

**The Semantics of Modals, Part 1:
Basics of the Quantificational Analysis, and the Appearance of ‘Ambiguity’¹**

(1) Overarching Question

What is the meaning of the ‘modal auxiliaries’ in English, exemplified in (1a)?
As illustrated by the sentences in (1b), they seem to have a rather varied and heterogeneous set of uses...

a. Illustrative Examples of English Modal Auxiliaries

may, might, can, must, have to...

b. Illustrative Examples of ‘Modal Sentences’

(i) John **must** be in New York.

(ii) George **might** have the flu.

(iii) Phil **can** slam dunk.

(iv) You **may** park in Lot 64.

(v) Dave **has to** write a report.

(2) Our Plan of Action (for This Handout)

- Let’s first develop a theory of the *truth-conditions* of a *limited* range of ‘modal sentences’. Using this, we’ll then develop some *preliminary* lexical entries for a limited range of these modal auxiliaries.
- Then, let us see how the insights gained from this initial exercise can be extended to a wider range of sentences/modals.
- Finally, let us evaluate/critique the overall picture that emerges from this ‘first stab’ at developing a semantics for modals.
- (Spoiler Alert: the failings/limitations of the system we put together here will point the way to the more advanced approach we will develop over the following two handouts...)

...So, let’s get started! Let’s begin (arbitrarily) with the modals *may* and *must*, under a particularly salient reading of theirs...

¹ These notes are based upon material in von Stechow (2007; Chapter 3) and Kratzer (1977, 2008).

1. Epistemic Modals

(3) Illustrative Use of the Modal ‘May’

- a. Sentence:
John **may** be in NYC.
- b. Context:
The police are looking for John, who has skipped town after an arrest. They know that he has a lot of family and friends in New York City. However, they also know that he has connections in Rhode Island.
- c. Paraphrase:
Given what we know, **an imaginable possibility** is that John is in NYC.

(4) Illustrative Use of the Modal ‘Must’

- b. Sentence:
John **must** be in NYC.
- b. Context:
The police are looking for John, who has skipped town after an arrest. They have been able to track his cell-phone activity. Just one minute ago, they tracked him making a call in the Empire State Building.
- c. Paraphrase:
Given what we know, **the only imaginable possibility** is that John is in NYC.

(5) The Content of the Modal Sentences: An Initial, Vague Idea

How, exactly, do the modal sentences in (3a) and (4a) differ in meaning from the non-modal sentence below?

- a. George is in NYC.
- b. Core Intuition:

The non-modal claim in (5a) is an assertion about how the world *really is*.

By contrast, the modal claims in (3a) and (4a) are principally claims about our knowledge; they are about ‘what is imaginable, given the facts we already know about the world’.

...Let us use the paraphrases in (3c)/(4c) and the ‘core intuition’ in (5b) to guide us towards a particular statement of the truth-conditions of (3a) and (4a)...

(6) **Truth Conditions of the Modal Sentences: A First Pass**

a. Sentence (3a)

“John may be in NYC” is T in w *iff*

(i) *Initial Paraphrase:*

Given what we know, **an imaginable possibility** is that John is in NYC.

(ii) *Informal Restatement:*

Our knowledge in w is **consistent with** it being the case that John is in NYC.

(iii) *More Formal Statement:*

There is a possible world w' such that **everything we know about w is also true in w'** , and John is in New York City in w' .

b. Sentence (4a)

“John must be in NYC” is T in w *iff*

(i) *Initial Paraphrase:*

Given what we know, **the only imaginable possibility** is that John is in NYC.

(ii) *Informal Restatement:*

- Our knowledge in w **necessitates** it being the case that John is in NYC.
- Our knowledge in w is **not consistent with** it **not** being the case that John is in NYC.

(iii) *More Formal Statement:*

For every possible world w' such that **everything we know about w is also true in w'** : John is in New York City in w' .

If we accept that (6aiii) and (6biii) are indeed the truth-conditions we wish our semantics to derive, let us then attempt to develop lexical entries for “may” and “must” which will serve to derive these truth-conditional statements...

An Important Aside:

The truth conditions in (6aiii) and (6biii) are not *entirely* accurate.

Consider, for example, the sentence under (4a) in the context under (4b).

- In context (4b), the relevant knowledge of world w seems to be the following:
John has just placed a cell-phone call, and the call originated from NYC.
- Now, according to the truth-conditions in (6biii), sentence (4a) can only be true if *in every* possible world w such that John has placed a cell-phone call from NYC in w , it is also the case that John is *in NYC* in w .
- However, these putative conditions don't seem to hold in the imagined scenario in (4b)
 - If we consider the *full* set of possible worlds, there are (far-fetched, but nevertheless imaginable) worlds where John's neck and arms can freely grow up to 100 miles in length.
 - In those (superhero) worlds, John can make a call from NYC without actually *being* in NYC (*i.e.*, the rest of his body is in Hoboken).

So, the truth-conditions stated in (6biii) would actually seem to wrongly predict that sentence (4a) is *false* in the imagined context...

So, how can we improve the truth conditions in (6biii) to avoid this problem? Here's one idea: *we further restrict the set of possible worlds w to those that are not 'far-fetched' possibilities.*

That is, while (6biii) is partially correct, the following might be a *fuller* statement of the T-conditions of (4a):

(7) Truth Conditions of the Modal Sentences: A Fuller Account

- a. "John **may** be in NYC" is T in w *iff*
There is a possible world w' such that **everything we know about w is also true in w' and nothing that is 'abnormal' in w occurs in w'** , and John is in New York City in w' .
- b. "John **must** be in NYC" is T in w *iff*
For every possible world w' such that **everything we know about w is also true in w' and nothing that is 'abnormal' in w occurs in w'** : John is in New York City in w' .

While the truth conditions in (7) do a better job than those in (6), let us for the moment put this matter aside, and carry on as if (6) is correct...

...we will, however, later return to this issue in 'Part 3' of our discussion of modality, where we'll augment a more complex theory of modals to capture these facts in a satisfying way...

(9) **Lexical Entries for “May” and “Must”**

- a. $[[\text{may}]]^w = \lambda p_{\langle \text{st} \rangle} . \exists w' \in W: \text{everything we know about } w \text{ is also true in } w' \ \& \ p(w') = T$
- b. $[[\text{must}]]^w = \lambda p_{\langle \text{st} \rangle} . \forall w' \in W: \text{everything we know about } w \text{ is also true in } w' \rightarrow p(w') = T$

It's easy to see that these lexical entries will derive the truth-conditions in (6):

(10) **Derivation of the Truth Conditions in (6biii)**

- a. **“John must be in New York City” is T in w** *iff* (by syntactic assumptions)
- b. $[[\text{ must } [\text{John be in NYC}]]]^w = T$ *iff* (by IFA)
- c. $[[\text{ must }]]^w ([[\text{John be in NYC}]]_{\epsilon}) = T$ *iff* (by 8b)
- d. $[[\text{ must }]]^w (\lambda w. \text{John is in NYC in } w) = T$ *iff* (by Lex.)
- e. $[\lambda p_{\langle \text{st} \rangle} . \forall w' \in W: \text{everything we know about } w \text{ is also true in } w' \rightarrow p(w') = T]$
 $(\lambda w. \text{John is in NYC in } w) = T$ *iff* (by LC)
- f. $\forall w' \in W: \text{everything we know about } w \text{ is also true in } w' \rightarrow$
 $[\lambda w. \text{John is in NYC in } w] (w') = T$ *iff* (by LC)
- g. **$\forall w' \in W: \text{everything we know about } w \text{ is also true in } w' \rightarrow \text{John is in NYC in } w'$**
For all possible worlds w':
if everything we know about w is also true in w', then
John is in NYC in w'

As we will soon see, the lexical entries in (9) are **far** from being the ‘whole story’ regarding the semantics of *may* and *must*...

However, this simple semantics already captures a number of ‘core logical relationships’ that these auxiliaries have to one another!

As illustrated below, the analysis captures these facts via its treatment of *may* and *must* as **quantifiers** over possible worlds.

Such a semantic analysis, then, we might dub a ‘quantificational analysis’ of modals...

1.1 Logical Properties of Modals, Captured by the ‘Quantificational Analysis’

(11) Entailment Relations

- a. Fact 1:
“John must be in NYC” *entails* “John may be in NYC”
- b. Fact 2:
“John may be in NYC” *does not entail* “John must be in NYC.”

The facts above together demonstrate that a sentence of the form “must VP” is *logically stronger* than a sentence of the form “may VP.”

(12) Consistency Relations

- a. Fact 1:
The following two sentences are consistent with each other; they could both be true at the same time:
 - (i) John may be in NYC.
 - (ii) John may not be in NYC.
- b. Fact 2:
The following two sentences are *not* consistent with each other; they could *not* both be true at the same time:
 - (i) John must be in NYC.
 - (ii) John must not be in NYC.

The facts above can serve as a diagnostic for whether a given modal has ‘universal force’ (like *must*) or merely ‘existential force’ (like *may*)

(13) Equivalence Relations

- a. Fact 1:
The following two sentences are logically equivalent; one is true *iff* the other is.
 - (i) John may be in NYC.
 - (ii) It’s not true that John must not be in NYC.
- b. Fact 2:
The following two sentences are logically equivalent; one is true *iff* the other is.
 - (i) John must be in NYC.
 - (ii) It’s not true that John may not be in NYC.

1.2 Interim Summary

Thus far, we've seen the following:

- The 'quantificational analysis' of modal auxiliaries in (9) is able to capture the truth-conditional statements in (6).
- Further preliminary evidence supports this 'quantificational analysis', namely, the fact that it can capture certain 'core' logical relationships between the modal auxiliaries.

At this point, however, we must now finally make contact with *the* fundamental, overarching puzzle for the analysis of modal auxiliaries...

- Under the semantics offered in (9), the modal auxiliaries 'may' and 'must' quantify over the set of possible worlds w with the following property:

everything we know about w is also true in w

- From now on, we'll refer to modals with the interpretation in (9) as 'epistemic modals'. Similarly, if a modal is being interpreted to have the meaning in (9), we'll say that the modal is being given an 'epistemic reading.'
- However, as we observed back in (1), modal auxiliaries seem to allow for interpretations *other* than this 'epistemic reading'...
- That is, the entries in (9) by no means exhaust the range of interpretations that modal auxiliaries (in English) can have!

In the following sections, we will examine three other principal 'readings' that modals appear to receive.

We will find that much of the insight gained by our 'quantificational analysis' of the epistemic modals in (9) can be extended to these other uses of modals...

...then we will take a *further* step back, and see that the system that emerges from our initial, piecemeal 'hacking' is missing some important generalizations...

... and then we will start building something better...

2. Circumstantial Modals

The first of the other three ‘principal types’ of reading we will consider is the so-called ‘circumstantial reading’.

Under this reading, modal sentences seem to making assertions about what possibilities are allowed/disallowed by the (physical) facts/circumstances of the world.

(14) Circumstantial Use of the Modal ‘Can’

- a. Sentence:
John **can** jump five inches.
- b. Context:
John is a very skilled jumper.
Though he isn’t jumping right now, and may not jump in the near future, his physical state certainly allows him to jump up to five inches in the air...
- c. Paraphrase:
Given the circumstances, a (physical) possibility is that John jumps five inches.

(15) Circumstantial Use of the Modal ‘Have To’

- a. Sentence:
John **has to** sneeze.
- b. Context:
A piece of dust has just flown up John's nose and triggered his 'sneeze' reflex.
Though he hasn't sneezed yet, his physical system is in a state where sneezing inevitable.
- c. Paraphrase:
Given the circumstances, the only (physical) possibility is that John sneezes.

Side-Note:

Note that we are dealing in (14) and (15) with a different pair of modals from *may* and *must*. Interestingly, *may* and *must* don’t allow for the ‘circumstantial’ readings above:

- (16) a. John **may** sneeze. (no circumstantial reading)
- b. John **must** sneeze. (circumstantial reading awkward)

A complementary generalization doesn’t appear to hold: while it is true that *can* doesn’t allow an ‘epistemic’ reading, *have to* does.

- (17) a. John **can** be in NYC. (no epistemic reading)
- b. John **has to** be in NYC. (epistemic reading definitely available)

Let us now attempt to further spell out the truth-conditions of the sentences in (14a) and (15a).

Given the ‘paraphrases’ in (14c) and (15c) – combined with our earlier ‘quantificational analysis’ of *may* and *must* – the following seem like likely initial candidates:

(18) **Truth Conditions of the ‘Circumstantial Reading’: A First Pass**

a. Sentence (14a)

“John can jump five inches” is T in w *iff*

(i) *Initial Paraphrase:*

Given the circumstances, a (physical) possibility is that John jumps five inches.

(ii) *Informal Restatement:*

The facts in w **are consistent with (do not rule out)** it being the case that John jumps five inches.

(iii) *More Formal Statement:*

There is a possible world w' such that **everything that is true in w is also true in w'** , and John jumps five inches in w' .

b. Sentence (15a)

“John must sneeze” is T in w *iff*

(i) *Initial Paraphrase:*

Given the circumstances, the only (physical) possibility is that John sneezes.

(ii) *Informal Restatement:*

- The facts in w **necessitate** it being the case that John sneezes.
- The facts in w **are not consistent with it not** being the case that John sneezes.

(iii) *More Formal Statement:*

For every possible world w' such that **everything that is true in w is also true in w'** : John sneezes in w' .

If we accept that (18a) and (18b) are indeed the truth-conditions we wish our semantics to derive, let us then attempt to develop lexical entries for “may” and “must” which will serve to derive these truth-conditional statements...

First Important Aside:

The truth conditions in (18aiii) and (18biii) are, in one glaring respect, rather crude.

- Given our metaphysics, the physical ‘facts’ that ‘hold in’ a particular world w include *everything that is ever the case in w until the end of time*.
- Thus, for the *actual world* w_0 , one of the physical ‘facts’ that hold ‘in w_0 ’ is whether or not John actually does jump five inches.
- Consequently, if at w_0 John in fact *doesn't* jump five inches, then the following is a fact that holds ‘in w_0 ’: John doesn't jump five inches.
- Consequently, any world w for which everything that is true at w_0 is also true at w , will be such that John *doesn't* jump five inches in w .
- Thus, if it happens to be true in the actual world w_0 that John never *does* jump five inches, it would follow that (under the T-conditions in (18aiii)) sentence (18a) is *false*.
- However, this is contrary to our English-speaker intuitions:
The sentence “John can jump five inches” can be T *even if he never does jump that high*.
(For example, if he simply decides never to jump again for the rest of his life.)

So, it seems that (given our metaphysical assumptions) the T-conditions in (18aiii) and (18biii) take in ‘too many facts’ about the actual world (namely, everything that occurs in the future)...

Solution:

When considering the ‘facts in w_0 ’, we consider only those facts that have held at w_0 *until the present*.

(19) Truth Conditions of Circumstantial Modals, A Fuller Account

- a. “John **can** jump five inches” is T in w *iff*
There is a possible world w' such that everything that is true in w **up to the present** is also true in w' , and John jumps five inches in w' .
- b. “John **has to** sneeze” is T in w *iff*
For every possible world w' such that everything that is true in w **up to the present** is also true in w' : John sneezes in w' .

If we follow the truth conditions above, then we predict that (in the imagined contexts), the only ‘facts in w_0 ’ that we consider are the facts that have held until the present.

Crucially, these facts include *John's physical state at the present*, but **don't** include what John actually goes on to do in the future!

While the treatment of ‘tense’ in these T-conditions is very crude, we'll retain it in what follows.

Second Important Aside:

The (augmented) truth conditions in (19) are still not *entirely* accurate.

Consider, for example, the sentence under (15a) in the context under (15b).

- In context (15b), the relevant ‘facts that have held in w_0 up to the present’ seems to be:
Those facts dealing with the state of John’s respiratory and nervous systems.
- Now, according to the truth-conditions in (19b), sentence (15a) can only be true if *in every possible world w' such that the facts in w_0 dealing with the state of John’s respiratory and nervous systems (up to the present) also hold in w' : John sneezes in w' .*
- However, these putative conditions don’t seem to hold in the imagined scenario in (15b)
 - If we consider the *full* set of possible worlds, there are (far-fetched, but nevertheless imaginable) worlds where John is about to sneeze (at the present), but is incinerated in a nuclear blast before he ever actually sneezes.
 - In those (apocalyptic) worlds, John can be in a physical state that would make sneezing immanent, but nevertheless *fail* to actually sneeze!

So, the truth-conditions stated in (19b) would actually seem to wrongly predict that sentence (15a) is *false* in the imagined context...

So, how can we improve the truth conditions in (19b) to avoid this problem? Here’s one idea:
Just as we did before, let us further restrict the set of possible worlds w' to those that are not ‘far-fetched’ possibilities.

(20) **Truth Conditions of Circumstantial Modals: A Still Fuller Account**

- a. “John **can** jump five inches” is T in w *iff*
There is a possible world w' **such that everything that is true in w up to the present is also true in w' and nothing that is ‘abnormal’ in w occurs in w'** , and John jumps five inches in w' .
- b. “John **has to** sneeze” is T in w *iff*
For every possible world w' **such that everything that is true in w up to the present is also true in w' and nothing that is ‘abnormal’ in w occurs in w'** : John sneezes in w' .

While the truth conditions in (20) do a better job than those in (19), let us for the moment put this matter aside, and carry on as if (19) were correct...

...we will, however, later return to this issue in ‘Part 3’ of our discussion of modality, where we’ll augment a more complex theory of modals to capture these facts in a satisfying way...

If we accept that (19a) and (19b) are indeed the truth-conditions we wish our semantics to derive, let us then attempt to develop lexical entries for “can” and “have to” which will serve to derive these truth-conditional statements...

- *We will continue to employ the background assumption regarding ‘reconstruction’.*
- *We will also appeal to the following stipulated intensions:*

(21) **Stipulated Intensions**

- a. $[[\text{John jump five inches}]]\epsilon = \lambda w. \text{John jumps five inches in } w$
 b. $[[\text{John sneeze}]]\epsilon = \lambda w. \text{John sneezes in } w$

(22) **Lexical Entries for “Can” and “Have-to”**

- a. $[[\text{can}]]^w = \lambda p_{\langle \text{st} \rangle} \exists w' \in W: \text{everything that is true in } w \text{ up to the present is also true in } w' \ \& \ p(w') = T$
 b. $[[\text{have to}]]^w = \lambda p_{\langle \text{st} \rangle} . \forall w' \in W: \text{everything that is true in } w \text{ up to the present is also true in } w' \rightarrow p(w') = T$

It’s easy to see that these lexical entries will derive the truth-conditions in (19):

(23) **Derivation of the Truth Conditions in (19b)**

- a. **“John has to sneeze” is T in w** *iff* (by syntactic assumptions)
 b. $[[\text{has-to [John sneeze]}]]^w = T$ *iff* (by IFA)
 c. $[[\text{has-to}]]^w ([[\text{John sneeze}]]\epsilon) = T$ *iff* (by 22b)
 d. $[[\text{has-to}]]^w (\lambda w. \text{John sneezes in } w) = T$ *iff* (by Lex.)
 e. $[\lambda p_{\langle \text{st} \rangle} . \forall w' \in W: \text{everything that is true in } w \text{ up to the present is also true in } w' \rightarrow p(w') = T]$
 $(\lambda w. \text{John sneezes in } w) = T$ *iff* (by LC)
 f. $\forall w' \in W: \text{everything that is true in } w \text{ up to the present is also true in } w' \rightarrow$
 $[\lambda w. \text{John sneezes in } w] (w') = T$ *iff* (by LC)
 g. **$\forall w' \in W: \text{everything that is true in } w \text{ up to the present is also true in } w' \rightarrow$**
John sneezes in w’

*For all possible worlds w’:
 if everything that is true in w up to the present is also true in w’, then
 John sneezes in w’*

3. Deontic Modals

The third reading of modals that we will consider here is the so-called ‘deontic reading’.

Under this reading, modal sentences seem to making assertions about what possibilities are allowed/disallowed by *the law* (or *the rules*, broadly construed).

(24) Deontic Use of the Modal ‘May’

- a. Sentence:
John **may** vote.
- b. Context:
The law states that a person below 18 cannot vote.
However, John has just turned 18, and so the law permits him to vote.
- c. Paraphrase:
Given what the law is, **an allowable possibility** is that John votes.

(25) Deontic Use of the Modal ‘Must’

- a. Sentence:
John **must** register with the Selective Service.
- b. Context:
The law requires that men register with the Selective Service when they turn 18.
John has just turned 18.
- c. Paraphrase:
Given what the law is, **the only allowable possibility** is that John registers with the Selective Service.

Side-Note:

Note that the modals above – *may* and *must* – are the *same* auxiliaries that we earlier saw carrying the ‘epistemic reading’. Note also that the ‘circumstantial modals’ from Section 2 – *can* and *have to* – also seem to allow the ‘deontic reading’ above.

- (26) a. John **can** vote. (deontic reading available)
b. John **has to** register with the Selective Service. (deontic reading available)

For the moment, let us treat the existence of these different readings as due to a genuine lexical ambiguity.

That is, we’ll provisionally assume that there are actually *two* different modal auxiliaries in English with the pronunciation /mej/ : the ‘epistemic’ modal in (9) and the ‘deontic’ modal above...

Let's now once again try to further spell out the T-conditions of the sentences in (24a) and (25a).

(27) **Truth Conditions of the 'Deontic Reading': A First Pass**

a. Sentence (24a)

“John may vote” is T in w iff

(i) *Initial Paraphrase:*

Given what the law is, **an allowable possibility** is that John votes

(ii) *Informal Restatement:*

The *law* in w **is consistent with (does not rule out)** it being the case that John votes.

(iii) *More Formal Statement:*

There is a possible world w' such that **law in w is being followed in w'** , and John votes in w' .

b. Sentence (25a)

“John must register with the Selective Service” is T in w iff

(i) *Initial Paraphrase:*

Given what the law is, **the only allowable possibility** is that John registers with the Selective Service.

(ii) *Informal Restatement:*

- The *law* in w **necessitates** it being the case that John registers with the Selective Service.
- The *law* in w **is not consistent with it not** being the case that John registers with the Selective Service.

(iii) *More Formal Statement:*

For every possible world w' such that **law in w is being followed in w'** : John registers with the Selective Service in w' .

If we accept the adequacy of the truth-conditions in (27a) and (27b), let us write out lexical entries for “may” and “must” which will derive them...

An Important Aside:

The truth conditions in (27) are (again) not *entirely* accurate.
Consider, for example, the sentence under (25a) in the context under (25b).

- In context (25b), the relevant ‘law in w' ’ seems to be:
All men register with the Selective Service when they turn 18.
- Now, according to the truth-conditions in (27biii), sentence (25a) can only be true if in every possible world w' such that *the law in w is being followed in w'* : John registers with the Selective Service in w' .
- However, these putative conditions don’t seem to hold in the imagined scenario in (25b)
 - If we consider the *full* set of possible worlds, there are possible worlds w' where John is *17* rather than 18 years old (*i.e.*, John was born one year later.)
 - In those worlds w' , the law in w (that 18 year olds register with the Selective Service) can be followed in w' , *without* it being the case that *John* registers with the Selective Service in w' (because John is only *17* in w').

So, our semantics in (27b) wrongly predicts that (25a) should be false in scenario (25b)...

What’s gone wrong here? Intuitively, when *must* receives the deontic reading, it doesn’t just quantify over the worlds where the law is followed, but rather *those worlds which are just like w (up to the present) and where the law is obeyed.*

So, how can we improve the truth conditions in (27b) to avoid this problem? Here’s one idea:
*Let us further restrict the set of possible worlds w' to those where **everything that is true in w up to the present is also true in w'***

(28) **Truth Conditions of Deontic Modals: A Fuller Account**

- a. “John **may** vote” is T in w *iff*
There is a possible world w' such that **law in w is being followed in w'** , **and everything that is true in w up to the present is also true in w'** , and John votes in w' .
- b. “John **must** register with the Selective Service” is T in w *iff*
For every possible world w' such that **law in w is being followed in w'** , **and everything that is true in w up to the present is also true in w'** : John registers with the Selective Service in w' .

Side-Note: Observe the similarity between the T-conditions in (28) and those in (20)!

While the truth conditions in (28) do a better job than those in (27), let us – as we’ve done before – momentarily put this matter aside, and carry on as if (27) were correct...

If we accept the adequacy of the truth-conditions in (27aiii) and (27biii), let us write out lexical entries for “may” and “must” which will derive them...

- We will continue to employ the background assumption regarding ‘reconstruction’.
- We will also appeal to the following stipulated intensions:

(29) **Stipulated Intensions**

- a. $[[\text{John vote}]]_{\epsilon}$ = $\lambda w. \text{John votes in } w$
- b. $[[\text{John register with the Selective Service}]]_{\epsilon}$ = $\lambda w. \text{John registers with the Selective Service in } w$

Now, recall that we are (momentarily) assuming that the ‘deontic’ *may* and *must* in (27) are separate lexical items from the ‘epistemic’ *may* and *must* that we saw earlier.

Thus, we may (and must) write out separate lexical entries for the ‘deontic’ *may* and *must*, in order to capture their observed truth-conditions in (27):

(30) **Lexical Entries for (Deontic) “May” and “Must”**

- a. $[[\text{may}]]^w = \lambda p_{\langle st \rangle} \exists w' \in W: \text{law in } w \text{ is being followed in } w' \ \& \ p(w') = T$
- b. $[[\text{must}]]^w = \lambda p_{\langle st \rangle} . \forall w' \in W: \text{law in } w \text{ is being followed in } w' \rightarrow p(w') = T$

It’s easy to see that these lexical entries will derive the truth-conditions in (27).

For example, by a proof similar to those in (10) and (23), our system derives the truth-conditional statements in (31) below:

(31) **Truth-Conditional Statements Derived by (30)**

- a. “John may vote” is T in *w* iff $\exists w' \in W: \text{law in } w \text{ is being followed in } w' \ \& \ \text{John votes in } w'$
- b. “John must register with the Selective Service” is T in *w* iff $\forall w' \in W: \text{law in } w \text{ is being followed in } w' \rightarrow \text{J. registers with the Sel. Ser. in } w'$

Side-Note:

Make sure not to confuse the following two sets of worlds:

- (i) worlds where the law in *w* is being *obeyed*
- (ii) worlds where the law is *the same* as it is in *w*

There can be worlds *w'* where (i) the law *in w* is obeyed, but where (ii) the law *in w'* is not obeyed (because *w'* actually has a different set of laws from *w*...)

4. Bouletic Modals

Thus far, we've seen that the English modal auxiliaries (*may, can, must, have to*) allow for at least one of three readings:

- (i) Epistemic
- (ii) Circumstantial
- (iii) Deontic.

The last of the four 'major types' of modal reading is the so-called 'bouletic reading'.

Under this reading, modal sentences seem to making assertions about what possibilities are allowed/disallowed by *a set of goals or desires*.

(32) Bouletic Use of the Modal 'Can'

- a. Sentence:
John **can** leave.
- b. Context:
We're having a meeting, and the subject has now turned to *working out the budget for next year*.
Since John doesn't work on the financial end of things, this discussion doesn't require his presence...
- c. Paraphrase:
Given what our goals are (i.e., working out the budget), an allowable possibility is that John leaves.

(33) Bouletic Use of the Modal 'Have To'

- a. Sentence:
John **has to** stay.
- b. Context:
We're having a meeting, and the subject has now turned to *working out the budget for next year*.
John is our Chief Financial Officer, and so our discussion requires his presence.
- c. Paraphrase:
Given what our goals are (i.e., working out the budget), the only allowable possibility is that John stays.

Side-Note:

As with the deontic readings, the modals *may* and *must* also allow for 'bouletic readings'...

... in general, as you may already intuit, the distinction between 'bouletic' and 'deontic' readings is a rather artificial one... (aren't all *laws* just a particular set of *goals/desires*?)

Let's now once again try to further spell out the T-conditions of the sentences in (32a) and (33a).

(34) **Truth Conditions of the 'Bouletic Reading': A First Pass**

a. Sentence (32a)

“John can leave” is T in w *iff*

(i) *Initial Paraphrase:*

Given what our goals are, **an allowable possibility** is that John leaves.

(ii) *Informal Restatement:*

Our goals in w **are consistent with (do not rule out)** it being the case that John leaves.

(iii) *More Formal Statement:*

There is a possible world w' such that **our goals in w are met in w'** , and John leaves in w' .

b. Sentence (33a)

“John has to stay” is T in w *iff*

(i) *Initial Paraphrase:*

Given what our goals are, **the only allowable possibility** is John stays.

(ii) *Informal Restatement:*

- *Our goals* in w **necessitate** it being the case that John stays.
- *Our goals* in w **are not consistent with it not** being the case that John stays.

(iii) *More Formal Statement:*

For every possible world w' such that **our goals in w are met in w'** : John stays in w' .

If we accept the adequacy of the truth-conditions in (34a) and (34b), let us write out lexical entries for “may” and “must” which will derive them...

An Important Aside:

The truth conditions in (34) are (again) not *entirely* accurate. Consider, for example, the sentence under (33a) in the context under (33b).

- In context (33b), the relevant ‘goal in w' ’ seems to be:
That we work out our budget for next year.
- Now, according to the truth-conditions in (34biii), sentence (33a) can only be true if in every possible world w' such that *our goals in w are met in w'* : John stays in w' .
- However, these putative conditions don’t seem to hold in the imagined scenario in (33b)
 - If we consider the *full* set of possible worlds, there are possible worlds w' where John is *not* our CFO (but, maybe, works in the mailroom).
 - In those worlds w' , our goal in w (that we work out the budget for next year) can be met in w' , *without* it being the case that John stays (at the meeting) in w'

So, our semantics in (34b) wrongly predicts that (33a) should be false in scenario (33b)...

What’s gone wrong here? Well, as with the deontic readings earlier, it seems that when *have to* receives the ‘bouletic reading’, it doesn’t just quantify over the worlds where our goals are met, but rather *those worlds which are just like w (up to the present) and where our goals are met.*

So, let’s adjust our truth-conditions in (34) just as we did earlier for the ‘deontic reading’...

(35) Truth Conditions of Bouletic Modals: A Fuller Account

- a. “John **can** leave” is T in w *iff*
There is a possible world w' such that **our goals in w are met in w' and everything that is true in w up to the present is also true in w'** , and John leaves in w'
- b. “John **must** stay” is T in w *iff*
For every possible world w' such that **our goals in w are met in w' and everything that is true in w up to the present is also true in w'** : John stays in w' .

Side-Note:

Again, note the similarity between this putatively distinct ‘bouletic reading’ and the ‘deontic reading’ we examined earlier...

While the truth conditions in (35) do a better job than those in (34), let us – as we’ve done before – momentarily put this matter aside, and carry on as if (34) were correct...

If we accept the adequacy of the truth-conditions in (34a-iii) and (34b-iii), let us write out lexical entries for “may” and “must” which will derive them...

- We will continue to employ the background assumption regarding ‘reconstruction’.
- We will also appeal to the following stipulated intensions:

(36) **Stipulated Intensions**

- a. $[[\text{John leave}]]_{\epsilon} = \lambda w. \text{John leaves in } w$
 b. $[[\text{John stay}]]_{\epsilon} = \lambda w. \text{John stays in } w$

Now, recall that we are (momentarily) assuming that the ‘bouletic’ *can* and *have-to* in (34) are separate lexical items from the ‘deontic’ and ‘circumstantial’ *can* and *have-to* from earlier.

Thus, we can (and have-to) write out separate lexical entries for the ‘bouletic’ *can* and *have-to* :

(37) **Lexical Entries for (Bouletic) “Can” and “Have-To”**

- a. $[[\text{can}]]^w = \lambda p_{\langle st \rangle} \exists w' \in W: \text{our goals in } w \text{ are met in } w' \ \& \ p(w') = T$
 b. $[[\text{have-to}]]^w = \lambda p_{\langle st \rangle} \forall w' \in W: \text{our goals in } w \text{ are met in } w' \rightarrow p(w') = T$

It’s easy to see that these lexical entries will derive the truth-conditions in (34).

For example, by a proof similar to those above, our system derives the truth-conditional statements in (38) below:

(38) **Truth-Conditional Statements Derived by (37)**

- a. “John can leave” is T in *w* iff
 $\exists w' \in W: \text{our goals in } w \text{ are met in } w' \ \& \ \text{John leave in } w'$
 b. “John has to stay” is T in *w* iff
 $\forall w' \in W: \text{our goals in } w \text{ are met in } w' \rightarrow \text{John stays in } w'$

Side-Note:

Just as with the deontic readings earlier, make sure not to confuse the following two sets of worlds:

- (i) worlds where our goals in *w* are *met*
 (ii) worlds where our goals are *the same* as they are in *w*

There can be worlds *w'* where (i) our goals in *w* are met, but where (ii) our goals *in w'* are not met (because in *w'* we actually have a different set of ‘goals’ from those in *w*...)

5. Assessing the System that We've Built

Throughout the preceding discussion, we've taken up the view that modal auxiliaries in English are *lexically ambiguous*, and can represent one of four very different types of logical operators.

(39) The Four Types of Readings that Modals Can Receive

a. Epistemic

$[[\text{may}]]^w =$
 $\lambda p_{\langle st \rangle} . \exists w' \in W: \text{everything we know about } w \text{ is also true in } w' \ \& \ p(w') = T$

$[[\text{must / have to}]]^w =$
 $\lambda p_{\langle st \rangle} . \forall w' \in W: \text{everything we know about } w \text{ is also true in } w' \rightarrow p(w') = T$

b. Circumstantial

$[[\text{can}]]^w =$
 $\lambda p_{\langle st \rangle} \exists w' \in W: \text{everything true in } w \text{ (up to now) is true in } w' \ \& \ p(w') = T$

$[[\text{have to}]]^w =$
 $\lambda p_{\langle st \rangle} . \forall w' \in W: \text{everything true in } w \text{ (up to now) is true in } w' \rightarrow p(w') = T$

c. Deontic

$[[\text{may / can}]]^w =$
 $\lambda p_{\langle st \rangle} \exists w' \in W: \text{law in } w \text{ is being followed in } w' \ \& \ p(w') = T$

$[[\text{must / have to}]]^w =$
 $\lambda p_{\langle st \rangle} . \forall w' \in W: \text{law in } w \text{ is being followed in } w' \rightarrow p(w') = T$

d. Bouletic

$[[\text{may / can}]]^w =$
 $\lambda p_{\langle st \rangle} \exists w' \in W: \text{our goals in } w \text{ are met in } w' \ \& \ p(w') = T$

$[[\text{must / have to}]]^w =$
 $\lambda p_{\langle st \rangle} . \forall w' \in W: \text{our goals in } w \text{ are met in } w' \rightarrow p(w') = T$

In this 'first pass' treatment of modals, then, English actually has – not *four* modal auxiliaries – but rather *thirteen* (or *nine*, if we collapse the 'deontic' and 'bouletic' readings...)

- (i) epistemic 'may', (ii) deontic 'may', (iii) bouletic 'may', (iv) circumstantial 'can',
- (v) deontic 'can', (vi) bouletic 'can', (vii) epistemic 'must', (viii) deontic 'must',
- (ix) bouletic 'must', (x) epistemic 'have to', (xi) circumstantial 'have to',
- (xii) deontic 'have to', (xiii) bouletic 'have to'

However, when we lay out *all* the postulated lexical entries in (39), the following problems with this (first pass) approach becomes obvious:

(40) **Problems for the View that Modal Auxiliaries are *Lexically Ambiguous***

a. The Multiplicity of Homophones

Are there really *four* different homophonous auxiliaries pronounced ‘have to’?
Do we really need to postulate all this lexical ambiguity?

b. The ‘Systematicity’ of the Semantic Variation

The thirteen separately stated lexical entries in (39) miss a glaring generalization about the meaning of the modal auxiliaries.

(i) *The General Meaning of “Can / May”*

All the lexical entries for *can* and *may* have the following abstract form...

$$\lambda p_{\langle st \rangle} . \exists w' \in W: R(w, w') \ \& \ p(w') = T$$

...where ‘R’ is some relation between the evaluation world w and the worlds w' where the proposition denoted by the VP holds.

(ii) *The General Meaning of “Must / Have-To”*

All the lexical entries for *must* and *have-to* have the following form...

$$\lambda p_{\langle st \rangle} \forall w' \in W: R(w, w') \rightarrow p(w') = T$$

...where ‘R’ is some relation between the evaluation world w and the worlds w' where the proposition denoted by the VP holds.

The point made in (40b) is a very important one, which bears some further discussion...

- Under all their different readings, the modals *can* and *may* make an **existential** claim:
That there is **some** world w' bearing a **given relation** to w , where the proposition p is T
- Under all their different readings, the modals *must* and *have to* make a **universal** claim:
That in **all** worlds w' bearing a **given relation** to w , the proposition p is T

Thus, it appears that across all their different ‘readings’, there is an invariant, ‘core’ meaning to the modals *can/may* and *must/have-to*, one that is not explicitly reflected in the mere list of readings in (39)...

Here's another way of stating the invariant, 'core' meanings of the different modals...

(41) **The Weak / Existential Modals (*Can / May*)**

a. Invariant Meaning:

$$\lambda p_{\langle st \rangle} . \exists w' \in \{ w' \in W : R(w, w') \} : p(w') = T$$

b. Characterization of (41a)

- (i) The modal quantifiers over a *restricted set of worlds*: the worlds w' bearing a **given relation** to w
- (ii) The modal asserts that *some member w' of that set* is such that $p(w') = 1$

(42) **The Strong / Universal Modals (*Must / Have-to*)**

a. Invariant Meaning:

$$\lambda p_{\langle st \rangle} . \forall w' \in \{ w' \in W : R(w, w') \} : p(w') = T$$

b. Characterization of (41a)

- (i) The modal quantifiers over a *restricted set of worlds*: the worlds w' bearing a **given relation** to w
- (ii) The modal asserts that *all members w' of that set* are such that $p(w') = 1$

Thus, under this characterization of the 'invariant core' of the modals, the so-called 'weak modals' (*can, may*) differ in their meaning from the so-called 'strong modals' (*must, have-to*) **solely with respect to the *quantificational force* of their assertion.**

- Weak modals are semantically identical to strong modals, except that they make an *existential* claim regarding the restricted set $\{ w' \in W : R(w, w') \}$ (namely: that *something* in that set satisfies p)
- Strong modals are semantically identical to weak modals, except that they make a *universal* claim regarding the restricted set $\{ w' \in W : R(w, w') \}$ (namely: that *everything* in that set satisfies p).

Finally, as illustrated below, we can see that each of the four different types of 'reading' cataloged above simply reflect different choices regarding the restricted set of worlds...

(43) **The Four ‘Readings’ of English Modals**

a. Epistemic Reading:

$$\{ w' \in W : \mathbf{R}(w, w') \} = \{ w' \in W : \mathbf{everything\ we\ know\ about\ w\ is\ also\ true\ in\ w'} \}$$

b. Circumstantial Reading:

$$\{ w' \in W : \mathbf{R}(w, w') \} = \{ w' \in W : \mathbf{everything\ true\ in\ w\ (up\ to\ now)\ is\ true\ in\ w'} \}$$

c. Deontic Reading

$$\{ w' \in W : \mathbf{R}(w, w') \} = \{ w' \in W : \mathbf{the\ law\ in\ w\ is\ being\ followed\ in\ w'} \}$$

d. Bouletic Reading

$$\{ w' \in W : \mathbf{R}(w, w') \} = \{ w' \in W : \mathbf{our\ goals\ in\ w\ are\ met\ in\ w'} \}$$

These considerations set a very clear goal before us:

(44) **New Analytic Goal**

a. Let us dispense with the separate lexical entries in (39).

b. Let us replace them with a single univocal semantics for
(i) the ‘weak modals’ and
(ii) the ‘strong modals’.

Under this semantics, we will *separate out* from the meaning of the modal the restricted set of worlds that it quantifies over...

c. Let us then develop a system where the different ‘readings’ of the modals can be derived via the semantic composition of
(i) the invariant meaning of the modal, with
(ii) other material in the sentence

Where this ‘other material’ contributes the restricted set of worlds that the modal quantifiers over...

In the next set of notes, we will begin working towards this new goal...

We will also start off with a number of *other* problems that the ‘ambiguity account’ in (39) faces [as if you needed any more...].