose that there is exactly one boy, Dave.
- (ii) Consequently, $[[\text{boy}]] = \{\text{Dave}\}$
- (iii) Recall that **for all x , $x \leq x$** (see p. 4)
- (iv) Consequently, $\text{MAX}(\{\text{Dave}\}) = \text{Dave}$
- (v) Consequently, $[[\text{the boy}]] = \text{Dave}$

b. When There is More than One Entity Satisfying the NP

- (i) Suppose that there are *two* boys, Dave and Bill.
- (ii) Consequently, $[[\text{boy}]] = \{\text{Dave}, \text{Bill}\}$
- (iii) Note that neither $\text{Dave} \leq \text{Bill}$ nor $\text{Bill} \leq \text{Dave}$.
- (iv) Consequently, there is no element in $\{\text{Dave}, \text{Bill}\}$ that contains all other elements in the set as component parts.
- (v) Consequently, $\text{MAX}(\{\text{Dave}, \text{Bill}\})$ is *undefined*
- (vi) Consequently, “the boy” is *undefined*

Prediction: A singular definite “the NP” will only be defined when there is only one entity satisfying the NP, in which case, it refers to that entity!

Our first pass system for plurals is almost complete...

...there is one last pressing issue though....

How do plurality-denoting phrases appear as arguments to Vs?

(25) **Definite Plurals as Arguments to Vs**

- a. **The boys** are running.
- b. Sue saw **the boys**.

Actually, there's no great mystery here. Recall that in our semantic system, even super-mega-complex plural entities like "the things in my house or in my yard" *are all of type e*

Consequently, there should be no problem with their combining with predicates of type $\langle et \rangle$, $\langle e \langle et \rangle \rangle$, or what have you!

(26) **An Illustration**

- a. Suppose that Sue sees the boys – Frank and Bill and Dave – standing together.
- b. We might, then, easily think of the extension of the verb "sees" as containing the following pair of entities:

$\langle \text{Sue, Frank+Bill+Dave} \rangle$

- c. Employing the very simplest of existential systems, then, we'd predict that "Sue saw the boys" should be true!

(27) **Another Illustration**

- a. Suppose that we're having a dance competition where both single dancers and team performances can enter and compete against each other.
- b. Suppose that at one point Sue (on her own) wins, and at another point Bill (on his own) wins, but then later, Dave and Frank (together, *as a team*) win.
- c. In a system where we have plural entities, we could think of the extension of "win" as being the following $\langle et \rangle$ function.

$\{ \langle \text{Sue, T} \rangle \ ; \ \langle \text{Bill, T} \rangle \ ; \ \langle \text{Dave+Frank, T} \rangle \ }$

(28) **The General Point**

In a semantic system where there are 'plural entities' all of type e , nothing stops functions of type $\langle e, \tau \rangle$ from taking such plural entities as arguments.

In such a semantic system, there is no deep metaphysical (or 'type-logical') difference between plural entities and singular entities... they are all the same kind of *thing*

... and so any predicate that can take *good ol' fashioned* singular entities as arguments should be able to directly take these newfangled plural entities as arguments too!

5. Some Further Observations Regarding the ‘Basic System’

5.1 Individual Entities are also Plural Entities

You may have noticed that in our semantic system individual entities like Bill and Dave are considered part of the domain *D of ‘plural entities’.

Relatedly, our system interprets the extension of a plural NP like “boys” as contain individual entities like Bill, Dave and Tom.

This is done for a very good reason!

(29) Proof that Plural NPs Contain ‘Individuals’ in their Extension

- a. Sentence:
No boys are in the room.
- b. Judgment:
False in a situation where there is *a single boy* in the room.
- c. Issue:
- (i) Suppose that $[[\text{boys}]]$ only contains groups of boys.
 $[[\text{ boy }]] = \{ \text{ Dave, Frank } \}$
 $[[\text{ boys }]] = \{ \text{ Dave+Frank } \}$
 - (ii) Suppose that $[[\text{no}]] = \lambda P. \lambda Q. P \cap Q = \emptyset$
 - (iii) Suppose that Dave (alone) is in the room.
 - (iv) Consequently, $[[\text{ in the room }]] = \{ \text{ Dave } \}$
 - (v) Clearly, $\{ \text{ Dave+Frank } \} \cap \{ \text{ Dave } \} = \emptyset$
 - (vi) Consequently, $[[\text{ No boys are in the room }]] = T$
- d. Conclusion:
- The sentence in (29a) quantifies over a set that also contains *individual* boys like Dave and Frank.**

Thus, plurals must also contain singular entities in their extension.

5.2 The Semantic Vacuity of Number Agreement

In the system laid out above, we interpret plural number on NPs. Plural number on an NP has an effect upon the truth-conditions of the sentence.

However, as is implicit in (25)-(28), our system doesn't semantically interpret plural number (agreement) morphology on the VP

(30) A Quick Illustration

- Let us imagine the dance competition scenario from example (27).
- In this scenario, intuitively, the sentences “Sue wins” and “Dave and Frank win” are true.
- Our system predicts this, as illustrated below.

a. The Truth of “Sue wins”

$$\begin{aligned} [[\text{Sue wins}]] &= [[\text{wins}]]([[\text{Sue}]]) \\ &= \{ \langle \text{Sue}, \text{T} \rangle ; \langle \text{Bill}, \text{T} \rangle ; \langle \text{Dave+Frank}, \text{T} \rangle \}(\text{Sue}) = \text{T} \end{aligned}$$

b. The Truth of “Dave and Frank win”

$$\begin{aligned} [[\text{Dave and Frank win}]] &= [[\text{win}]]([[\text{Dave and Frank}]]) \\ &= \{ \langle \text{Sue}, \text{T} \rangle ; \langle \text{Bill}, \text{T} \rangle ; \langle \text{Dave+Frank}, \text{T} \rangle \}(\text{Dave+Frank}) \\ &= \text{T} \end{aligned}$$

- Note, however, that in this system, the singular-marked VP “wins” is assigned the exact same extension as the plural-marked VP “win”
- Thus, this system doesn't treat plural number *agreement* on the V as being semantically contentful

Thus, this system adds some independent semantic weight to the claim, well-known in syntactic theory, that ‘phi-features’ on verbs are not interpreted...

5.3 ‘Plurality-Seeking’ Predicates

There appear to be some predicates that can *only* take plural entities as arguments.

(31) Some ‘Plurality-Seeking’ Predicates

- a. Dave and Frank / The boys / * Sue / * The girl **met in the park.**
- b. Dave and Frank / The boys / * Sue / * The girl **gathered outside.**
- c. Dave and Frank / The boys / * Sue / * The girl **split up.**

This property clearly follows from the lexical semantics of these predicates, the *kinds* of events/actions that they describe.

If we really want our theory to capture these facts, we could add a so-called ‘meaning postulate’ to our semantic theory of English.

(32) A Meaning Postulate that Will Capture the Facts in (31)

$[[\text{meet} / \text{gather} / \text{split up}]] \subseteq *D - D$

The extensions of “meet”, “gather” and “split up” have to consist solely of ‘non-atomic’ (strictly plural) entities.