

The Semantics of Modals, Part 2: The Modal Base¹

1. On Our Last Episode...

(1) The Failure of a ‘Lexical Ambiguity’ Account

Postulating separate lexical entries for *all* the different ‘readings’ of a modal misses a glaring generalization about their meaning:

- a. The General Meaning of a Weak / Existential Modal (*Can / May*)
 $\lambda p_{\langle st \rangle} . \exists w' \in \{ w' \in W : R(w, w') \} : p(w') = T$
- b. The General Meaning of a Strong / Universal Modal (*Must / Have-to*)
 $\lambda p_{\langle st \rangle} . \forall w' \in \{ w' \in W : R(w, w') \} : p(w') = T$

We obtain the different ‘readings’ of a modal by simply varying the set of worlds that the quantification is restricted to:

c. The Different ‘Readings’ of the Modals

Epistemic: $\{ w' \in W : \text{everything we know about } w \text{ is also true in } w' \}$

Circumstantial: $\{ w' \in W : \text{everything true in } w \text{ (up to now) is true in } w' \}$

Deontic: $\{ w' \in W : \text{the law in } w \text{ is being followed in } w' \}$

Bouletic: $\{ w' \in W : \text{our goals in } w \text{ are met in } w' \}$

(2) New, Overarching Analytic Goal

Let’s develop an explicit compositional semantics for modals, where their various ‘readings’ can be derived via the semantic composition of

- (i) the invariant meaning of the modal, with
- (ii) ‘other material’ in the sentence (that contributes the restricted set of worlds that the modal quantifies over)

Our work towards the goal in (2) will be guided, in part, by a certain additional problems faced by the ‘lexical ambiguity’ account we are looking to replace.

¹ These notes are based upon material in von Stechow (2007; Chapter 3), Kratzer (1977, 2008), and Kratzer (1991).

2. Some Additional Problems for the ‘Lexical Ambiguity’ Account

2.1 The Existence of ‘Neutral’ Readings of Modals (Kratzer 1977, 2008)

Consider the following sentences, each of which contains a modal.

(3) Neutral Readings of Modals

- a. In view of the evidence, John **must** be in NYC.
- b. In view of what the law states, John **may** vote.
- c. In view of our goals for the meeting, John **can** leave.
- d. In view of his physical state, John **must** sneeze.

(4) Question

In the sentences above, which of the readings from ‘Part 1’ (*epistemic, deontic, bouletic, circumstantial*) are modals in (3) receiving?

(5) First Answer

Well, in (3a), the modal is *epistemic*...in (3b), the modal is *deontic*...in (3c), the modal is *bouletic*...in (3d), the modal is *circumstantial*! ...right?

(6) Problem with the ‘First Answer’ in (5)

If the meanings of the modals in (3) are as stated in (5), then *what does the adverbial contribute?*

- For example, if the modal in (3a) really were receiving an epistemic reading, then that would seem to make the adverbial ‘in view of the evidence’ *redundant*.

(7) Another Answer to the Question in (4)

- a. In each of the sentences in (3), the modal receives a ‘*neutral reading*’, where the set of worlds that it quantifies over is not lexically specified.
- b. What the ‘in view of...’ phrase contributes is *that set of worlds that the modal quantification is restricted to*.

(8) The Issue that This Raises for a ‘Lexical Ambiguity’ Account

- a. Besides all the other readings, we must also admit this ‘neutral reading’...
- b. **And isn’t it ‘funny’ that this neutral reading can – in combination with other phrasal material – yield *all the other readings observed for the modal!*...**

(9) **A Better Picture that Emerges from the Sentences in (3)**

- a. Instead of all the other lexical entries for the modal, *let's suppose that the modal only has the neutral reading observed in (3).*
- b. Let's suppose that the sentences in (3) overtly reveal the logical structure of *all* modal statements, *even ones without overt 'in view of...' phrases.*
- c. That is, the picture that the sentences in (3) move us towards is the following:
 - (i) The meaning of a modal like 'must' isn't a simple $\langle st, t \rangle$ function, of the kind postulated in 'Part 1'.
 - (ii) Rather, it is a function which:
 - first, takes the meaning of the 'in view of...' phrase as argument
 - then, returns an $\langle st, t \rangle$ function (generally, one similar to one of the 'lexical entries' postulated in Part 1).
 - (iii) In sentences where there does not appear to be an 'in view of...' phrase overtly on the surface, *context alone* supplies a value for this argument.

(10) **A More Concrete Sketch of the Picture in (9) of 'Modal Sentences'**

Modal	Restriction	Scope
Weak (<i>Can / Might</i>) Strong (<i>Must / Have to</i>)	Overt (<i>In view of the law...</i>) Covert / Phonologically Empty/ Supplied by Context	VP (<i>Dave vote</i>)

- A modal possesses *only* a 'neutral reading', where the set of worlds that it quantifies over (its *restriction*) is not lexically specified.
- The restriction of the modal is supplied by other phrasal material in the sentence.
 - This material can be overt, such as an 'in view of...' phrase.
 - This material can also be *covert* (*phonologically empty*)
- Having combined semantically with its *restriction*, the modal then combines with material providing its *scope*, typically the VP of the sentence.

An Immediate Question:

How *exactly* do we represent the meaning of the modal's 'restriction'?...

How *exactly* does such a restriction get 'provided by context alone'?...

...*We might arrive at an answer to these questions by considering another serious problem for a 'lexical ambiguity account'...*

2.2 The Infinitude of Modal ‘Readings’ (Kratzer 1977, 2008)

In the system of lexical entries from ‘Part 1’, we treat the so-called ‘deontic reading’ as if it were a *single, specific* reading of the modal.

(11) Putative Lexical Entry for ‘Deontic Modals’

$$[[\text{may / can}]]^w = \lambda p_{\langle st \rangle} \exists w' \in W: \text{‘the law’ in } w \text{ is being followed in } w' \ \& \ p(w') = T$$
$$[[\text{must / have to}]]^w = \lambda p_{\langle st \rangle} . \forall w' \in W: \text{‘the law’ in } w \text{ is being followed in } w' \rightarrow p(w') = T$$

Such an approach, however, is clearly inadequate, *since there is never a single set of laws in a given world.*

For example, consider sentence (12a), which appears to admit of the both the readings in (12b).

(12) Multiple Deontic Readings

- a. Sentence:
Grad students **must** register for LING610 in their first semester.
- b. Readings:
- (i) True Reading:
Given the laws/rules of the linguistics PhD program, grad students must register for LING610 in their first semester.
- (ii) False Reading:
Given the laws/rules of the state of Massachusetts, grad students must register for LING610 in their first semester.

(13) Crucial Point of (12)

The readings in (12b) are clearly two separate readings, as one is (intuitively) *true*, while the other is (intuitively) *false*.

However, both the readings in (12b) also seem to qualify as ‘deontic readings’, since both are concerned with what a given set of laws/rules requires!

(14) General Conclusion

The so-called ‘deontic reading’ is not a single reading, but an *infinite set* of readings!

- For any set of rules/laws you can imagine, a modal possesses a ‘deontic reading’ that makes reference to that set of rules/laws.

Does the 'general conclusion' in (14) effectively demolish a 'lexical ambiguity account'?

- Well... It's true that we can't very well assume that there is an *infinite* number of lexical entries, one for each of the deontic readings...
- But, suppose we take up the following (conservative) view...

(15) **Towards a Theory of the 'Context Dependency' of Modals**

There is a kind of 'context-dependency' at play in the so-called 'deontic reading'.

Intuitively, this is connected with the very 'context-dependency' of the phrase *the law* that appears within the lexical entries in (11).

Therefore:

- (i) Let's suppose that the semantics of a 'deontic modal' does not itself inherently specify the set of laws/rules that it makes reference to...
- (ii) Instead, we might suppose instead that the particular set of laws/rules is set by the *context* somehow...

What is the point of this digression?...

(16) **A Project / Thought-Experiment**

- Let's suppose, for argument's sake, that the idea in (15) is correct. That is, let us try to:
 - (i) retain a separate, specific treatment of the 'deontic reading', but
 - (ii) alter our semantic analysis so that 'the context' is able to supply the exact set of laws/rules that it makes reference to...
- By doing so, we'll see how to capture, via a single semantic analysis of the 'deontic reading', the full potential *infinity* of different, specific 'deontic readings'.
- *Why would we want to do that?*
As we'll see, the insights gained from this little 'thought experiment' will *immediately generalize to modals as a whole!*

Once it's clear how we capture the full set of 'deontic readings', it will also be clear how we can dispense with the special lexical entry for the 'deontic readings' as a whole...

... and then it will also be clear how we can dispense with *all* the special lexical entries from 'Part 1', and replace them with a single 'context dependent' semantics for modals!

3. Towards a ‘Context Dependent’ Semantics for Deontic Modals

Let us now construct step-by-step a ‘context dependent’ semantics for ‘deontic modals’. We begin with a characterization of the two readings we wish to derive:

(17) Our Targeted Truth-Conditions

a. Sentence:
Grad students **must** register for LING610 in their first semester.

b. Truth Conditions:

(i) *True ‘Rules-of-Grad-Program’ Reading*

$\forall w' \in \{ w' : \text{the law/rules of the UMass Linguistics program in } w_0 \text{ is being followed in } w' \}$:
grad students register for LING610 in their first semester in w'

(ii) *False ‘Laws-of-Massachusetts’ Reading*

$\forall w' \in \{ w' : \text{the law/rules of the state of Massachusetts in } w_0 \text{ is being followed in } w' \}$:
grad students register for LING610 in their first semester in w'

(18) Some Starting Observations

a. What is crucially different between these two readings is the underlined material. (*i.e.*, the set of laws being considered)

b. So, let us attempt to separate out this content from the remainder of the deontic modal...
That is, let's see if we can't analyze a deontic modal as being a function that takes as argument a set of laws.

3.1 The Postulation of a Special Null Category (‘LAW’)

First, let us assume that there exists a phonologically null pronoun ‘LAW’.

Given our background assumptions regarding pronouns (see handouts on ‘Extensional Semantics’), the following holds:

(19) The Syntax and Semantics of ‘LAW’, Part 1

a. The pronoun LAW bears an index i .

b. The interpretation of LAW_i is provided by the assignment function g as follows:
 $[[LAW_i]]^{w,g} = g(i)$

... But, what kind of 'thing' does 'LAW_i' have as its semantic value?

3.2 The Semantics of the Null Pronoun 'LAW'

Let us digress and reflect upon the English expression "the law". The following seems to be an accurate claim regarding the content of this expression:

(20) The Semantics of "The Law"

At a particular world (w_0), in a particular *context* at that world, the phrase "the law" refers to a *set of propositions*.

- a. "the law" = "the law/rules of the UMass linguistics program in w_0 " =
 $\{ \{ w : \text{students file both their GPs by the end of their third year in } w \},$
 $\{ w : \text{students register for 610 in their second semester in } w \}, \dots \}$
- b. "the law" = "the law of the state of Massachusetts in w_0 " =
 $\{ \{ w : \text{there is no greyhound racing in } w \},$
 $\{ w : \text{no one below the age of 17 has a driver's license in } w \},$
 $\{ w : \text{liquor is not sold on Christmas in } w \}, \dots \}$

(21) Crucial Observation

Suppose you take one of these sets of propositions in (20) (constituting 'the law') and you intersect together all the propositions it contains. What do you get?

Well... thinking of 'propositions' as sets of worlds, you get:

- Those worlds that are members of each of the propositions in 'the law'.
- Thus, those worlds where each of the propositions in 'the law' is true.
- Thus, (intuitively) **those worlds where all of 'the laws' are followed.**

Illustration: Intersection of the Set in (20a)

$$\begin{aligned} \cap \{ \{ w : \text{students file both their GPs by the end of their third year in } w \}, \\ \{ w : \text{students register for 610 in their second semester in } w \}, \dots \} &= \\ \{ w : w \in \{ w' : \text{students file both their GPs by the end of their third year in } w' \} \& \\ w \in \{ w' : \text{students register for 610 in their second semester in } w' \} \& \dots \} &= \\ \{ w : \text{students file both their GPs by the end of their third year in } w \& \\ \text{students register for 610 in their second semester in } w \& \dots \} &= \\ \{ w : \text{the law/rules of the UMass Linguistics program in } w_0 \text{ is being followed in } w \} \end{aligned}$$

(23) **The Key Conclusion**

From the sets of propositions in (20) – which we could intuitively think of as ‘the law’ in a given context – we can *construct* the restricted set of worlds that our truth-conditions in (17) make reference to!

- Take the value of ‘the law’ in a context C (construed as a set of propositions)
- Intersect all the proposition in ‘the law’ at C together
- What you obtain is *the set of worlds where ‘the law’ in C is being followed.*

Thus, we might be able to build a compositional semantics for deontic modals from the notion that ‘the law’ at a given context is a set of propositions!

(24) **The Syntax and Semantics of ‘LAW’, Part 2**

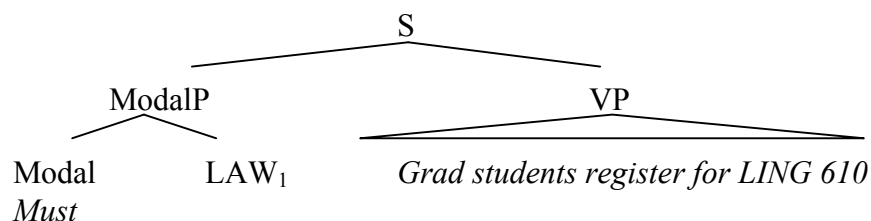
- a. The pronoun ‘LAW_i’ is of type <s, <st,t>>.
- b. That is, given any variable assignment g, the value of g taking the index of ‘LAW’ as an argument is **a function from worlds to sets of propositions**
- c. *Examples:*
- (i) $[[\text{LAW}_1]]^w, g = g(1) =$
[λw. the laws/rules of the UMass linguistics program in w]
- (ii) $[[\text{LAW}_2]]^w, g = g(2) =$
[λw. the laws/rules of the state of Massachusetts in w]

We can now write a lexical entry for the deontic modals whereby they take as argument the value of ‘LAW_i’ and the intension of the VP to yield the targeted truth conditions in (17)!

3.3 A Compositional, ‘Context-Dependent’ Semantics for Deontic Modals

(25) **Syntactic Assumptions**

Let’s assume that the (deontic) modals *may/must* take the null pronoun ‘LAW’ as a complement. The resulting phrase *then* combines syntactically with the VP.



The most crucial step towards our new system, though, is the introduction of the following new lexical entries for “may” and “must”:

(26) **New Semantics for (Deontic) Modals**

- a. $[[\text{may / can}]]^w = \lambda B_{\langle s, \langle st, t \rangle \rangle} . \lambda p. \exists w' \in \cap B(w): p(w') = T$
- b. $[[\text{must / have-to}]]^w = \lambda B_{\langle s, \langle st, t \rangle \rangle} . \lambda p. \forall w' \in \cap B(w): p(w') = T$

How do these lexical entries (in combination with our other assumptions) yield the desired ‘deontic’ truth conditions?

(27) **How This System Works**

- Given our semantics for ‘LAW’ in (24), its value at any world w and variable assignment g will be a function from worlds to sets of propositions
- Given our lexical entries in (26) and our syntactic assumptions in (25), the meaning of the (deontic) modal takes as its first argument the type $\langle s, \langle st, t \rangle \rangle$ function provided by ‘LAW’
- Given our lexical entries in (26), the following then ‘takes place’ to the function contributed by ‘LAW’
 - (i) The function is fed the ‘evaluation world’ w as its argument, producing a set of propositions (constituting one of the many ‘bodies of law’ in w)
 - (ii) This set of propositions is then intersected together. Given our earlier discussion, this generalized intersection yields the set of worlds w' where ‘the relevant law’ in w is being followed.
- Given our lexical entries in (26), the meaning of the entire ‘ModalP’ then takes as argument the intension of the VP. The value which is returned depends upon the identity of the Modal head.
 - (i) If the modal is *may/can*, then the value is T *iff*:
 There is **some** world w' in the set of worlds where ‘the relevant law’ in w is being followed such that $[[VP]]_g(w') = 1$
 - (ii) If the modal is *must/have-to*, then the value is T *iff*:
 For **all** worlds w' in the set of worlds where ‘the relevant law’ in w is being followed : $[[VP]]_g(w') = 1$

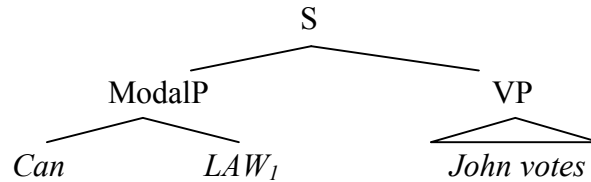
The following derivation illustrates more concretely how the proposed system functions:

(28) **Derivation of Deontic Truth Conditions**

- a. Sentence: John can vote.
- b. Deontic Paraphrase: *According to federal law*, John can vote.
- c. Targeted Truth-Conditions

“John can vote” is T in w *iff*
 $\exists w' \in \{ w'' : \text{the federal law in } w \text{ is being followed in } w'' \} : \text{John votes in } w'.$

- d. Assumed Syntax of (28a)



- e. Additional Semantic Assumption:

$g(1) = \lambda w. [\lambda p. p \in \text{Federal Law in } w]$
 ‘function from a world w to the set of propositions constituting *federal law in w*.’

- f. Derivation of Truth-Conditions in (28c)

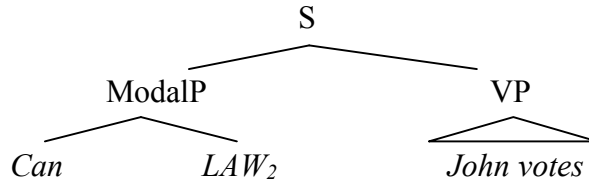
- i. $[[S]]^{w,g} = T$ *iff* (by IFA)
- ii. $[[\text{ModalP}]]^{w,g} ([[VP]]_c) = T$ *iff* (by ‘Part1’ HO)
- iii. $[[\text{ModalP}]]^{w,g} (\lambda w'. \text{John votes in } w') = T$ *iff* (by FA)
- iv. $[[[\text{can}]]^{w,g} ([[LAW_1]]^{w,g})](\lambda w'. \text{John votes in } w') = T$ *iff* (by (19b))
- v. $[[[\text{can}]]^{w,g} (g(1))](\lambda w'. \text{John votes in } w') = T$ *iff* (by (28e))
- vi. $[[[\text{can}]]^{w,g} (\lambda w''. \lambda p. p \in \text{Federal Law in } w'')](\lambda w'. \text{John votes in } w') = T$ *iff* (by Lex.)
- vii. $[[\lambda B_{\langle s, \langle st, t \rangle \rangle}. \lambda p. \exists w''' \in \cap B(w): p(w''') = T](\lambda w''. \lambda p. p \in \text{Federal Law in } w'')](\lambda w'. \text{John votes in } w') = T$ *iff* (by LC)
- viii. $[\lambda p. \exists w''' \in \cap [\text{Federal Law in } w]: p(w''') = T](\lambda w'. \text{John votes in } w') = T$ *iff* (by LC)
- ix. $\exists w''' \in \cap [\text{Federal Law in } w]: \text{John votes in } w'''$ *iff* (by General Reasoning)
- x. $\exists w''' \in \{ w'' : \text{the federal law in } w \text{ is being followed in } w'' \} : \text{John votes in } w'''$

Importantly, the overall system above is able to predict ‘deontic’ readings for (28a) *other than* the specific ones listed in (28b,c).

As illustrated below, the way that all these other deontic readings are generated is *simply by varying the index on the null pronoun ‘LAW’!*

(29) **Derivation of Deontic Truth Conditions**

- a. Sentence: John can vote.
- b. Deontic Paraphrase: *According to state law*, John can vote.
- c. Targeted Truth-Conditions
 $\exists w' \in \{ w'' : \text{the state law in } w \text{ is being followed in } w'' \} : \text{John votes in } w'$
- d. Assumed Syntax of (29a)



- e. Additional Semantic Assumption:

g(2) = $\lambda w. [\lambda p . p \in \text{State Law in } w]$
 ‘function from a world w to the set of propositions constituting *state law in } w*.’

f. Derivation of Truth-Conditions in (29c)

- i. $[[S]]^{w,g} = T$ *iff* (by IFA)
- ii. $[[\text{ModalP}]]^{w,g} ([[VP]]_e) = T$ *iff* (by ‘Part1’ HO)
- iii. $[[\text{ModalP}]]^{w,g} (\lambda w'. \text{John votes in } w') = T$ *iff* (by FA)
- iv. $[[[\text{can}]]^{w,g} ([[LAW_2]]^{w,g})] (\lambda w'. \text{John votes in } w') = T$ *iff* (by (19b), (29e))
- v. $[[[\text{can}]]^{w,g} (\lambda w''. \lambda p. p \in \text{State Law in } w'')] (\lambda w'. \text{John votes in } w') = T$ *iff* (by Lex.)
- vi. $[[\lambda B_{\langle s, t \rangle} . \lambda p. \exists w''' \in \cap B(w) : p(w''') = T] (\lambda w''. \lambda p. p \in \text{State Law in } w'')]$
 $(\lambda w'. \text{John votes in } w') = T$ *iff* (by LC)
- vii. $\exists w''' \in \cap [\text{State Law in } w] : \text{John votes in } w'''$ *iff* (by General Reasoning)
- viii. $\exists w''' \in \{ w'' : \text{the state law in } w \text{ is being followed in } w'' \} : \text{John votes in } w'''$

3.4 Discussion: The Value of This Exercise

So, as demonstrated in (28) and (29) our semantic analysis of ‘deontic modals’ is able to accomplish the following desiderata:

- It is able to capture the ‘context dependency’ of the deontic reading.
- It is able to capture the potentially infinite number of different ‘deontic readings’

Crucially, it is able to accomplish these properties via the postulation of the pronoun ‘LAW_i’

- Since it’s a pronoun, the value of ‘LAW’ is set by the context (*i.e.* the assignment function g).
- Since it is a pronoun, the value of ‘LAW’ is permitted to be any type $\langle s, \langle st, t \rangle \rangle$ function.
 - Thus, for any set of ‘laws/rules’ you care to imagine, there is an $\langle s, \langle st, t \rangle \rangle$ function that can serve as the value of ‘LAW’ (and which will ultimately yield a ‘deontic reading’ that makes reference to that set of ‘laws/rules’)

But, in addition to its meeting the desiderata that we began with, there is one other, especially notable property of this semantic system:

(30) A Significant Achievement of this Analysis

Under this analysis of the ‘deontic reading(s)’, *there is nothing inherently ‘deontic’ in the meaning of the modal head itself*:

- a. $[[\text{may / can}]]^w = \lambda B_{\langle s, \langle st, t \rangle \rangle} . \lambda p. \exists w' \in \cap B(w): p(w') = T$
 b. $[[\text{must / have-to}]]^w = \lambda B_{\langle s, \langle st, t \rangle \rangle} . \lambda p. \forall w' \in \cap B(w): p(w') = T$

Rather, the specific (deontic) reading that the sentence possesses is determined entirely by the $\langle s, \langle st, t \rangle \rangle$ function that the modal takes as argument.

Thus, the analysis above of the ‘deontic reading’ adheres very closely to the ‘overarching analytic goal’ in (2), repeated below:

(31) Overarching Analytic Goal

Let’s develop a semantics for modals, where their various ‘readings’ are derived via the semantic composition of

- a. the invariant meaning of the modal, with
- b. other (possibly covert) material in the sentence (which contributes the restricted set of worlds that the modal quantifies over)

Finally, recall the questions that were earlier raised regarding our ‘general picture’ of the semantics of modals in (9) and (10):

(32) Questions Regarding the Semantic Picture in (9) and (10)

- a. How *exactly* do we represent the meaning of the modal’s ‘restriction’?
- b. How *exactly* does such a restriction get ‘provided by context alone’?

The analysis of ‘deontic readings’ above provides some potential answers to these questions.

(33) Answers to the Questions in (32)

- a. The ‘restriction’ of the modal can be thought of as a function of type $\langle s, \langle st, t \rangle \rangle$ (from worlds to sets of propositions)
- b. The context is able to ‘provide’ a restriction via the existence of a (phonologically null) *pronoun*, which has functions of type $\langle s, \langle st, t \rangle \rangle$ as its values! [That is, the context determines a particular value for this pronoun.]

It becomes quite apparent, then, that our treatment of ‘deontic modals’ above should pave the way to a more general treatment of the semantics of modals...

... in what follows, we will generalize the ‘core insights’ underlying our analysis in (24) – (27) to each of the other three ‘principal readings’ from ‘Part I’.

A Terminological Aside:

(34) The ‘Modal Base’

- a. The $\langle s, \langle st, t \rangle \rangle$ function B which the modal head takes as its first argument. (*i.e.*, the variable ‘B’ in the lexical entries in (30)) (*cf.* Kratzer 1977, 1991, 2008)
- b. The set of propositions that the function B yields when fed the evaluation world as its argument (*cf.* von Stechow 2007, *et multia alia*) (*e.g.*, for deontic modals, the propositions constituting ‘the relevant body of law’.)
- c. The set of worlds which is obtained by intersecting together the set of propositions B(w) (*cf.* von Stechow 2007, *et multia alia*) (*e.g.* for deontic modals, the set of worlds w’ such that ‘the relevant body of law at w’ is being obeyed in w’.)

(35) The Conversational Background

For now, this is identical to the ‘Modal Base’ (though it won’t be later...)

4. Generalizing Our Treatment of Deontic Modals

Our goal now is to take what we've learned from our treatment of 'deontic modals' above, and apply it to the other 'principal readings' from 'Part 1'.

Since the so-called 'bouletic reading' is so similar to the 'deontic reading', we'll begin with that.

4.1 The Bouletic Reading

To begin, it's useful to note that – as with the 'deontic reading' – the so-called 'bouletic reading' is actually *an infinite number of distinct readings*.

(36) Multiple Deontic Readings

- a. Sentence:
John **must** leave.
- b. Context:
We need John to stay for the meeting. John's wife needs him to come home.
- c. Readings:
 - (i) True Reading:
Given John's wife's goals, John must leave.
 $\forall w' \in \{ w'' : \text{John's wife's goals in } w \text{ are met in } w'' \} : \text{John leaves in } w'.$
 - (ii) False Reading:
Given our goals for the meeting, John must leave.
 $\forall w' \in \{ w'' : \text{our goals in } w \text{ are met in } w'' \} : \text{John leaves in } w'.$

Thus, let's directly import our treatment of 'deontic modals' to the analysis of 'bouletic modals.' That is, let's make the following additional assumption:

(37) The Syntax and Semantics of 'GOALS'

- a. There exists a phonologically empty pronoun, GOALS_i
- b. $[[\text{GOALS}_i]]^w, g = g(i)$
- c. As with the pronoun 'LAW', the pronoun 'GOALS_i' is of type $\langle s, \langle st, t \rangle \rangle$.
- d. *Examples:*
 - (i) $[[\text{GOALS}_1]]^w, g = g(1) =$
 $[\lambda w. \text{the goals of the John's wife in } w]$
 - (ii) $[[\text{GOALS}_2]]^w, g = g(2) =$
 $[\lambda w. \text{the goals of our meeting in } w]$

(38) **On ‘Goals’ as Sets of Propositions**

Just as with ‘laws’, it’s intuitively natural to think of ‘goals’ as sets of propositions:

a. Example:

$$\begin{aligned} \text{‘the goals of our meeting in } w_0\text{’} &= \\ \{ & \{ w : \text{our budget is finalized in } w \}, \\ & \{ w : \text{the date for the summer BBQ is set in } w \}, \dots \} \end{aligned}$$

b. Crucial Observation:

Suppose you take one of these sets of propositions (constituting ‘some set of *goals*’) and you intersect together all the propositions it contains. What results?

Well... thinking of ‘propositions’ as sets of worlds, you get:

- Those worlds that are members of each of the propositions in ‘the goals’.
- Thus, those worlds where each of the propositions in ‘the goals’ is true.
- Thus, (intuitively) **those worlds where all of ‘the goals’ are met.**

c. Illustration: Intersection of the Set in (38a)

$$\begin{aligned} \cap \{ & \{ w : \text{our budget is finalized in } w \}, \\ & \{ w : \text{the date for the summer BBQ is set in } w \}, \dots \} &= \\ \{ w : & w \in \{ w' : \text{our budget is finalized in } w' \} \ \& \\ & w \in \{ w' : \text{the date for the summer BBQ is set in } w' \} \ \& \dots \} &= \\ \{ w : & \text{our budget is finalized in } w \ \& \\ & \text{the date for the summer BBQ is set in } w \ \& \dots \} &= \\ \{ w : & \text{the goals of our meeting in } w_0 \text{ are met in } w \} \end{aligned}$$

(39) **The Key Conclusion (Again)**

From the sets of propositions in (38a) – which we could intuitively think of as ‘the goals’ in a given context – we can *construct* the restricted set of worlds that our truth-conditions in (36c) make reference to!

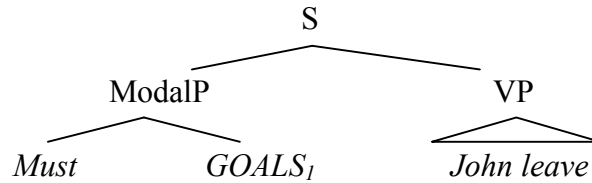
- Take the value of ‘the goals’ in a context C (construed as a set of propositions)
- Intersect all the proposition in ‘the goals’ at C together
- What you obtain is *the set of worlds where ‘the goals’ in C are met!*

Thus, we might be able to build a compositional semantics for bouletic modals from the notion that ‘the goals’ at a given context is a set of propositions!

In fact, we already have all the ingredients for capturing the so-called ‘bouletic readings’ in precisely this way!

(40) **Derivation of Bouletic Truth Conditions**

- a. Sentence: John must leave.
- b. Bouletic Paraphrase: Given *John’s wife’s goals*, John must leave.
- c. Targeted Truth-Conditions
 “John must leave” is T in w iff
 $\forall w' \in \{w'' : \text{John's wife's goals in } w \text{ are met in } w''\} : \text{John leaves in } w'$.
- d. Assumed Syntax of (28a)



- e. Additional Semantic Assumption:

$g(1) = \lambda w. [\lambda p. p \in \text{John's wife's goals in } w]$
 ‘function from w to the set of propositions constituting *John’s wife’s goals in } w*.’

f. Derivation of Truth-Conditions in (40c)

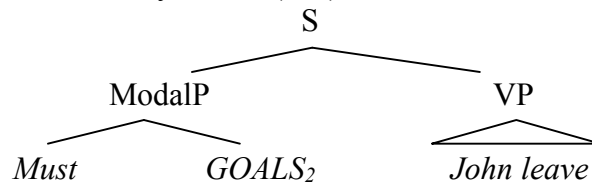
- i. $[[S]]^{w,g} = T$ iff (by IFA)
- ii. $[[\text{ModalP}]]^{w,g} ([[VP]]_c) = T$ iff (by ‘Part1’ HO)
- iii. $[[\text{ModalP}]]^{w,g} (\lambda w'. \text{John leaves in } w') = T$ iff (by FA)
- iv. $[[[\text{must}]]^{w,g} ([[GOALS_1]]^{w,g})] (\lambda w'. \text{John leaves in } w') = T$ iff (by (37))
- v. $[[[\text{must}]]^{w,g} (\lambda w'' . \lambda p. p \in \text{John's wife's goals in } w'')]$
 $(\lambda w'. \text{John leaves in } w') = T$ iff (by Lex.)
- vi. $[[\lambda B_{<s,t>} . \lambda p. \forall w''' \in \cap B(w) : p(w''') = T]$
 $(\lambda w'' . \lambda p. p \in \text{John's wife's goals in } w'')] (\lambda w'. \text{John leaves in } w') = T$ iff (by LC)
- vii. $\forall w''' \in \cap [\text{John's wife's goals in } w] : \text{John leaves in } w'''$ iff (by General Reasoning)
- viii. $\forall w''' \in \{w'' : \text{John's wife's goals in } w \text{ are met in } w''\} : \text{John leaves in } w'''$

The derivation below shows that our system *also* predicts that (40a) can have the truth-conditions in (36cii):

(40) **Derivation of Bouletic Truth Conditions**

- a. Sentence: John must leave.
- b. Bouletic Paraphrase: *Given our goals for the meeting, John must leave.*
- c. Targeted Truth-Conditions: “John must leave” is T in w *iff*
 $\forall w' \in \{ w'' : \text{our goals in } w \text{ are met in } w'' \} : \text{John leaves in } w'.$

d. Assumed Syntax of (28a)



e. Additional Semantic Assumption:

$g(2) = \lambda w. [\lambda p. p \in \text{our goals in } w]$
‘function from w to the set of propositions constituting *our goals in } w*.’

f. Derivation of Truth-Conditions in (40c)

- i. $[[S]]^{w,g} = T$ *iff* (by IFA)
- ii. $[[\text{ModalP}]]^{w,g} ([[VP]]_c) = T$ *iff* (by ‘Part1’ HO)
- iii. $[[\text{ModalP}]]^{w,g} (\lambda w'. \text{John leaves in } w') = T$ *iff* (by FA)
- iv. $[[[\text{must}]]^{w,g} ([[GOALS_2]]^{w,g})] (\lambda w'. \text{John leaves in } w') = T$ *iff* (by (37))
- v. $[[[\text{must}]]^{w,g} (\lambda w''. \lambda p. p \in \text{our goals in } w'')]$
 $(\lambda w'. \text{John leaves in } w') = T$ *iff* (by Lex.)
- vi. $[[\lambda B_{\langle s, \langle st, t \rangle \rangle} . \lambda p. \forall w''' \in \cap B(w): p(w''') = T]$
 $(\lambda w''. \lambda p. p \in \text{our goals in } w'')] (\lambda w'. \text{John leaves in } w') = T$ *iff* (by LC)
- vii. $\forall w''' \in \cap [\text{our goals in } w] : \text{John leaves in } w'''$ *iff* (by General Reasoning)
- viii. $\forall w''' \in \{ w'' : \text{our goals in } w \text{ are met in } w'' \} : \text{John leaves in } w'''$

(41) **Achievements of this Analysis of ‘Bouletic Modals’**

- a. Captures the ‘context dependency’ of the bouletic reading.
- b. Captures the potentially infinite number of different ‘deontic readings’
- c. **Provides a semantics for the ‘bouletic reading’ that is in line with our ‘overarching analytic goal’ in (31).**

Following this analysis, a ‘bouletic reading’ can be obtained simply by combining:

- (i) the ‘invariant’ (*i.e.*, neutral) meaning of the modal auxiliary
- (ii) the ‘modal base’ provided by the null pronoun ‘GOALS’.

As long as the modal base in the sentence is a function from worlds to a set of ‘goals’, we obtain a ‘bouletic’ reading for the modal sentence.

4.1.1 Intermission: An Important Technical Improvement

In the system as it stands, we have two null pronouns:

- A pronoun ‘LAW’ (used to derive the ‘deontic readings’)
- A pronoun ‘GOALS’ (used to derive the ‘bouletic readings’)

However, there is very little in our system that actually distinguishes these two pronouns:

- Both these pronouns are of the same semantic type: $\langle s, \langle st, t \rangle \rangle$
- Though we’ve been speaking as if ‘LAW’ denotes functions that yield ‘laws’, while ‘GOALS’ denotes functions that yield ‘goals’, *there is actually nothing in our semantics that can enforce such a condition.*

So, let’s dispense with this artificial (silly) distinction between ‘LAW’ and ‘GOALS’...

(42) **The (Phonologically Empty) Pronoun ‘BASE’**

From now on, our system will postulate only a *single* phonologically empty pronoun of type $\langle s, \langle st, t \rangle \rangle$. This pronoun is ‘BASE_i’

There are no restrictions on the meaning of ‘BASE_i’ other than it be of type $\langle s, \langle st, t \rangle \rangle$. Thus, ‘BASE_i’ can take all the following functions as its value:

- a. $[\lambda w. \text{the laws/rules of the UMass linguistics program in } w]$
- b. $[\lambda w. \text{the laws/rules of the state of Massachusetts in } w]$
- c. $[\lambda w. \text{the goals of the John’s wife in } w]$
- d. $[\lambda w. \text{the goals of our meeting in } w]$

The introduction of the pronoun ‘BASE’ leads to the following picture of the alleged ‘deontic’-‘bouletic’ ambiguity in the modal system.

(43) **The Distinction Between So-Called ‘Bouletic’ and ‘Deontic’ Readings**

Whether a modal sentence receives a ‘deontic’ reading or a ‘bouletic’ reading, *it’s syntactic structure is exactly the same.*

Under *either* category of reading, there are the same two key components to the sentence:

- a. The semantically invariant (neutral) modal head, which provides the *quantificational force* of the modal statement.
- b. The phonologically empty pronoun ‘BASE’, which provides the ‘modal base’ of the modal statement.

Whether the modal sentence (as a whole) is interpreted as being ‘deontic’ or ‘bouletic’ is simply a matter of the value that ‘BASE’ takes in the context in question.

- c. If ‘BASE’ is a function from worlds to ‘goals’, then the reading is one we categorize as ‘bouletic’.
- d. If ‘BASE’ is a function from worlds to ‘laws’, then the reading is one we categorize as ‘deontic’.

Thus, in the proposed system, ***there is no grammatical distinction at all between so-called ‘bouletic’ and ‘deontic’ readings.***

Indeed, from the point of view offered by this analysis, the very distinction between so-called ‘bouletic’ and ‘deontic’ readings is rather artificial.

Consider:

- The pronoun ‘it’ in English is able to take both animate and inanimate entities as its value
- However, our theory of English grammar makes no significant distinction between these types of interpretation of ‘it’:

For example, we don’t claim that sentences containing ‘it’ have both an ‘animate’ and an ‘inanimate’ reading.

- Similarly, under the analysis offered above, *there is no interesting or significant difference between the so-called ‘bouletic’ and ‘deontic’ readings of modal sentences.*

These labels ‘bouletic’ and ‘deontic’ (and, as we’ll soon see, ‘epistemic’ and ‘circumstantial’) simply represent an ‘artificial taxonomy’ we employ to categorize different types of interpretations that modal sentences can have.

They do not necessarily correspond to ‘linguistically real’ categories...

4.2 The Epistemic Reading

Let us now consider how we might similarly capture the so-called ‘epistemic reading’ of modal sentences. The truth conditions we’re targeting are the following:

(44) The Targeted ‘Epistemic’ Truth Conditions

“John must be in NYC” is T in w_0 iff
 $\forall w' \in \{ w' \in W : \text{everything we know about } w_0 \text{ is also true in } w' \} : p(w') = T$

Interestingly, our introduction of the phonologically null pronoun ‘BASE’ already allows our system to predict such a reading.

After all, one of the values that ‘BASE’ can receive is the following $\langle s, \langle st, t \rangle \rangle$ function:

(45) The ‘Epistemic’ Base

$[\lambda w. \text{ the propositions } \textit{known} \text{ to be true in } w]$

(46) ‘Knowledge’ as Sets of Propositions

Just as with ‘laws’, it’s intuitively natural to think of ‘knowledge’ as sets of propositions:

a. Example:

‘the propositions known to be true in w_0 ’ =
 $\{ \{ w : \text{John has family in NYC in } w \}, \{ w : \text{John has placed a cell-phone call from NYC in } w \}, \dots \}$

b. Crucial Observation:

Suppose you take one of this set of propositions (constituting ‘our *knowledge*’) and you intersect together all the propositions it contains. What results?

Well... thinking of ‘propositions’ as sets of worlds, you get:

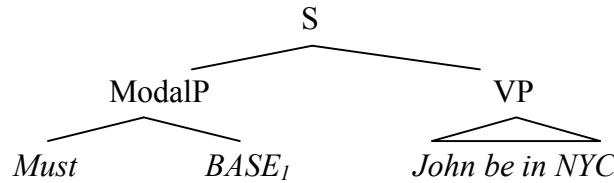
- The worlds that are members of each of the propositions in ‘our knowledge in w_0 ’
- Thus, those worlds where each of the propositions in ‘our knowledge in w_0 ’ is true.
- Thus, (intuitively) **those worlds w' such that everything that we know about w_0 is also true in w' !**

Thus, we find that the intersection of the propositions yielded by the function in (45) is again equivalent to the set of worlds quantified over in the ‘targeted truth conditions’ in (44).

(47) Derivation of ‘Epistemic’ Truth Conditions

- a. Sentence: John must be in NYC.
- b. Bouletic Paraphrase: Given *what we know*, John must be in NYC.
- c. Targeted Truth-Conditions:
“John must be in NYC” is T in w iff
 $\forall w' \in \{w'' : \text{what we know in } w \text{ is true in } w''\} : \text{John is in NYC in } w'.$

d. Assumed Syntax of (28a)



e. Additional Semantic Assumption:

$g(1) = [\lambda w. \text{the propositions known to be true in } w]$
‘function from w to the set of propositions known to be true in w .’

f. Derivation of Truth-Conditions in (47c)

- i. $[[S]]^{w,g} = T$ iff (by IFA)
- ii. $[[\text{ModalP}]]^{w,g} ([[VP]]_c) = T$ iff (by ‘Part1’ HO)
- iii. $[[\text{ModalP}]]^{w,g} (\lambda w'. \text{John is in NYC in } w') = T$ iff (by FA)
- iv. $[[[\text{must}]]^{w,g} ([[BASE_1]]^{w,g})] (\lambda w'. \text{John is in NYC in } w') = T$ iff (by (47e))
- v. $[[[\text{must}]]^{w,g} (\lambda w'' . \lambda p. p \in \text{the propositions known to be true in } w'')]$
 $(\lambda w'. \text{John is in NYC in } w') = T$ iff (by Lex.)
- vi. $[[\lambda B_{<s, <st, t>>}. \lambda p. \forall w''' \in \cap B(w): p(w''') = T]$
 $(\lambda w'' . \lambda p. p \in \text{the propositions known to be true in } w'')] (\lambda w'. \text{John is in NYC in } w') = T$
iff (by LC)
- vii. $\forall w''' \in \cap [\text{the propositions known to be true in } w] : \text{John is in NYC in } w'''$
iff (by General Reasoning)
- viii. $\forall w''' \in \{w'' : \text{what we know in } w \text{ is true in } w''\} : \text{John is in NYC in } w'''$

(48) **General Point**

Given that the null pronoun ‘BASE’ can take as its value the $\langle s, \langle st, t \rangle \rangle$ function in (45), the system we previously developed to capture the ‘context-dependency’ of the ‘deontic’ and ‘bouletic’ readings *automatically captures the existence of the ‘epistemic reading’!*

In a parallel way, the system also captures the existence of the so-called ‘circumstantial reading’:

4.3 The Circumstantial Reading

(49) **The Targeted ‘Epistemic’ Truth Conditions**

“John has to sneeze” is T in w_0 *iff*
 $\forall w' \in \{ w' \in W: \text{everything true in } w_0 \text{ (up to now) is true in } w' \} : \text{John sneezes in } w'$

Once again, our introduction of the phonologically null pronoun ‘BASE’ already allows our system to predict such a reading, as one value it can receive is the following $\langle s, \langle st, t \rangle \rangle$ function:

(50) **The ‘Circumstantial’ Base**

[λw . the ‘circumstances’ in w (up to the present)]

(46) **‘Circumstances’ as Sets of Propositions**

We can think of ‘circumstances’ as sets of propositions

a. Example:

‘the circumstances in w_0 (up to the present) =
 $\{ \{ w : \text{a piece of dust is in John’s nose in } w \}, \{ w : \text{John’s sneeze reflex is triggered in } w \}, \dots \}$

b. Crucial Observation:

Suppose you take one of this set of propositions (constituting ‘the *circumstances*’) and you intersect together all the propositions it contains. What results?

Following our earlier reasoning, we obtain:

Those worlds w' such that everything that is true in w_0 (up to the present) is also true in w' !

We can easily follow the approach laid out above to show that:

IF we allow the pronoun ‘BASE’ to have the value in (50)

THEN we derive the targeted ‘circumstantial’ truth-conditions in (49)

5. Achievements of – and Challenges for – the Analysis

We have clearly met our ‘overarching analytic goal’:

(47) Overarching Analytic Goal

A semantics for modals where the various ‘readings’ are derived via the semantic composition of

- a. the invariant meaning of the modal, with
- b. other (possibly covert) material in the sentence (which contributes the restricted set of worlds that the modal quantifies over)

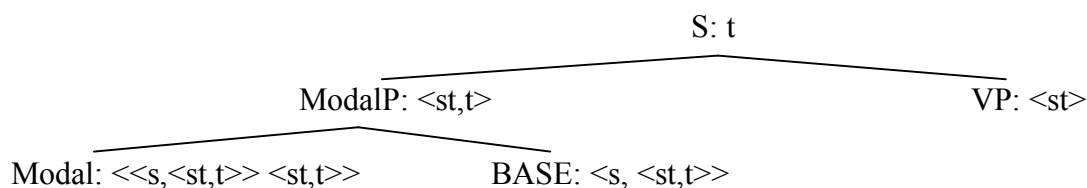
(48) The Invariant Meaning of the Modal Heads

- a. $[[\text{may / can}]]^w = \lambda B_{\langle s, \langle st, t \rangle \rangle} . \lambda p. \exists w' \in \cap B(w): p(w') = T$
- b. $[[\text{must / have-to}]]^w = \lambda B_{\langle s, \langle st, t \rangle \rangle} . \lambda p. \forall w' \in \cap B(w): p(w') = T$

(49) The Null Pronoun ‘BASE’: The Covert Material That Provides the ‘Restriction’

- a. ‘BASE’ is complement to the modal head, and provides its first argument.
- b. ‘BASE’ is a pronoun, and so its value is provided by ‘context’ (the function g)
- c. The contextually-determined value of ‘BASE’ is a function of type $\langle s, \langle st, t \rangle \rangle$.
 - When fed the evaluation world as argument, it yields a set of propositions.
 - When these propositions are intersected together, it yields a set of worlds.
 - **This resulting set of worlds provides the ‘restricted’ set of worlds that the modal ‘quantifies over’.**
 - **Thus, by varying the identity of the $\langle s, \langle st, t \rangle \rangle$ function provided by ‘BASE’, we vary the set of worlds the modal ends up quantifying over, and thus we vary the ‘reading’ that the modal receives.**

(50) A Schematic of the Syntax and Semantics Proposed



(Compare the sketch above to the picture earlier laid out in (10)!)

(51) **An Additional Benefit: The Infinitude of Readings**

Besides meeting the desiderata laid out in Section 2.1, the account also avoids the ‘problem’ noted in Section 2.2.

That is, the overall account is entirely consistent with the existence of an *infinite number* of different ‘readings’ (*deontic, bouletic*) for a given modal.

(52) **Another Additional Benefit: The Unimportance of our Taxonomy of ‘Readings’**

Under this semantics, the classification of ‘readings’ introduced in ‘Part 1’ is of no real, deep grammatical significance. Rather:

- ‘BASE’ is permitted to be any $\langle s, \langle st, t \rangle \rangle$ function you can imagine.
- Some of these functions yield propositions identifiable as ‘the law’. Others yield propositions identifiable as ‘knowledge’, *etc.*
- In this picture, then, the taxonomy of readings is rather artificial. *And, we needn’t suppose that it in any way offers a complete and exhaustive categorization of modal sentences.*

This does indeed seem to be a happy result, since it is possible to imagine cases where this taxonomy ‘breaks down’.

This is especially so for actual, naturally occurring English corpus data. In many examples drawn from actual English texts, it is *very* difficult to categorize the reading in question in terms of the four-way division from ‘Part 1’ (Chris Potts, p.c.)

Nevertheless, there are some potential problems faced by our proposed analysis...

(53) **Problem 1: Predictions for Inconsistent Bases**

As pointed out by Kratzer (1977, 1991, 2008), it is possible to spell out scenarios where the ‘modal base’ constitutes an *inconsistent* set of propositions.

Unfortunately, under such scenarios, our system seems to make predictions that are quite clearly incorrect.

Kratzer (1977, 1991, 2008) shows how to remedy the problem, by augmenting the semantics of the modal head itself.

Check out Kratzer (2008: 15-27) for more details!

(54) **Problem 2: The Possible Importance of our Taxonomy of Readings**

As noted in (52), our analysis claims that there is no real, deep grammatical distinction between the different ‘readings’ of the modals.

However, recall that there is at least one way in which the grammar of English does seem to ‘notice’ the difference between these readings:

Not every modal (in English) allows every ‘reading’

- a. “may” : Cannot have circumstantial reading.
- b. “can” : Cannot have epistemic reading.
- c. “must” : Cannot have epistemic reading.

Conclusion: *Something* in the lexical semantics of ‘may’ prevents it from having a circumstantial reading.

Problem: *How* do we possibly add this condition to the lexical entry in (48)??

And there are more challenges besides this!...

In the next (and final) part of our discussion, we will consider a number of these additional challenges to the picture in (48) – (50)....

...we will then see that a solution to these additional challenges may be gained if we hypothesize that – in addition to the ‘modal base’ – the modal also combines with yet another (potentially covert) argument...