

**The Semantics of Modals, Part 3:
The Ordering Source ¹**

1. On Our Last Episode...

We developed a semantics for modal auxiliaries in English, that achieved the goals in (1).

(1) Overarching Analytic Goal

A semantics that obtains the various modal ‘readings’ via productive composition of:

- a. the invariant meaning of the modal, with
- b. other (possibly covert) material in the sentence (which contributes the restricted set of worlds that the modal quantifies over)

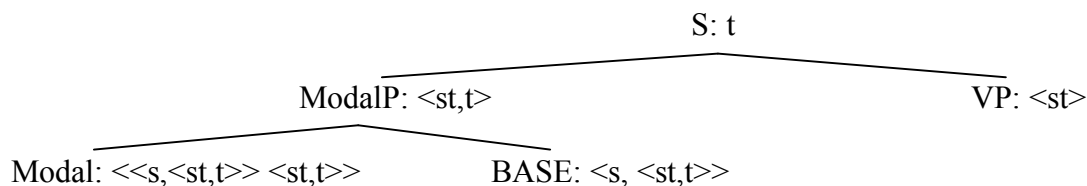
(2) The Invariant Meaning of the Modal Heads

- a. $[[\text{may / can}]]$ ^w = $\lambda B_{\langle s, \langle st, t \rangle \rangle} . \lambda p. \exists w' \in \cap B(w): p(w') = T$
- b. $[[\text{must / have-to}]]$ ^w = $\lambda B_{\langle s, \langle st, t \rangle \rangle} . \lambda p. \forall w' \in \cap B(w): p(w') = T$

(3) The Null Pronoun ‘BASE’: The Covert Material That Provides the ‘Restriction’

- a. ‘BASE’ is complement to the modal head, and provides its first argument.
- b. ‘BASE’ is a pronoun, and so its value is provided by ‘context’ (the function *g*)
- c. The contextually-determined value of ‘BASE’ is a function of type $\langle s, \langle st, t \rangle \rangle$.
 - When fed the evaluation world as argument, it yields a set of propositions.
 - When these propositions are intersected together, it yields a set of worlds.
 - **This resulting set of worlds provides the ‘restricted’ set of worlds that the modal ‘quantifies over’.**
 - **Thus, by varying the identity of the $\langle s, \langle st, t \rangle \rangle$ function provided by ‘BASE’, we vary the set of worlds the modal ends up quantifying over, and thus we vary the ‘reading’ that the modal receives.**

(4) A Schematic of the Syntax and Semantics Proposed



¹ These notes are based upon material in von Stechow (2007; Chapter 5) and Kratzer (1991).

- In this third (and final) part of our discussion of modals, we will consider a couple challenges to the picture above.
 - Moreover, we will see that these challenges can be overcome (in an interesting way), if we assume that modal auxiliaries have *one more* argument place...
That is, we will propose that – besides ‘BASE’ – there is another (potentially covert) $\langle s, \langle st, t \rangle \rangle$ function that the meaning of the modal combines with...
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2. Some Challenges to the Analysis from ‘Part 2’

2.1 The Hypothesized ‘Restriction’ is not Restricted Enough

Although we developed a more sophisticated means of deriving them, the truth-conditions our account predicts for the various ‘modal readings’ are *still* those that appear at the end of ‘Part 1’

(5) The Truth-Conditions Predicted for “Must VP”

a. Epistemic ‘Must’

(i) *Formulation at the End of Part 1*

$\forall w' \in W$: **everything we know about w is also true in w'** $\rightarrow p(w') = T$

(ii) *Equivalent Formulation in the System from Part 2*

$\forall w' \in \cap \{ p : \text{we know that } p \text{ in } w \}$: $p(w') = T$

b. Circumstantial ‘Have-to’

(i) *Formulation at the End of Part 1*

$\forall w' \in W$: **everything true in w (up to now) is true in w'** $\rightarrow p(w') = T$

(ii) *Equivalent Formulation in the System from Part 2*

$\forall w' \in \cap \{ p : p \text{ is true in } w \text{ (up to the present)} \}$: $p(w') = T$

c. Deontic ‘Must’

(i) *Formulation at the End of Part 1:*

$\forall w' \in W$: **‘the law’ in w is being followed in w'** $\rightarrow p(w') = T$

(ii) *Equivalent Formulation in the System from Part 2*

$\forall w' \in \cap \{ p : p \text{ is ‘the law’ in } w \}$: $p(w') = T$

d. Bouletic ‘Must’

(i) *Formulation at the End of Part 1:*

$\forall w' \in W$: **‘our goals’ in w are met in w'** $\rightarrow p(w') = T$

(ii) *Equivalent Formulation in the System from Part 2*

$\forall w' \in \cap \{ p : p \text{ is one of ‘our goals’ in } w \}$: $p(w') = T$

However, recall from 'Part 1' that these truth conditions above are not entirely correct. They don't truly capture the full content of each of the four types of 'modal readings'.

(6) **More Complete Statement of the Truth-Conditional Contribution of the Modal**

a. Epistemic Reading (cf. p. 4 of 'Part 1')

$[[\text{may VP}]]^w =$

$\exists w' \in W$: everything we know about w is also true in w' **and nothing that is 'abnormal' in w occurs in w'** & $[[VP]]_c(w') = T$

$[[\text{must / have to VP}]]^w =$

$\forall w' \in W$: everything we know about w is also true in w' **and nothing that is 'abnormal' in w occurs in w'** $\rightarrow [[VP]]_c(w') = T$

b. Circumstantial Reading (cf. p. 11 of 'Part 1')

$[[\text{can VP}]]^w =$

$\exists w' \in W$: everything true in w (up to now) is true in w' **and nothing that is 'abnormal' in w occurs in w'** & $[[VP]]_c(w') = T$

$[[\text{have to VP}]]^w =$

$\forall w' \in W$: everything true in w (up to now) is true in w' **and nothing that is 'abnormal' in w occurs in w'** $\rightarrow [[VP]]_c(w') = T$

c. Deontic Reading (cf. p. 16 of 'Part 1')

$[[\text{may / can VP}]]^w =$

$\exists w' \in W$: law in w is being followed in w' **and everything true in w (up to now) is true in w'** & $[[VP]]_c(w') = T$

$[[\text{must / have to VP}]]^w =$

$\forall w' \in W$: law in w is being followed in w' **and everything true in w (up to now) is true in w'** $\rightarrow [[VP]]_c(w') = T$

d. Bouletic Reading (cf. p. 20 of 'Part 1')

$[[\text{may / can VP}]]^w =$

$\exists w' \in W$: our goals in w are met in w' **and everything true in w (up to now) is true in w'** & $[[VP]]_c(w') = T$

$[[\text{must / have to VP}]]^w =$

$\forall w' \in W$: our goals in w are met in w' **and everything true in w (up to now) is true in w'** $\rightarrow [[VP]]_c(w') = T$

That is, as noted above and in 'Part 1', the set of worlds quantified over in each of the four 'principal readings' seems to be a more restricted set than what appears in our currently-predicted truth-conditions under (5)...

...Thus, we should seek to augment our analysis from 'Part 2' so that it predicts (something akin) to these 'more complete' truth-conditions in (6)...

2.2 Crime and Punishment with Deontic Modals

A second problem with our analysis from 'Part 2' concerns an acute failing of our proposed semantics for the 'deontic reading'...

First, note that given the context in (7a), both the (deontic) sentences in (7b) seem to be true.

(7) Deontic Modals and 'The Law'

a. Context:

- (i) The law consists of the following two propositions:
- *Nobody commits murder. (Murder is a crime.)*
- *Anyone who commits murder goes to jail.*
(- *Anyone who doesn't commit murder doesn't go to jail*)
- (ii) Moreover, **John has just committed murder.**

b. True Sentences (Containing 'Deontic Modals')

- (i) (Given what the law is,) Dave **must** not commit murder.
(ii) (Given what the law is,) John **must** go to jail.

Now, consider the truth conditions that our semantics from 'Part 2' predicts for the relevant 'deontic reading' of these sentences.

(8) Predicted Truth Conditions of Sentences in (7b)

a. Truth-Conditions of (7bi)

$\forall w' \in \cap \{ p : p \text{ is 'the law' in } w \}$: Dave does not commit murder in w' OR
 $\forall w' \in \cap \{$ { w : no one commits murder in w },
{ w : anyone who commits murder in w goes to jail in w },
{ w : anyone who doesn't commit murder doesn't go to jail in w } }:
Dave does not commit murder in w' .

b. Truth-Conditions of (7bii)

$\forall w' \in \cap \{ p : p \text{ is 'the law' in } w \}$: John goes to jail in w' OR
 $\forall w' \in \cap \{$ { w : no one commits murder in w },
{ w : anyone who commits murder in w goes to jail in w },
{ w : anyone who doesn't commit murder doesn't go to jail in w } }:
John goes to jail in w' .

(9) **Critical Problem!**

Following the truth-conditions predicted in (8b), sentence (7bii) should be *false* in context (7a).

- Take any world w' from the following set:
 $\cap \{ \{ w: \text{no one commits murder in } w \},$
 $\{ w: \text{anyone who commits murder in } w \text{ goes to jail in } w \},$
 $\{ w: \text{anyone who doesn't commit murder doesn't go to jail in } w \} \}$
- World w' will necessarily be a world such that:
No one commits murder in w' *and*
No one goes to jail in w' who has not committed murder in w' .
- Thus, in world w' , *John has not committed murder.*
- Consequently, in world w' , *John does not go to jail.*
- Thus, the truth-conditions in (8b) do not hold in the imagined context, and sentence (7bii) is predicted to be false in that context.

(10) **A Non-Solution**

Recall from Section 2.1 that the restriction of a deontic modal appears to not simply be the worlds where the law is being followed, but rather is the following:

- a. A More Complete Statement of the Deontic Modal Base:
 $\{w' : \text{law in } w \text{ is being followed in } w' \text{ and everything true in } w \text{ (up to now) is true in } w'\}$

Perhaps if we adopt this more accurate – more *restricted* – characterization of the modal base we can avoid the problem?...

Why This Won't Work:

If we assume that 'the law' is as stated in (7a), then the 'more restricted' base in (10a) is actually the *empty set!*

- The 'more restricted' base in (10a) will be the following set:
 $\cap \{ \{ w: \text{no one commits murder in } w \},$
 $\{ w: \text{anyone who commits murder in } w \text{ goes to jail in } w \},$
 $\{ w: \text{anyone who doesn't commit murder doesn't go to jail in } w \}, \dots$
 $\{ w: \text{John commits murder in } w \}, \dots \}$
- Since there are no worlds where *both* John commits murder *and* no one commits murder, it follows that the 'more restricted' base above will be the empty set.
- Thus, under this semantics, all the following end up being (trivially) true in context (7a).
John must go to jail; John must eat pizza; Dave must go to jail.

(11) **Another Non-Solution**

Clearly, the problem here is connected with the fact that one of the propositions we ‘intersect together’ to produce the modal base is “*No one commits murder.*”

- a. Perhaps, then, we’re wrong to think that ‘the law’ consists of such propositions.
- b. Perhaps the way that natural language ‘conceptualizes’ the ‘law’ in a given context is simply as a set of conditional punishments, like ‘anyone who commits murder in w goes to jail in w ’.
- c. If this were the case, then the restriction of the modal in (7bii) would simply be the following set:

$$\cap \{ \begin{array}{l} \{w: \text{anyone who commits murder in } w \text{ goes to jail in } w \}, \\ \{w: \text{anyone who doesn't commit murder doesn't go to jail in } w \}, \dots \\ \{w: \text{John commits murder in } w \}, \dots \} \end{array}$$

- d. Clearly, *this* is a non-empty set of worlds where John does indeed commit murder... and so the problems in (9) and (10) don’t seem to obtain!

Why This Won’t Work:

If we *remove* the proposition “*No one commits murder*” from the modal base, then we *fail* to predict the truth of sentence (7bi) in context (7a).

- Note that there are worlds in the base in (11c) where *Dave* commits murder. (No proposition in the intersected set would seem to rule such worlds out)
- Thus, if we assume that the restriction of the deontic modal in context (7a) is the set in (11c), we wrongly predict that sentence (7bi) is *false*.

(12) **Towards a Real Solution**

Reflecting upon the issues in (9) – (11), it seems like the following might be a better statement of what the ‘restriction’ of a deontic modal is. It’s those worlds w' such that:

- a. Everything true in the actual world (up to the present) is true in w'
- b. **Nothing illegal happens ever in w' that doesn't already happen in w_0**
(i.e., after the present, nothing illegal ever happens)

Consider:

In all such worlds John commits murder
(though it violates the law, it's already happened in w_0)

In all such worlds, *Dave does not commit murder.*
(since it violates the law and *doesn't* already happen in w_0)

In all such worlds, *John goes to jail.*
(given that he's a murderer in all these worlds, his *not* going to jail would be something that violates the law, and which *doesn't* already happen in w_0)

Consequently:

The following would seem to be an analysis of the truth conditions of sentences (7bi, ii) which *correctly predicts both to be true in context (7a)!*

- a. “John must go to jail” is T in w_0 *iff*
For all worlds w' in the following RESTRICTION, John goes to jail in w' :

RESTRICTION = Those worlds w' such that:
 - a. Everything that is true in w_0 (up to the present) is true in w'
 - b. Nothing illegal ever happens in w' that hasn't already happened in w_0 (up to the present)
- b. “Dave must not commit murder is T in w_0 *iff*
For all worlds w' in the above RESTRICTION, Dave doesn't murder in w'

The Challenge:

How do we augment our semantics from ‘Part 2’ so that it predicts the T-conditions in (12a,b)?

(13) **Summary: Problems for the System in (2) – (4)**

- a. Postulated restrictions for the various readings are *too broad*.
Must somehow incorporate the additional restrictive information reflected in (6).
- b. Postulated restrictions for the deontic reading are (in one respect) *too narrow*.
Must somehow amend the restriction to the set sketched in (12).

We will see that a solution to all three of the above problems can be gained *via the introduction of an additional ‘character’ into our semantic analysis of modals: the “ordering source”!*

We’ll introduce this new character gradually...

... And, as we did for ‘the modal base’, we’ll begin with the problems surrounding deontic modals...

3. A New Approach to the Semantics of the ‘Deontic Reading’

Recall that, following our discussion in Section 2.2., we would like to somehow revise our semantic analysis of the ‘deontic reading’ in the following way:

(16) **A New Semantics for ‘Deontic Modals’ (First Pass)**

“John must go to jail” is T in w_0 *iff*

For all worlds w' in the following RESTRICTION, John goes to jail in w' :

RESTRICTION =

Those worlds w'' such that:

- a. Everything that is true in w_0 (up to the present) is true in w''
- b. Nothing illegal ever happens in w'' that hasn’t already happened in w_0 (up to the present)

As a first step to a fully formalized account, consider that the following set is equivalent to the RESTRICTION in (16).

(17) **A Restatement of the ‘RESTRICTION’ in (16)**

RESTRICTION-2 =

Those worlds w' from

the set $S = \{ w'' : \text{everything that is true in } w_0 \text{ (up to the present) is true in } w'' \}$

which satisfy the *greatest number of propositions* in the set:

LAW = $\{ p : p \text{ is ‘the law’ in } w_0 \}$

(18) **Claim: RESTRICTION-2 is equivalent to RESTRICTION**

Proof: All the worlds w' in RESTRICTION-2 have the following properties:

- a. Everything that is true in w_0 (up to the present) is true in w' (trivial)
- b. *Nothing illegal happens in w' that hasn't already happened in w_0*

- Let $w' \in \text{RESTRICTION-2}$.
- For any $w \in W$, let $\text{LAW}(w) = \{ p : p \in \text{LAW} \ \& \ p(w) = 1 \}$
- By definition, there is no $w'' \in S$ such that $\text{LAW}(w') \subset \text{LAW}(w'')$
- **Now (for a contradiction) suppose** that *something illegal occurs in w' that hasn't already occurred in w_0*
- Clearly, $\text{LAW}(w') \subset \{ p : p \in \text{LAW} \ \& \ p \text{ is satisfied in } w_0 \text{ up to the present} \}$
- **Now consider** any world $w'' \in S$ such that *nothing illegal ever occurs in w'' that hasn't already occurred in w_0*
- Clearly, $\text{LAW}(w'') = \{ p : p \in \text{LAW} \ \& \ p \text{ is satisfied in } w_0 \text{ up to the present} \}$
- Thus, $\text{LAW}(w') \subset \text{LAW}(w'')$
- Thus, **contrary to hypothesis**, there is a $w'' \in S$ such that $\text{LAW}(w') \subset \text{LAW}(w'')$

...So, in order to improve our analysis of the deontic reading, we need to somehow augment our semantics so that the set RESTRICTION-2 serves as the restriction of the modal.

In the following sub-sections, we'll see step-by-step how we can employ our 'set-theoretic machinery' to more formally construct the set of worlds in (17).

3.1 Step 1: Sets of Propositions Define an Ordering of Possible Worlds

The first step is to observe that any set of propositions P defines an *ordering relation* on any set of possible worlds:

Intuitively, the ordering relation ' w satisfies more propositions in P than w'' '

(19) **Ordering Relation Defined by Set of Propositions P**

Let P be a set of propositions $\{ p_1, \dots, p_n \}$, and X be any set of worlds.

For any two worlds $w, w' \in X$:

$$w \prec_P w' \quad \text{iff} \quad \{ p : p \in P \ \& \ p(w') = 1 \} \subset \{ p : p \in P \ \& \ p(w) = 1 \}$$

Read $w \prec_P w'$ as *w is closer to 'the ideal set by P ' than w'* .

3.2 Step 2: Maximal Elements of an Ordering

The second step is to introduce a function that – given a set S and an ordering O of that set – picks out the members of S that are ‘maximal’ with respect to O .

(20) The Function ‘ $MAX_{<P}$ ’

Let X be any set of worlds, and let ‘ $<_P$ ’ be an ordering on X .

$$MAX_{<P}(S) = \{ w \in X : \neg \exists w' \in X . w' <_P w \}$$

Read $MAX_{<P}(S)$ as those worlds that are ‘maximal’ with respect to P
those worlds from X that satisfy the most propositions from P
those worlds from X that come closest to ‘the ideal set by P ’

3.3 Step 3: A More Formal Statement of the Targeted-Truth Conditions

We can now use the tools introduced above to provide a more formal statement of the truth-conditions we are targeting in (16) and (17):

(21) The Targeted Truth-Conditions

“John must go to jail” is T in w_0 *iff*

$$\forall w' \in MAX_{<[\text{the-law-in-}w_0]} (\cap \{ p : p \text{ is true in } w_0 \text{ (up to the present)} \}): \\ \text{John goes to jail in } w'.$$

(22) What These Truth-Conditions Say

- Take those worlds that are just like w_0 (up to the present).
(*i.e.* the set ‘ $\cap \{ p : p \text{ is true in } w_0 \text{ (up to the present)} \}$ ’)
- Order these worlds according to how many propositions from ‘the law in w_0 ’ they satisfy.
- Now, look at those worlds that satisfy *the most* propositions from ‘the law in w_0 ’
(*i.e.*, those where nothing illegal ever occurs that hasn’t already occurred in w_0)
(*i.e.*, those worlds where, after the present, the law is followed perfectly)
- In all *those* (maximal) worlds, *John goes to jail*.
- THAT IS TO SAY:
John goes to jail in all those worlds which
 - are just like w_0 (up to the present)
 - nothing illegal ever occurs that hasn’t already occurred in w_0
(*i.e.*, after the present, the law is followed to the letter)

Conclusion:

The more formally stated truth-conditions in (21) equate to the informally stated truth conditions in (12) and (16).

Thus, the truth-conditions in (21) will avoid the problem from Section 2.2...

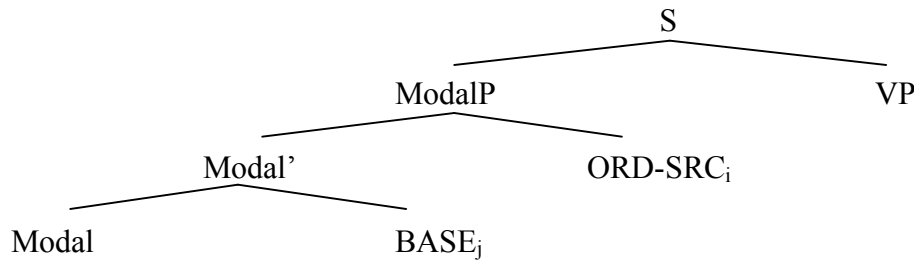
So, let's develop a semantic analysis that allows us to compositionally derive them!!

3.4 A Compositional Treatment of the Hypothesized Truth-Conditions

(23) Syntactic Assumptions

In addition to the null pronoun 'BASE', there is another null pronoun: 'ORD-SRC'.

- Like 'BASE', 'ORD-SRC' bears an index i .
- 'ORD-SRC' is sister to the phrase containing the modal and 'BASE'



(24) Semantic Assumptions

a. Assumptions Regarding 'ORD-SRC'

- Given that it is a pronoun, the meaning of 'ORD-SRC' is provided by the assignment function g
- Like 'BASE', the value of 'ORD-SRC' is a function of type $\langle s, \langle st, t \rangle \rangle$
- Thus: $[[\text{ORD-SRC}_i]]^{w,g} = g(i) \in D_{\langle s, \langle st, t \rangle \rangle}$

b. Lexical Entries for the Modals

(i) $[[\text{may / can}]]^w = \lambda B_{\langle s, \langle st, t \rangle \rangle} . \lambda O_{\langle s, \langle st, t \rangle \rangle} . \lambda p_{\langle st \rangle} . \exists w' \in \text{MAX}_{\langle O(w) \rangle} (\cap B(w)) : p(w') = 1$

*There is some world w' in the following set where p is true:
Of the possible worlds that satisfy all the propositions in $B(w)$,
those that satisfy the most propositions in $O(w)$.*

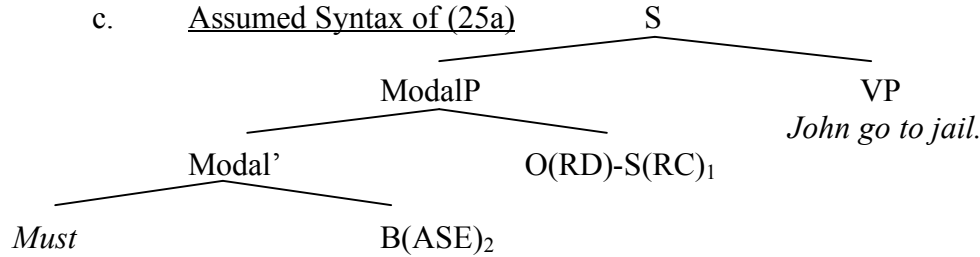
(ii) $[[\text{must / have-to}]]^w = \lambda B_{\langle s, \langle st, t \rangle \rangle} . \lambda O_{\langle s, \langle st, t \rangle \rangle} . \lambda p_{\langle st \rangle} . \forall w' \in \text{MAX}_{\langle O(w) \rangle} (\cap B(w)) : p(w') = 1$

*All worlds w' in the following set are worlds where p is true:
Of the possible worlds that satisfy all the propositions in $B(w)$,
those that satisfy the most propositions in $O(w)$.*

(25) Derivation of Deontic Truth Conditions

a. Sentence: John must go to jail.

b. Targeted Truth-Conditions “John must go to jail” is T in w iff
 $\forall w' \in \text{MAX}_{\langle \text{the-law-in-}w \rangle} (\cap \{ p : p \text{ is true in } w \text{ (up to the present)} \})$: John goes to jail in w' .



d. Additional Semantic Assumptions:

$$\begin{aligned} g(1) &= \lambda w. [\lambda p. p \in \text{the Law in } w] \\ g(2) &= \lambda w. [\lambda p. p \text{ is true in } w \text{ (up to the present)}] \end{aligned}$$

e. Derivation of Truth-Conditions in (25b)

- i. $[[S]]^{w,g} = T$ iff (by IFA, FA)
- ii. $[[[[\text{Modal}]]^{w,g} ([[B_2]]^{w,g})] ([[O-S_1]]^{w,g})] ([[VP]]_c) = T$ iff (by Ass.)
- iii. $[[[[\text{Modal}]]^{w,g} ([[B_2]]^{w,g})] ([[O-S_1]]^{w,g})] (\lambda w'. \text{John goes to jail in } w') = T$ iff (by (24a))
- iv. $[[[[\text{Modal}]]^{w,g} (g(2))] (g(1))] (\lambda w'. \text{John goes to jail in } w') = T$ iff (by (25d))
- v. $[[[[\text{Modal}]]^{w,g} (\lambda w'''. [\lambda p. p \text{ is true in } w''' \text{ (up to the present)}])] (\lambda w''. [\lambda p. p \in \text{the Law in } w''])] (\lambda w'. \text{John goes to jail in } w') = T$ iff (by (24bii))
- vi. $[[[\lambda B. \lambda O. \lambda p. \forall w'''' \in \text{MAX}_{\langle O(w) \rangle} (\cap B(w)): p(w''') = 1] (\lambda w'''. [\lambda p. p \text{ is true in } w''' \text{ (up to the present)}])] (\lambda w''. [\lambda p. p \in \text{the Law in } w''])] (\lambda w'. \text{John goes to jail in } w') = T$ iff (by LC)
- vii. $[[\lambda O. \lambda p. \forall w''' \in \text{MAX}_{\langle O(w) \rangle} (\cap \{ p' : p' \text{ is true in } w \text{ (up to the present)} \}): p(w''') = 1] (\lambda w''. [\lambda p. p \in \text{the Law in } w''])] (\lambda w'. \text{John goes to jail in } w') = T$ iff (by LC)
- viii. $[\lambda p. \forall w'' \in \text{MAX}_{\langle p' : p' \in \text{the Law in } w \rangle} (\cap \{ p' : p' \text{ is true in } w \text{ (up to the present)} \}): p(w'') = 1] (\lambda w'. \text{John goes to jail in } w') = T$ iff (by LC)
- ix. $\forall w' \in \text{MAX}_{\langle p : p \in \text{the Law in } w \rangle} (\cap \{ p : p \text{ is true in } w \text{ (up to the present)} \})$:
John goes to jail in w'

3.5 Some Discussion

The semantics developed above for the ‘deontic reading’ of *must/may (can/have-to)* has the following three (immediately apparent) advantages:

(26) **Advantage 1:**

This semantic system circumvents the problem from Section 2.2.
(*Obvious at this point.*)

(27) **Advantage 2:**

This semantic system *partly* circumvents the problem from Section 2.1

- Under this analysis, a deontic modal doesn’t simply quantify over the set of worlds where the law is followed.
- Rather, the worlds it quantifies over are *also* required to *match* the facts of the actual world (up to the present).
- Thus, this additional ‘restrictive’ information reflected in (6c) is also reflected in the semantics in (24b).

(28) **Advantage 3:**

There is nothing inherently ‘deontic’ in the semantics given for the modal heads themselves in (24b)

- As before, we obtain the ‘deontic reading’ by combining a ‘neutral’ meaning for the modal head with two other elements in the sentence:
 - a ‘circumstantial’ base : ‘ $\lambda w. [\lambda p . p \text{ is true in } w \text{ (up to the present) }]$ ’
 - a ‘deontic’ ordering source: ‘ $\lambda w. [\lambda p . p \in \text{the Law in } w]$ ’
- *However, nothing in the system requires that ‘BASE’ or ‘ORD-SRC’ take on the values that they do in (25)!*

Project for the Next Few Sections:

Let us see how varying the value of ‘BASE’ and ‘ORD-SRC’ can yield each of the other four ‘principal modal readings’.

In each case, we’ll see that the meaning generated by our system incorporates the ‘more restrictive’ information reflected in the entries under (6).

Thus, in the end, our augmented system will be able to completely overcome the problem from Section 2.1...

A Terminological Aside:

(29) **The ‘Ordering Source’**

- a. The $\langle s, \langle st, t \rangle \rangle$ function O which the modal head takes as its second argument. (*i.e.*, the variable ‘ O ’ in the lexical entries in (24b))(*cf.* Kratzer 1977, 1991, 2008)
- b. The set of propositions that the function O yields when fed the evaluation world as its argument (*cf.* von Stechow 2007, *et multia alia*) (*e.g.*, for deontic modals, the propositions constituting ‘the relevant body of law’.)

(30) **The Conversational Background**

- a. A function (any function) of type $\langle s, \langle st, t \rangle \rangle$
- b. A set of propositions obtained by feeding the evaluation world to a function of type $\langle s, \langle st, t \rangle \rangle$
- c. The meaning of the ‘in-view-of’ phrase.

Key Quote (Kratzer 1991):

“In modal reasoning, a conversational background may function as a modal base or as an ordering source. The modal base determines the set of accessible worlds (for a given world). The ordering source imposes an ordering on this set...” (Kratzer 1991: 645-646)

4. The Bouletic Reading

Let us see how we might, in the system developed above, capture the ‘bouletic reading(s)’.

- Let’s start off with a refresher on the truth-conditions we are seeking to capture.
- Let’s also follow the ‘more complete’ truth-conditions stated in (6)...

(31) **Bouletic Truth-Conditions**

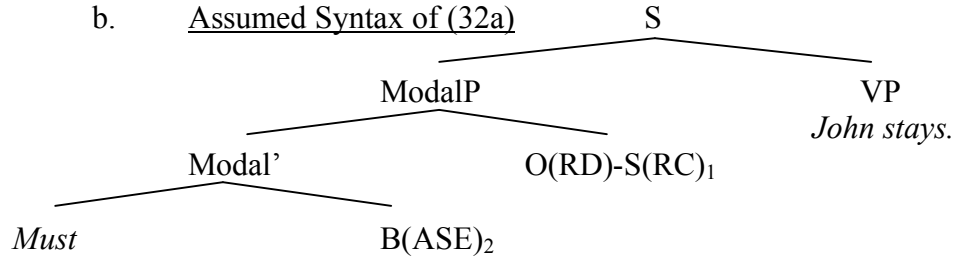
“John must stay” is true in a world w *iff*
 $\forall w' \in W$: **our goals in w are met in w' and everything true in w (up to now) is true in w'** \rightarrow John stays in w' .

Now, with (31) in the background as the ‘truth-conditions’ of the bouletic reading, *let’s just see what reading our system yields under the following conditions:*

- $[[\text{ORD-SRC}_1]]$ ^{w,g} = $g(1)$ = $\lambda w. [\lambda p . p \in \text{our goals in } w]$
- $[[\text{BASE}_2]]$ ^{w,g} = $g(2)$ = $\lambda w. [\lambda p . p \text{ is true in } w \text{ (up to the present) }]$

(32) **Derivation of Bouletic Truth Conditions**

a. Sentence: John must stay.



c. Additional Semantic Assumptions:

$$\begin{array}{l}
 g(1) = \lambda w. [\lambda p. p \in \text{our goals in } w] \\
 g(2) = \lambda w. [\lambda p. p \text{ is true in } w \text{ (up to the present)}]
 \end{array}$$

d. Derivation of Truth-Conditions for (32a)

- i. $[[S]]^{w,g} = T$ *iff* (by IFA, FA)
- ii. $[[[[\text{Modal}]]^{w,g} ([[B_2]]^{w,g})] ([[O-S_1]]^{w,g})] ([[VP]]_{\epsilon}) = T$ *iff* (by Ass.)
- iii. $[[[[\text{Modal}]]^{w,g} ([[B_2]]^{w,g})] ([[O-S_1]]^{w,g})] (\lambda w'. \text{John stays in } w') = T$ *iff* (by (24a))
- iv. $[[[[\text{Modal}]]^{w,g} (g(2))] (g(1))] (\lambda w'. \text{John stays in } w') = T$ *iff* (by (25d))
- v. $[[[[\text{Modal}]]^{w,g} (\lambda w'''. [\lambda p. p \text{ is true in } w''' \text{ (up to the present)}])] (\lambda w''. [\lambda p. p \in \text{our goals in } w''])] (\lambda w'. \text{John stays in } w') = T$ *iff* (by (24bii))
- vi. $[[[\lambda B. \lambda O. \lambda p. \forall w'''' \in \text{MAX}_{<O(w)}(\cap B(w)): p(w''') = 1] (\lambda w'''. [\lambda p. p \text{ is true in } w''' \text{ (up to the present)}])] (\lambda w''. [\lambda p. p \in \text{our goals in } w''])] (\lambda w'. \text{John stays in } w') = T$ *iff* (by LC)
- vii. $[[\lambda O. \lambda p. \forall w''' \in \text{MAX}_{<O(w)}(\cap \{p' : p' \text{ is true in } w \text{ (up to the present)}\}): p(w''') = 1] (\lambda w''. [\lambda p. p \in \text{our goals in } w''])] (\lambda w'. \text{John stays in } w') = T$ *iff* (by LC)
- viii. $[\lambda p. \forall w'' \in \text{MAX}_{<\{p'' : p'' \in \text{our goals in } w\}}(\cap \{p' : p' \text{ is true in } w \text{ (up to the present)}\}): p(w'') = 1] (\lambda w'. \text{John stays in } w') = T$ *iff* (by LC)
- ix. $\forall w' \in \text{MAX}_{<\{p : p \in \text{our goals in } w\}}(\cap \{p : p \text{ is true in } w \text{ (up to the present)}\}):$
John stays in } w'

*For all worlds w' in the following set, John stays in w'
Those worlds – from the set of worlds identical to w up to the present –
which satisfy the greatest number of our goals in w .*

(33) **Interim Result**

If we allow the ordering source to be the set $\{p : p \in \text{our goals in } w_0\}$ and the base to be the ‘circumstantial base’ $\{p : p \text{ is true in } w_0\}$, then we derive:

“John must stay” is T in w_0 *iff*

$$\forall w' \in \text{MAX}_{\langle \{p : p \in \text{our goals in } w\} \rangle} (\cap \{p : p \text{ is true in } w \text{ (up to the present)}\}):$$

John stays in w'

(34) **Crucial Question**

Do the derived truth-conditions in (33) equate to the targeted ‘bouletic truth-conditions’ in (31), repeated below:

“John must stay” is true in a world w' *iff*

$$\forall w' \in W: \text{our goals in } w \text{ are met in } w' \text{ and everything true in } w \text{ (up to now)}$$

is true in } w' \rightarrow \text{John stays in } w'.

(35) **Answer, Part 1**

YES – as long as nothing in the actual world w *prevents* our satisfying all our goals in w

- Since nothing in the actual world w *prevents* our satisfying all our goals, there will certainly exist worlds w' in the set $\cap \{p : p \text{ is true in } w \text{ (up to the present)}\}$ such that they satisfy *all* propositions in $\{p : p \in \text{our goals in } w\}$
- Clearly, for any such world w' , there is no other world in the set $\cap \{p : p \text{ is true in } w \text{ (up to the present)}\}$ that satisfies *more* propositions from $\{p : p \in \text{our goals in } w\}$.
- Thus, $w' \in \text{MAX}_{\langle \{p : p \in \text{our goals in } w\} \rangle} (\cap \{p : p \text{ is true in } w \text{ (up to the present)}\})$
- Moreover, for any world w'' that doesn't satisfy all of $\{p : p \in \text{our goals in } w\}$, it's clear that $w'' \notin \text{MAX}_{\langle \{p : p \in \text{our goals in } w\} \rangle} (\cap \{p : p \text{ is true in } w \text{ (up to the present)}\})$
- Thus, all $w' \in \text{MAX}_{\langle \{p : p \in \text{our goals in } w\} \rangle} (\cap \{p : p \text{ is true in } w \text{ (up to the present)}\})$, are such that w' satisfies all the propositions in $\{p : p \in \text{our goals in } w\}$
- **Thus:** $\text{MAX}_{\langle \{p : p \in \text{our goals in } w\} \rangle} (\cap \{p : p \text{ is true in } w \text{ (up to the present)}\}) =$
 $\{w' : \text{everything that is true in } w \text{ (up to the present) is true in } w' \text{ and}$
 $\text{all our goals in } w \text{ are met in } w'\}$

...OK, but what happens if facts in the actual world **do** prevent us from meeting all our goals?
Happily, our derived/predicted truth-conditions in (33) out-perform those in (34)!

(36) **A Situation Where the World Prevents Us Meeting All Our Goals**

a. Context:

(i) The goals for our meeting are the following:

- *Jim finds a way to raise \$46 million.*

- *If Jim can't find a way to do that, we plan our escape to the Caymans.*

(ii) Moreover, **Jim has just committed suicide (and hence cannot find a way to raise \$46 million.)**

b. True Sentence (Containing 'Bouletic Modal')

We **have to** plan our escape to the Caymans.

(37) **Fact 1: The Failure of the ('Conjunctive') Truth-Conditions in (31) / (34)**

Consider the truth-conditions that the treatment in (31) / (34) would predict for (36b):

a. "We have to plan our escape to the Caymans" is T in w *iff*

$\forall w' \in W$: **our goals in w are met in w' and everything true in w (up to now) is true in w' \rightarrow we plan our escape in w' .**

However note that – as we noted earlier in similar circumstances for 'deontic modals' – the restriction in (37a) is the *empty set*.

- There *is no* world that is just like w up to the present and where we meet *all* our goals

(because Jim has just died in w , any worlds that is just like w up to the present will necessarily be world that can't satisfy the goal 'Jim finds a way to raise \$46 million.')

Thus, for reasons similar to what we saw earlier for 'deontic modals', the truth conditions in (37a) will fail to yield the right result...

Basically, because the truth-conditions in (37a) predict that the modal restriction in context (36a) is the *null set*, those truth-conditions also predict that all sorts of (intuitively false) sentences are true in that context:

- We must eat chocolate with our feet.
- We must give ourselves up to the police...etc.

(38) **Fact 2: The Success of the Truth-Conditions in (32) / (33)**

Now consider the truth-conditions that our analysis in (32) would predict for (36b):

- a. “We have to plan our escape to the Caymans” is T in w_0 iff
 $\forall w' \in \text{MAX}_{<\{p : p \in \text{our goals in } w\}} (\cap \{p : p \text{ is true in } w \text{ (up to the present)}\})$:
we plan our escape in w'

According to this analysis, the modal quantifies over the following restriction:

- b. Those worlds from the set $\{w' : w' \text{ is just like } w \text{ (up to the present)}\}$ which satisfy *the most* propositions from the set ‘our goals in w' ’

Clearly, all such worlds (in the restriction) will satisfy the following propositions:

- Jim can’t find a way to raise \$46 million dollars.
(Because Jim has just died in w , any world in the restriction will necessarily be a world where Jim has just died.)
- If Jim can’t find a way to raise \$46 million, we plan our escape to the Caymans.
(Nothing that has happened in the real world w prevents this goal from being true.)

Thus, (by modus ponens), all worlds in the restriction satisfy the following proposition:

- We plan our escape to the Caymans.

Therefore: *Under the truth-conditions derived in (32), sentence (36b) is correctly predicted to be (non-trivially) true in context (36a)!*

(39) **Conclusion (Answer, Part 2)**

The ‘bouletic’ truth-conditions predicted by our account in (32) make the right predictions in cases where the actual world prevents us from satisfying all of ‘our goals’.

So: Our account in (32) provides a (more accurate) semantics for the ‘bouletic reading’!

(40) **One Final Point**

The account in (32) avoids the problem from Section 2.1

- Under this analysis, a bouletic modal doesn’t simply quantify over the set of worlds where our goals are *met*.
- Rather, the worlds it quantifies over are *also* required to *match* the facts of the actual world (up to the present).

5. The Epistemic Reading

(41) Epistemic Truth-Conditions (from (6a))

“John must be in NYC” is true in a world w *iff*
 $\forall w' \in W$: everything we know about w is also true in w' and nothing that is
 ‘abnormal’ in w occurs in w' \rightarrow John is in NYC in w' .

Now, with (41) in the background as the ‘truth-conditions’ of the bouletic reading, *let’s just see what reading our system yields under the following conditions:*

- $[[\text{BASE}_1]]^{w,g} = g(1) = \lambda w. [\lambda p . p \text{ is known in } w]$
- $[[\text{ORD-SRC}_2]]^{w,g} = g(2) = \lambda w. [\lambda p . p \text{ is a ‘reasonable expectation’ in } w]$

Side-Note:

A proposition p is a ‘reasonable expectation’ in w *iff* $\neg p$ would be ‘abnormal’ in w .

If we follow a derivation akin to that in (32), we obtain the following truth conditions:

(42) Derived Truth Conditions

“John must be in NYC” is true in a world w *iff*
 $\forall w' \in \text{MAX}_{\langle \{p : p \text{ is a ‘r.e.’ in } w \} \cap \{p : p \text{ is known in } w \} \rangle}$: John is in NYC in w'

*For all worlds w' in the following set, John is in NYC in w'
 Those worlds, from the set of worlds where everything we know about w is true
 which satisfy the greatest number of our ‘reasonable expectations’ in w .*

(43) Crucial Question

Do the derived truth-conditions in (42) equate to the targeted ‘epistemic truth-conditions’ in (41)?

(44) Answer, Part 1

YES – as long as no known proposition is in conflict with our ‘reasonable expectations’

- Since nothing we know conflicts with our ‘reasonable expectations’, there are worlds w' within the set $\cap \{p : p \text{ is known in } w \}$ such that they satisfy *all* the propositions in $\{p : p \text{ is a ‘reasonable expectation’ in } w \}$.
- Thus (following earlier reasoning)

$$\text{MAX}_{\langle \{p : p \text{ is a ‘r.e.’ in } w \} \cap \{p : p \text{ is known in } w \} \rangle} =$$

$\{ w' : \text{everything we know in } w \text{ is true in } w' \text{ and nothing that is ‘abnormal’ in } w \text{ occurs in } w' \}$

OK...but (again), what happens when a known proposition is in conflict with one of our 'reasonable expectations'?...

(45) **A Situation Where our Knowledge Conflicts with Our Expectations**

a. Context:

(i) Our 'reasonable expectations' are the following.

- *John cannot fly.*

- *John's arms and neck cannot freely grow arbitrarily long.*
(*John is not 'Plasticman'*)

(ii) What we know is the following:

- John has just made a cell-phone call that originated in NYC.

- **John is, in fact, Plasticman. (We just got a call from the military.)**

b. True Sentence (Containing 'Epistemic Modal')

John **might** be in Hoboken.

(46) **Fact 1: The Failure of the ('Conjunctive') Truth-Conditions in (41)**

Consider the truth-conditions that the treatment in (41) would predict for (45b):

a. "John might be in Hoboken" is T in w iff

$\exists w' \in W$: **everything we know about w is also true in w' and nothing that is 'abnormal' in w occurs in w'** & John is in Hoboken in w'

However, note that – as in earlier examples – the restriction in (46a) is the *empty set*.

- Given that our knowledge of w (namely, that John is Plasticman) contradicts our reasonable expectations of w , there is no world where everything we know about w is true and nothing 'abnormal' happens.

Thus, the truth-conditions in (46a) actually predict that (45b) is *false* in context (45a)!

(47) **Fact 2: The Success of the Truth-Conditions in (42)**

Now consider the truth-conditions that our analysis in (42) would predict for (45b):

- a. “John might be in Hoboken” is true in a world w iff
 $\exists w' \in \text{MAX}_{\langle \{p : p \text{ is a 'r.e.' in } w\} \rangle} (\cap \{p : p \text{ is known in } w\})$:
John is in NYC in w'

According to that analysis, the modal quantifies over the following restriction:

- b. Those worlds from the set $\{w' : \text{everything we know in } w \text{ is true in } w'\}$ which satisfy *the most* propositions from the set ‘our reasonable expectations in w' ’.

Clearly, all such worlds (in the restriction) will satisfy the following propositions:

- John’s arms can freely grow arbitrarily long.
(Because we’ve just learned that w is a world where John is Plasticman).
- John cannot fly.
(Because it’s a reasonable expectation not contradicted by our knowledge of w .)
- John has just made a cell-phone call that originated in NYC.

Now, clearly there are also worlds that satisfy all three of the above propositions *and* where John is in Hoboken.

(Because in such worlds, ‘the rest’ of John is in Hoboken, while his head and arms have stretched over to NYC.)

Thus, there are worlds w' in the restriction of (47b) such that John is in Hoboken in w' .

Therefore: *Under the truth-conditions derived in (42), sentence (45b) is correctly predicted to be true in context (45a)!*

(48) **Conclusion (Answer, Part 2)**

The ‘epistemic’ truth-conditions predicted by our account in (42) make the right predictions in cases where our knowledge contradicts our ‘reasonable expectations’.

Our account in (42) actually provides a *better* semantics for the ‘epistemic reading’!

(49) **One Final Point**

The account in (42) avoids the problem from Section 2.1

The restriction is not just the worlds where what we know is true, but also those worlds that meet (the greatest possible number of) our reasonable expectations.

6. Circumstantial Reading

(50) Circumstantial Truth-Conditions (from (6b))

“John has to sneeze” is true in a world w *iff*
 $\forall w' \in W$: everything true in w (up to now) is true in w' and nothing that is
 ‘abnormal’ in w occurs in w' \rightarrow John sneezes in w' .

Now, with (50) in the background as the ‘truth-conditions’ of the bouletic reading, *let’s just see what reading our system yields under the following conditions:*

- $[[\text{BASE}_1]]$ ^{w,g} = $g(1)$ = $\lambda w. [\lambda p . p \text{ is true in } w \text{ (up to the present) }]$
- $[[\text{ORD-SRC}_2]]$ ^{w,g} = $g(2)$ = $\lambda w. [\lambda p . p \text{ is a ‘reasonable expectation’ in } w]$

If we follow a derivation akin to that in (32), we obtain the following truth conditions:

(51) Derived Truth Conditions

“John has to sneeze” is true in a world w *iff*
 $\forall w' \in \text{MAX}_{\langle \{ p : p \text{ is a ‘r.e.’ in } w \} \cap \{ p : p \text{ is true in } w \text{ (up to the present) } \} \rangle}$:
John sneezes in w'

For all worlds w' in the following set, John sneezes in w'
Those worlds – from the set of worlds that are just like w (up to the present) -
which satisfy the greatest number of our ‘reasonable expectations’ in w .

(52) Crucial Question

Do the derived truth-conditions in (52) equate to the targeted ‘circumstantial truth-conditions’ in (50)?

(53) Answer, Part 1

YES – as long as nothing that is true in w (up to the present) is in conflict with our ‘reasonable expectations’

- Since nothing that has happened ‘so far’ in w conflicts with our ‘reasonable expectations’, there are worlds w' within the set $\cap \{ p : p \text{ is true in } w \text{ (up to the present) } \}$ such that they satisfy *all* the propositions in $\{ p : p \text{ is a ‘reasonable expectation’ in } w \}$.
- Thus (following earlier reasoning)

$$\text{MAX}_{\langle \{ p : p \text{ is a ‘r.e.’ in } w \} \cap \{ p : p \text{ is true in } w \text{ (up to the present) } \} \rangle} =$$

$\{ w' : \text{everything true in } w \text{ (up to the present) is true in } w'$
and nothing that is ‘abnormal’ in w occurs in } w'

OK...but (again), what happens when ‘the circumstances’ in w are in conflict with one of our ‘reasonable expectations’ for w ?...

...Left as an exercise to the reader...

(54) **Conclusion**

- The truth-conditions derived by our account in (51) match the intuitive truth-conditions of sentences containing ‘circumstantial modals’.
- Thus, our augmented system is able to capture the ‘circumstantial reading’ of modals.

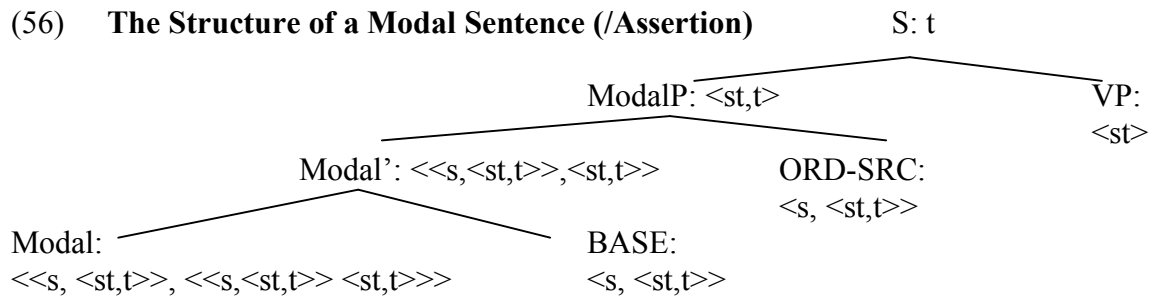
(55) **One Final Point**

The account in (51) avoids the problem from Section 2.1

The restriction is not just the worlds that match w up to the present, but also those worlds that meet (the greatest possible number of) our expectations.

7. **Summary Discussion**

In the system developed above, a ‘modal sentence’ has the following syntactic and logical form:



- a. For any given modal sentence, there are – in addition to the proposition provided by the VP – two further (possibly phonologically null) arguments to the modal.
 - (i) **BASE:** A function of type $\langle s, \langle st, t \rangle \rangle$, which ultimately provides a set of worlds.
 - (ii) **ORD-SRC:** A function of type $\langle s, \langle st, t \rangle \rangle$, which ultimately provides an ordering over the set of worlds provided by BASE. (An ‘ideal’ that the worlds in the BASE are ranked against)
- b. These two additional arguments – BASE and ORD-SRC – together yield the restricted set of possible worlds that the modal quantifies over:

Those worlds w' which:

- Of the worlds that satisfy all the propositions in BASE
- Come closest to the ‘ideal’ set by the ORD-SRC

Furthermore, in this system, each of the four principal types of modal reading can be derived via a particular combination of ‘modal base’ and ‘ordering source’.

(57) **The Four Principle Readings**

Epistemic

| | | |
|----------|-----------------|--|
| Base: | ‘Epistemic’ | $\lambda w. [\lambda p . p \text{ is known in } w]$ |
| Ord-Src: | ‘Stereotypical’ | $\lambda w. [\lambda p . p \text{ is a ‘reasonable expectation’ in } w]$ |

Circumstantial:

| | | |
|----------|------------------|---|
| Base: | ‘Circumstantial’ | $\lambda w. [\lambda p . p \text{ is true in } w \text{ (up to the present) }]$ |
| Ord-Src: | ‘Stereotypical’ | $\lambda w. [\lambda p . p \text{ is a ‘reasonable expectation’ in } w]$ |

Deontic:

| | | |
|----------|------------------|---|
| Base: | ‘Circumstantial’ | $\lambda w. [\lambda p . p \text{ is true in } w \text{ (up to the present) }]$ |
| Ord-Src: | ‘Deontic’ | $\lambda w. [\lambda p . p \text{ is a ‘the law (of X)’ in } w]$ |

Bouletic:

| | | |
|----------|------------------|---|
| Base: | ‘Circumstantial’ | $\lambda w. [\lambda p . p \text{ is true in } w \text{ (up to the present) }]$ |
| Ord-Src: | ‘Bouletic’ | $\lambda w. [\lambda p . p \text{ is a ‘a goal (of X)’ in } w]$ |

As we’ve seen, this extended system resolves all the ‘problems’ from Section 2.

- Kratzer (1991) also describes a number of other advantages that are enjoyed by a system where modals are ‘doubly parameterized (relativized)’ in this way...

(58) **Modal Sentences are Not Logically Stronger than Non-Modal Sentences**

Note that, intuitively, sentence (58a) does *not* entail (58b).

- John must be in NYC. (epistemic reading)
- John is in NYC.

PROBLEM:

The systems in ‘Part 1’ and ‘Part 2’ predict that (58a) *does* entail (58b).

- Under those analyses, (58a) is T in w *iff* **in all worlds where what we know about w is true**, John is in NYC.
- Clearly, **w itself** is a world where what we know about w is true.
- Thus, if (58a) is T at w , then (58b) must be true at w too.

SOLUTION:

The system developed here for epistemic modals (correctly) *doesn’t* make this prediction.

- Under this new analysis, the restriction isn’t *all* the worlds where what we know about w is true...
- Rather is only those worlds that *also* satisfy the greatest number of our expectations.
- *However, the actual world needn’t be a world that satisfies the greatest number of our expectations...*

However, there does remain (at least) *one* issue...

(59) **An Outstanding Puzzle / Problem**

a. Puzzling Fact:

Not every modal auxiliary allows for every reading:

- (i) have: epistemic, deontic, bouletic, circumstantial (all four readings)
- (ii) can: deontic, bouletic, circumstantial * epistemic
- (iii) may: epistemic, deontic, bouletic * circumstantial
- (iv) must: epistemic, deontic, bouletic * circumstantial

b. The General Challenge:

How does one limit the ‘kinds’ of bases / ordering sources that particular modals can take as argument?

In our current system, the grammar just sees them all as <s, <st,t>> functions!

c. A More Acute Challenge:

- (i) The fact that ‘can’ only allows the deontic, bouletic and circumstantial readings can, in the current system, be captured via the following elegant generalization:

The base for modal ‘can’ must be ‘circumstantial’.

- (ii) However, the fact that ‘may/must’ allow only the deontic, bouletic and *epistemic* readings cannot be captured via a similarly elegant generalization.

- Clearly the base for ‘may/must’ can be circumstantial, since it allows deontic and bouletic readings.
- Clearly, the ordering-course for ‘may/must’ can be ‘stereotypical’, since it allows the epistemic reading.
- **However, ‘may/must’ cannot take at the *same time* a ‘circumstantial’ base and a ‘stereotypical’ ordering source!** (Otherwise, it would allow the ‘circumstantial’ reading...)

Is there an answer?...