The Semantics of Conditionals, Part 2:
Intensional Treatments of Conditionals[1]

1. Towards an Intensional Treatment of Conditionals

(1) **General Conclusion From Last Time**

An 'extensional' semantics for conditionals is plagued with the following problems:

a. The conditions of 'falsehood' it imposes are too strict.
Conditionals can be false without the antecedent being true or the consequent being false.

b. An extensional semantics validates inferences that are not formally valid for natural language conditionals.
Natural language conditionals fail to validate:
(i) *'Strengthening the Antecedent'*
(ii) *'Transitivity'*
(ii) *'Contraposition'*

c. An extensional semantics cannot capture the truth-conditions of conditional sentences whose consequents contain modals.

So, let's attempt to see if an ***intensional*** *analysis of conditionals can fare better on (1a-c).*

Let's start things off here by recalling some of our earlier 'introspections' regarding the conditions under which conditionals are ***false****.*

(2) **Some Introspection Regarding the Falsehood of Certain Conditionals**

The conditional in (2a) below is intuitively false. *Why?*

a. If the sun explodes tomorrow, then Alpha Centauri will explode tomorrow too.

b. <u>Some Introspective Reflections</u>

- Sentence (2a) is false because it seems plausible (given our knowledge of physics) that the sun's exploding will *not* bring about the explosion of A.C.

- That is, the following kind of situation seems *possible*: the sun explodes and Alpha Centauri doesn't.

- **That is, *there is at least one possible world* where the sun explodes and Alpha Centauri doesn't also explode.**

[1] These notes are based upon material in von Fintel (2007; Chapter 4, Chapter 5).

Let's roll with the introspective reflections in (2b), and make the following hypothesis regarding the 'falsehood conditions' of conditional statements.

(3) **Falsehood Conditions of Conditionals**

"If S_1 (then) S_2" is FALSE at world w iff
There exists a possible world $w' \in W$ such that
$[[S_1]]_{¢}(w') = T$ and $[[S_2]]_{¢}(w') = F$

These 'falsehood' conditions in (3) are, of course, equivalent to the truth-conditions in (4).

(4) **Truth Conditions of Conditionals (First Pass)**

"If S_1 (then) S_2" is TRUE at world w iff
$\forall w' \in [[S_1]]_{¢} : [[S_2]]_{¢}(w') = T$
Every world at which "S_1" is True, is also a world at which "S_2" is true.

There are clearly some things we're going to have to fix in (4), but let's pause for a moment and notice the things it gets *right*…

1.1 Advantages of the (Simple) Intensional Semantics

1.1.1 Captures the 'Guiding Intuitions' That Motivated the Extensional Semantics

Recall that our extensional semantics was initially motivated by the following 'guiding intuitions'.

(5) **Intuitions that 'Motivated' the Extensional Semantics**

(a) In a conditional sentence "if S_1 , (then) S_2", the *truth* of S_2 is asserted to 'depend upon' the *truth* of S_1. (When S_1 is *true*, S_2 is also (thereby) made *true* as well…)

(b) The sentence "If S_1 then S_2" is FALSE ***if*** S_1 is TRUE and S_2 is FALSE

Happily, our semantics in (4) is able to capture both these 'guiding intuitions'.

(6) **Captures the 'Guiding Intuition' in (5a)**
(trivial)

(7) **Captures the 'Guiding Intuition' in (5b)**

- Suppose that, in the actual world w_0, "S_1" is T and "S_2" is F.
- Then clearly, there is a possible world w' such that $[[S_1]]_{¢}(w') = T$ and $[[S_2]]_{¢}(w') = F$
- Thus, our truth-conditions in (3)/(4) predict that "If S_1 (then) S_2" is False (at w_0)

1.1.2 Avoids the Problem in (1a)

Our extensional semantics from Part 1 strengthened the (true) generalization in (5b) to the much stronger statement below.

(8) **(Obsolete) Extensional Falsehood Conditions**

The sentence "If S_1 then S_2" is FALSE ***iff*** S_1 is TRUE and S_2 is FALSE

As we noted in Part 1, this stronger *bi*-conditional statement in (8) is falsified by sentences like the following.

(9) **False Conditionals Without True Antecedents or False Consequents**

a. If the sun explodes tomorrow, then Alpha Centauri will explode tomorrow too.
b. If I wear the same sweater every day, then the sun will not explode next week.

Happily, our intensional semantics in (3)/(4) doesn't predict the incorrect statement in (8), and has no problem with the sentences in (9).

(10) **Falsehood Conditions of Conditionals (Repeated from Above)**

"If S_1 (then) S_2" is FALSE at world w iff
There exists a possible world w' $\in$ W such that
$[[S_1]]_¢(w') = T$ and $[[S_2]]_¢(w') = F$

Under these truth-conditions, a conditional sentence is false so long as there is at least *one* possible world (not necessarily the *actual world*) where the antecedent is T and the consequent F.

(11) **Correctly Predicts that (9a) is False**

Even though the sun won't *actually* explode tomorrow (and so the antecedent is false in the actual world), there *are* possible worlds where this will happen (and so the antecedent is true in those worlds).

Moreover, it seems that among these worlds, there are some where the consequent is F. Thus, our semantics in (10) correct predicts that (9a) is false.

(12) **Correctly Predicts that (9b) is False**

Even though the sun *won't* explode tomorrow (and so the consequent is *true* in the actual worlds), there *are* worlds where it does (and so the consequent is *false* in those worlds.)

Moreover, it seems that among those worlds, there are some where the antecedent is T. Thus, our semantics in (10) correctly predicts that (9b) is false.

1.2 Problem for the (Simple) Intensional Semantics

So, the simple intensional truth-conditions in (3)/(4) provide a solution to the problem in (1a):

> *In this semantics, a conditional can be false without the antecedent actually being true and the consequent actually being false.*

But, how well does this simple semantics fare with respect to the problem in (1b)? ***Not well!***

> ***As illustrated below, our intensional truth-conditions in (4) still validate all the problematic inferences in (1b).***

(13) **Inferences Validated by the Truth-Conditions in (3)/(4)**

a. Strengthening the Antecedent

(i) *Inference Pattern*
If p then q. Therefore, if p and z, then q.

(ii) *Proof of Validity*
- Suppose "If p then q" is true.
- Then in all worlds where p is true, q is also true.
- Clearly, then, in all worlds where p *and z* is true, q is also true.
- Thus, by our truth-conditions, "if p and z then q" is also true.

b. Transitivity

(i) *Inference Pattern*
If p, then q. If q, then z. Therefore, if p, then z.

(ii) *Proof of Validity*
- Suppose the following are true, "if p then q" and "if q then z".
- Thus, all worlds where p is true are worlds where q is true.
- And, all worlds where q is true are also worlds where z is true.
- Clearly, then, all worlds where p is true are worlds where z is true.

c. Contraposition

(i) *Inference Pattern*
If p then q. Therefore, if not q, then not p.

(ii) *Proof of Validity*
- Suppose the following is true: "if p then q".
- Thus, in every world where p is true, q is true too.
- Suppose, now, that q is false is some world w'.
- Clearly, w' cannot be a world where p is true.
- Thus, in every world where 'not q' is true, 'not p' is also true.

> *So, how can we amend our intensional semantics so that it doesn't face these problems?...*

2. Introducing the Ordering Source Again

(14) **The Problem, Restated**

Our intensional semantics predicts that the inference patterns in (13) are (universally) valid for natural language conditionals. *But, certain examples suggest otherwise…*

Let's try to get a handle on this problem by considering a concrete 'counterexample' to the validity of (13a), 'strengthening the antecedent'.

(15) **Failure of 'Strengthening the Antecedent'**

a. If kangaroos had no tails, they would fall over. **T**
b. If kangaroos had no tails, **and had crutches instead**, they would fall over. **F**

(16) **Preliminary Goal**

We want a semantics for conditionals that allows (15a) to be true and (15b) to be false.

But, before we start towards the goal in (16), we should probably take note of the following:

(17) **Acute Problem for the Semantics in (3)/(4)**

According to our first-pass intensional semantics in (3)/(4), conditional (15a) is *false*.

a. If we consider the full set of possible worlds, there are all sorts of crazy worlds, including worlds like the following:

   (i) Worlds where kangaroos have three legs instead of two.
   (ii) Worlds where kangaroos have jet packs.
   (iii) *Worlds where kangaroos have crutches.*

b. Thus, if we consider the *full* set of worlds where kangaroos don't have tails, there are clearly worlds in this set where – because all sorts of crazy stuff happens – *kangaroos don't fall over*.

c. Thus, it's not true that
   Every world at which "Kangaroos have not tails" is True, is also a world at which "Kangaroos fall over" is true.

d. Thus, our truth-conditions in (4) predict that (15a) is *false*.

What's going wrong here?...
This is clearly similar to the problems that motivated the introduction of the 'ordering source' into our semantics for modals…

(18) **Another Guiding Intuition**

- When we judge that the conditional in (15a) is true, we aren't considering the crazy, whacked-out worlds in (17a).
- Rather, we're only considering 'normal' worlds; worlds where kangaroos don't have crutches or jet-packs or whatever…

This intuition suggests something like the following revision of our truth-conditions in (4).

(19) **Revised Intensional Semantics for Conditionals, Part 1**

"If S_1 (then) S_2" is TRUE at world w iff

$\forall$w' $\in$ [[S_1]]$_¢$ $\cap$ {p: p is a 'reasonable expectation' in w}: [[S_2]]$_¢$(w') = T

Every world at which "S_1" is True ***and where nothing happens that is 'abnormal' in w****, is also a world at which "S_2" is true.*

Well, we can go ahead and rule this idea out!...

(20) **Problem for the Revised Semantics**

Under the revised semantics in (19), the conditional in (15a) is *trivially true*.

- Consider: one of our reasonable expectations about w_0 is that *kangaroos have tails!*
- Thus, there is no world w' which satisfies the proposition "kangaroos don't have tails", *and* which satisfies *all* our reasonable expectations about w_0

Given what we did in 'Part 3' of our discussion of modals, the solution here is obvious:
We introduce an 'ordering source' into the semantics of the conditional

(21) **Revised Intensional Semantics for Conditionals, Part 2**

"If S_1 (then) S_2" is TRUE at world w iff

$\forall$w' $\in$ MAX$_{<\{\text{p: p is a 'reasonable expectation' in w}\}}$([[$S_1$]]$_¢$): [[S_2]]$_¢$(w') = T

Every world w' in the following set is such that S_2 is true at w':
Those worlds, from the set of worlds at which S_1 is true,
Which satisfy <u>the most</u> of our 'reasonable expectations' in w.

Under the truth-conditions in (21), we now correctly predict that (15a) is (intuitively) true!

(22) **Predicted Truth of Conditional (15a)**

a. Consider a world w with the following properties:
- Kangaroos do not have tails.
- Aside from this, all our 'reasonable expectations' about the world are fulfilled

b. In such a world, their absence of tails would make kangaroos liable to fall over.

- After all, since all our other 'reasonable expectations' are filled, there is nothing else that kangaroos can use to balance themselves
(Since they don't have crutches or jet-packs or anything…)

c. Thus, in such a world, kangaroos would fall over.

d. Thus, in *all* the following worlds, the sentence "kangaroos fall over" is true:
Those worlds which satisfy the proposition "kangaroos don't have tails", and which satisfy the *most* of our reasonable expectations about the world.

e. Thus, truth-conditions in (21) predict that conditional (15a) is true.

But that's not all! Under the truth-conditions in (21), we <u>also</u> correctly predict that conditional (15b) is (intuitively) false!

(23) **Predicted Falsity of Conditional (15b)**

a. Consider a world w with the following properties.
- Kangaroos do not have tails.
- Kangaroos have crutches.
- Aside from that, all our 'reasonable expectations' about w_0 are fulfilled.

b. In such a world, kangaroos would not be likely to fall over.

- After all, since all our other 'reasonable expectations' are fulfilled, they could use their crutches to balance themselves.

c. Thus, in such a world, kangaroos would not fall over.

d. Thus, there is a world from the following set where "kangaroos fall over" is *false*:
Those worlds which satisfy the propositions "kangaroos don't have tails" and "kangaroos have crutches", and which satisfy the *most* of our reasonable expectations about the world.

e. Thus, the truth-conditions in (21) predict that conditional (15b) is *false*.

(24) **What Just Happened, In Abstract**

By introducing an 'ordering source' into our semantics for conditionals, as in (21), we defeat the reasoning in (13a) that validates 'strengthening the antecedent'.

- The reasoning in (13a) goes through because the worlds in $[[S_i]]_¢$ are necessarily a subject of the worlds in $([[S_i]]_¢ \cap [[S_j]]_¢)$.

- Crucially, however, the worlds in $[[S_i]]_¢$ that satisfy the most propositions from P ***needn't be a subset*** of the worlds in $([[S_i]]_¢ \cap [[S_j]]_¢)$ that satisfy the most propositions from P!
  (Since $[[\text{NOT } S_j]]_¢$ might be one of the propositions *in P*)

- And, to use a specific example, the worlds in [[kangroos don't have tails]]$_¢$ that satisfy the most of our 'reasonable expectations' ***aren't a subset*** of the worlds in ([[kangaroos don't have tails]]$_¢$ $\cap$ [[kangaroos have crutches]]$_¢$) that satisfy the most propositions from P!
  (Since one of our 'reasonable expectations' is that *kangaroos don't have crutches!*)

Thus, our augmented semantics in (21) achieves our 'preliminary goal' in (16)!

Ok, so (21) correctly predicts that the inference in (13a) is invalid…
…but what about the inferences in (13b) and (13c)?

Let's start with inference (13b), 'Transitivity', and consider a counterexample to its validity.[2]

(25) **Failure of 'Transitivity'**

a. (Even) if Goethe hadn't died in 1832, he would still be dead today. (True)
b. If Goethe were alive today, then he wouldn't have died in 1832. (True)
c. If Goethe were alive today, then he would still be dead today. (False)

(26) **Preliminary Goal**

We want a semantics for conditionals which does the following:
- Predicts that (25a) is (intuitively) true.
- Predicts that (25b) is (intuitively) true.
- Predicts that (25c) is (intuitively) false.

[2] The counter-example in (25) differs from the one in 'Part 1' of our discussion on conditionals (which had to do with Hoover being a communist). The reason is that there is possibly 'more going on' in that earlier example than the one in (25). I refer the interested reader to:
von Fintel (2001) "Counterfactuals in a Dynamic Context." In Kenstowicz, M. (ed) *Ken Hale: A Life in Language.* MIT Press.

(27) **Predicted Truth of Conditional (25a)**

a. Consider a world w with the following properties:
- Goethe didn't die in 1832
- Aside from this, all our 'reasonable expectations' about the world are fulfilled

b. In such a world, Goethe would definitely be dead today.
After all, since all our other 'reasonable expectations' are filled, Goethe *couldn't* have lived from 1832 until today.

c. Thus, in *all* the following worlds, "Goethe would be dead today" is true:
Those worlds which satisfy the proposition "Goethe didn't die in 1832", and which satisfy the *most* of our reasonable expectations about the world.

d. Thus, truth-conditions in (21) predict that conditional (25a) is true.

(28) **Predicted Truth of Conditional (25b)**

a. Consider a world w with the following properties:
- Goethe is alive today.
- Aside from this, all our 'reasonable expectations' about the world are fulfilled

b. In such a world, Goethe would definitely *not* have died in 1832
After all, since all our other 'reasonable expectations' are filled, people don't rise from the dead in such a world…

c. Thus, in *all* the following worlds, "Goethe didn't die in 1832" is true:
Those worlds which satisfy the proposition "Goethe is alive today", and which satisfy the *most* of our reasonable expectations about the world.

d. Thus, truth-conditions in (21) predict that conditional (25b) is true.

(29) **Predicted *Falsity* of Conditional (25c)**

a. Consider a world w with the following properties:
- Goethe is alive today.
- Aside from this, all our 'reasonable expectations' about the world are fulfilled

b. In such a world, Goethe would definitely *not* be dead today.

c. Thus, there is a world from the following set where "Goethe would be dead today" is *false*:
Those worlds which satisfy the proposition "Goethe is alive today", and which satisfy the *most* of our reasonable expectations about the world.

d. Thus, truth-conditions in (21) predict that conditional (25c) is *false.*

(30) **What Just Happened, In Abstract**

By introducing an 'ordering source' into our semantics for conditionals as in (21) we defeat the reasoning in (13b) that validates 'transitivity'.

a. How The Semantics in (4) Validates the Reasoning in (13b)

(i) *The Semantics in (4):*
[[If S_i then S_j]] = T *iff* $[[S_i]]_¢ \subseteq [[S_j]]_¢$

(ii) *The Reasoning That This Validates*

- If [[If S_1 then S_2]] = T *then* $[[S_1]]_¢ \subseteq [[S_2]]_¢$
- If [[If S_2 then S_3]] = T *then* $[[S_2]]_¢ \subseteq [[S_3]]_¢$
- Clearly, then $[[S_1]]_¢ \subseteq [[S_3]]_¢$, and so [[If S_1 then S_3]] = T

b. How the Semantics in (21) *In*validates the Reasoning in (13b)

(i) *The Semantics in (21):*
[[If S_i then S_j]] = T *iff* $MAX_{<P} ([[S_i]]_¢) \subseteq [[S_j]]_¢$

(ii) *The Invalidation of (13b)*

- If [[If S_1 then S_2]] = T *then* $MAX_{<P} ([[S_1]]_¢) \subseteq [[S_2]]_¢$
- If [[If S_2 then S_3]] = T *then* $MAX_{<P} ([[S_2]]_¢) \subseteq [[S_3]]_¢$

- *But from this, it* ***doesn't*** *follow that* $MAX_{<P} ([[S_1]]_¢) \subseteq [[S_3]]_¢$
And so it ***doesn't*** *follow that* [[If S_1 then S_3]] = T

- Why does it not follow?

Simply because X is a subset of $[[S_2]]_¢$, *it doesn't follow* that X is a subset of **the worlds from $[[S_2]]_¢$ that satisfy the most propositions from P**
(Since one of the propositions in P might be one that conflicts with X)

- To use a specific example:

The worlds in [[Goethe is alive today]]$_¢$ which satisfy the most of our 'reasonable expectations' are clearly a subset of the worlds in [[Goethe didn't die in 1832]]$_¢$

However, they *aren't* a subset of **the worlds in [[Goethe didn't die in 1832]]$_¢$** ***which satisfy the most of our 'reasonable expectations'.***
(Since one of our 'reasonable expectations' is that people alive in 1832 didn't survive until today).

Thus, our augmented semantics in (21) achieves our 'preliminary goal' in (26)!

Finally, let's turn to (13c), 'Contraposition', and consider a counterexample to its validity.

(31) **Failure of Contraposition**

a. (Even) if Goethe hadn't died in 1832, he wouldn't be alive today. (True)
b. If Goethe were alive today, then he would have died in 1832. (False)

(32) **Preliminary Goal**

We want a semantics for conditionals which does the following:
- Predicts that (31a) is (intuitively) true.
- Predicts that (31b) is (intuitively) false.

(33) **Predicted Truth of Conditional (31a)**

a. Consider a world w with the following properties:
- Goethe didn't die in 1832
- Aside from this, all our 'reasonable expectations' about the world are fulfilled

b. In such a world, Goethe would definitely be dead today.
After all, since all our other 'reasonable expectations' are filled, Goethe *couldn't* have lived from 1832 until today.

c. Thus, in *all* the following worlds, "Goethe would be dead today" is true:
Those worlds which satisfy the proposition "Goethe didn't die in 1832", and which satisfy the *most* of our reasonable expectations about the world.

d. Thus, truth-conditions in (21) predict that conditional (31a) is true.

(34) **Predicted Falsity of Conditional (31b)**

a. Consider a world w with the following properties:
- Goethe is alive today.
- Aside from this, all our 'reasonable expectations' about the world are fulfilled

b. In such a world, Goethe would definitely *not* have died in 1832
After all, since all our other 'reasonable expectations' are filled, people don't rise from the dead in such a world…

c. Thus, there is a world from the following set where "Goethe died in 1832" is *false*:
Those worlds which satisfy the proposition "Goethe is alive today", and which satisfy the *most* of our reasonable expectations about the world.

d. Thus, truth-conditions in (21) predict that conditional (31b) is *false.*

(35) **What Just Happened, In Abstract**

By introducing an 'ordering source' into our semantics for conditionals as in (21) we defeat the reasoning in (13c) that validates 'transitivity'.

a. <u>How The Semantics in (4) Validates the Reasoning in (13c)</u>

(i) *The Semantics in (4):*
[[If S_i then S_j]] = T *iff* $[[S_i]]_¢ \subseteq [[S_j]]_¢$

(ii) *The Reasoning That This Validates*

- If [[If S_1 then S_2]] = T *then* $[[S_1]]_¢ \subseteq [[S_2]]_¢$
- Clearly, then $\sim[[S_2]]_¢ \subseteq \sim[[S_1]]_¢$, and so [[If not S_2 then not S_1]] = T

b. <u>How the Semantics in (21) *In*validates the Reasoning in (13b)</u>

(i) *The Semantics in (21):*
[[If S_i then S_j]] = T *iff* $MAX_{<P} ([[S_i]]_¢) \subseteq [[S_j]]_¢$

(ii) *The Invalidation of (13b)*

- If [[If S_1 then S_2]] = T *then* $MAX_{<P} ([[S_1]]_¢) \subseteq [[S_2]]_¢$
- *But from this, it* ***doesn't*** *follow that* $MAX_{<P} (\sim[[S_2]]_¢) \subseteq \sim[[S_1]]_¢$
  And so it ***doesn't*** *follow that* [[If not S_2 then not S_1]] = T

- Why does it not follow?

Simply because the worlds from X **that satisfy the most propositions in P** is a subset of Y, it *doesn't follow* that the worlds from -Y **that satisfy the most propositions in P** are a subset of -X.

(Since one of the propositions in P might be one that conflicts with -X)

- To use a specific example:

The worlds in $[[\text{Goethe didn't die in 1832}]]_¢$ that satisfy the most of our 'reasonable expectations' are clearly a subset of the worlds in $[[\text{Goethe is dead today}]]_¢$

However, the worlds in $[[\text{Goethe is alive today}]]_¢$ that satisfy the most of our 'reasonable expectations' are *not* a subset of the worlds in $[[\text{Goethe died in 1832}]]_¢$
(Since one of our 'reasonable expectations' is that folks don't rise from the dead)

Thus, our augmented semantics in (21) achieves our 'preliminary goal' in (32)!

(36) **Summary**

Our augmented intensional semantics for conditionals – which incorporates an 'ordering source' into the meaning of the conditional – provides a solution to both the problems in (1a) and (1b):

- It allows conditionals to be false without the antecedent being true or the consequent being false.

- It predicts that the following inferences are not 'formally valid' for natural language conditionals:
  (i) *'Strengthening the Antecedent'*
  (ii) *'Transitivity'*
  (ii) *'Contraposition'*

But, how well does the account fare with respect to the problem in (1c), ***the meaning of conditionals that contain modals in their consequent?...***

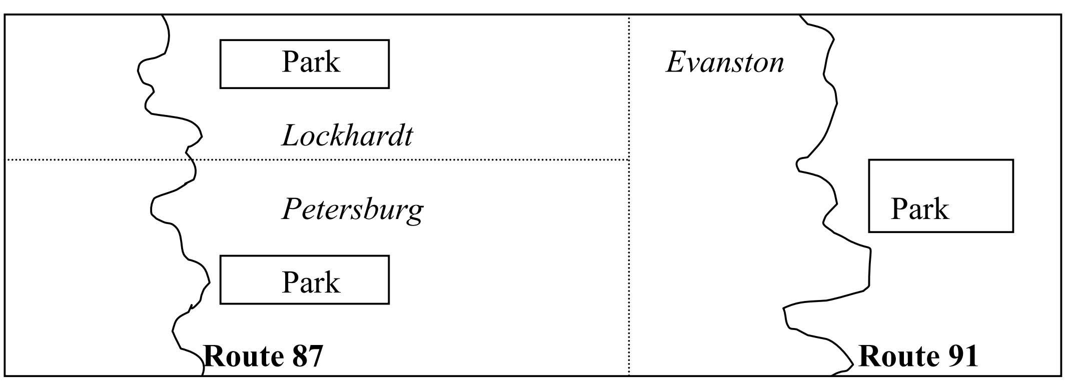
The answer is: NOT WELL!

2.1 Conditionals Containing Modals: Still a Problem

To see the problem, let us recall our crucial scenario from 'Part 1', along with the crucial data:

(37) **The Scenario: Lost on the Highway**

Sandy and Kim are driving to the town of Lockhardt for a party.
They've gotten lost, though, and no longer know (exactly) what road they are on.
However, they do know the following:
They are either on Route 87 or Route 91
They've just passed a park.
Sandy checks the map and sees the following picture:



- o Route 87 passes by two parks, one in Lockhardt and one in Petersburg
- o Route 91 posses by one park, in Evanston

(38) **The Crucial Data**

If, after looking at the map, Sandy uttered (38a), she would be saying something *true.*
If, after looking at the map, Sandy uttered (38b), she would be saying something *false*.

a. If we are on Route 87, then we might be in Lockhardt. (True in (37))
b. If we are on Route 91, then we might be in Lockhardt (False in (37))

... So, clearly, we need our theory of conditionals to predict that (38a) is <u>true</u> in context (37).
... and that (38b) is <u>false</u> in context (37).

Let's start off with Sentence (38b).
Let's also start off by assuming that our semantics just directly interprets the surface form (38b)

(39) **Truth-Conditions Predicted by Our Revised Intensional Semantics, Part 1**

a. <u>Structure at LF:</u>
[[If we are on Route 91] [then we might be in Lockhardt]]

b. <u>Predicted Truth Conditions</u>
$\forall$w' $\in$ MAX$_{<\{p:\ p \text{ is a 'reasonable expectation' in } w\}}$({w'': we are on Rte 91 in w''}):
$\exists$w''' $\in$ {w'''': what we know in w' is true in w''''} :
We are in Lockhardt in w'''

Every world w' in the following set is such that our knowledge in w' is consistent with our being in Lockhardt.
Those worlds, from the set of worlds at which we are on Rte 91
Which satisfy <u>the most</u> of our 'reasonable expectations' in w.

c. <u>Predicted Truth Conditions (Rough, Informal Statement</u>
In the (most plausible) worlds where we happen to *be* on Route 91, our *knowledge* is consistent with our being in Lockhardt.

If we happen to *be* on Route 91 now (whether or not we know it), then what we *know* (now) is consistent with our being in Lockhardt.

(40) **Problems with the Truth Conditions in (39c): General Problem**

These truth-conditions are 'weird', and don't correspond to what we intuitively interpret (38b) as saying.

- These truth-conditions assert that in all worlds where Sandy and Kim *are* on Route 91 (even if they don't know it), it's consistent with their knowledge that they are in Lockhardt.
- Thus, they assert a modal relationship between *just being* in a certain place and *having a certain knowledge state…*

(41) **Problems with the Truth Conditions in (39c): More Acute Problem**

- In the scenario as sketched out in (37), it was never said *whether in fact* the girls were on Route 87 or Route 91.
- Intuitively, then, if they happen to *actually be* on Route 91, it's still the case that their knowledge (at the time (38b) is uttered) is consistent with their being in Lockhardt.
- Thus, (at the time (38b)) is uttered), the worlds where Sandy and Kim are on **Route 91** *and* which satisfy the most of their 'reasonable expectations' (and so match the scenario sketched in (37)), are all worlds where it's consistent with their knowledge that they are in Lockahardt.
- **Thus, in scenario (37), the predicted truth-conditions in (39c) *hold*, and so our semantics wrongly predicts that (38b) is *true.***

So, our revised intensional semantics for conditionals (in (21)) does not assign to LF (39a) the intuitive meaning of (38b).

Let's try something else, though… Let's assume that by LF, the modal moves and takes scope over the entire conditional. *Does our semantics <u>now</u> give us the right truth-conditions for the sentences in (38)?...*

Again, the answer seems to be 'NO'
Here, the problem is the truth-conditions for sentence (38a)….

(42) **Truth-Conditions Predicted by Our Revised Intensional Semantics, Part 2**

a. <u>Structure at LF:</u>
MIGHT [[If we are on Route 87] [then we be in Lockhardt]]

b. <u>Predicted Truth Conditions</u>
$\exists$w' $\in$ {w'': what we know in w is true in w''} :
$\forall$w'' $\in$ $MAX_{<\{p:\ p \text{ is a 'reasonable expectation' in } w'\}}$({w''': we are on Rte 87 in w'''}):
We are in Lockhardt in w''

The following is consistent with our knowledge in w:
Every world w' in the following set is such that we are in Lockhardt in w':
Those worlds which, from the set of worlds where we are on Rte 87
Satisfy <u>the most</u> of our 'reasonable expectations' in w.

c. <u>Predicted Truth Conditions (Rough, Informal Statement</u>
The following might be true:
If we are on Route 87, then we are in Lockhardt.

(43) **The Problem with the Truth-Conditions in (42)**

- In the scenario sketched in (37), it's clear from the map that simply being on Route 87 *does not necessitate* that Sandy and Kim are in Lockhardt (since Petersburg also goes by 87 and has a park).

- Thus, consider any world w' that is consistent with Sandy and Kim's knowledge.

- Now consider those worlds w'' where they are on Route 87 *and* their 'reasonable assumptions' in *w'* are fulfilled (i.e., the scenarion in (37) holds)…

- *Some* of those worlds will be ones where they are Petersburg (since nothing about *w'* (*i.e.,* their knowledge in w) necessitates that they *aren't*).

- Thus, at *w'*, the conditional "if we are on Route 87 , then we are in Lockhardt" is *false*.

- Thus, at *any* possible world w' (consistent with Sandy and Kim's knowledge), the conditonal "if we are on Route 87, then we are in Lockhardt" is *false*.

- Therefore, the derived truth-conditions in (42b) fail to hold, and so our semantics in (21) (wrongly) predicts that sentence (38a) is *false* in scenario (37)…

(44) **Conclusion**

Even our improved intensional truth-conditions in (21) *still* won't derive the correct truth-conditions for conditionals that have modals in their consequents…

So, how do we fix this?...

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3. Conditional Antecedents as Modal Restrictors

In our quest for a 'better mousetrap', let's keep in mind a certain 'intuition' regarding the meaning of (38a,b) that we took note of back in 'Part 1' of our discussion:

(45) **A 'Guiding Intuition' Regarding (38)**

Sentence (38a) is *true* in context (37) because:
If Sandy and Kim ***learned*** *that they were on Route 87,*
they would conclude (given the map) that they might be in Lockhardt.

Sentence (38b) is *false* in context (37) because:
If Sandy and Kim ***learned*** *that they were on Route 91,*
they wouldn't conclude (given the map) that they might be in Lockhardt.

Following this intuition, we might paraphrase the (intuitive) meaning of (38a,b) as follows:

(46) **Paraphrases of the Truth-Conditions of (38a,b)**

a. If we are on Route 87, then we might be in Lockhardt.

If we add to our knowledge that we are on Route 87,
then our knowledge would be consistent with our being in Lockhardt.
(Thus, (38a) is *true* in scenario (37)…)_

b. If we are on Route 91, then we might be in Lockhardt.

If we add to our knowledge that we are on Route91,
then our knowledge would be consistent with our being in Lockhardt.
(Thus, (38b) is *false* in scenario (37)…)_

Finally, if we accept this 'paraphrase' of the truth-conditions of the sentences in (38), the following might be a more formal means of stating them:

(47) **Formal Statement of the Targeted Truth Conditions**

a. "If we are on Route 87, then we might be in Lockhardt" is T (at world w) *iff*

∃w' ∈ {w'': what we know in w is true in w'' *and* **we are on Route 87 in w''**}:
we are in Lockhardt in w'.

There is some world w' that is consistent with what we know and where we are on Route 87, and we are in Lockhardt in w'.

b. "If we are on Route 91, then we might be in Lockhardt" is T (at world w) *iff*

∃w' ∈ {w'': what we know in w is true in w'' *and* **we are on Route 91 in w''**}:
we are in Lockhardt in w'.

There is some world w' that is consistent with what we know and where we are on Route 91, and we are in Lockhardt in w'.

(48) **New Goal**

- Let us accept the truth-conditions in (47) as accurate.
- **Let us, then, develop a system that derives these truth-conditions…**

3.1 Step One: Simplifying Assumptions Regarding Modals (and the Modal Base)

Following our class discussion on 3/10 (regarding the homework on 'The Conversational Background'), let us assume the following model of the semantics of modals.

(49) **The Contextual Parameter *f***

In addition to the evaluation world *w* and the assignment function *g*, the semantic valuation function "[[ ]]" is parameterized to a given **conversational background,** written as *f*.

$[[XP]]^{w,g,f}$ = the extension of XP at w, relative to g and f

(50) **The Logical Type of *f***

The conversational background *f* is a function of type <s, <st>>, from possible worlds to sets of possible worlds.

Thus, the following are possible 'conversational backgrounds':

a. f_{EPIS} = λw. λw'. everything that is known in w is true in w'
b. f_{DEON} = λw. λw'. the law in w is followed w'

(51) **The Semantics of the Pronoun 'BASE'**

The value of the pronoun 'BASE' is determined – not by the assignment function *g* – but by both the conversational background *f* and the evaluation world *w*.

$[[BASE]]^{w,g,f}$ = *f* (w)

Thus, the pronoun BASE is of type <s, t>.

(52) **The Semantics of Modal Auxiliaries**

A modal auxiliary is of type < <st> <st,t>>. [3]

a. $[[might]]^{w,g,f}$ = $\lambda B_{<st>} . \lambda p_{<st>} . \exists w' \in B . p(w') = T$

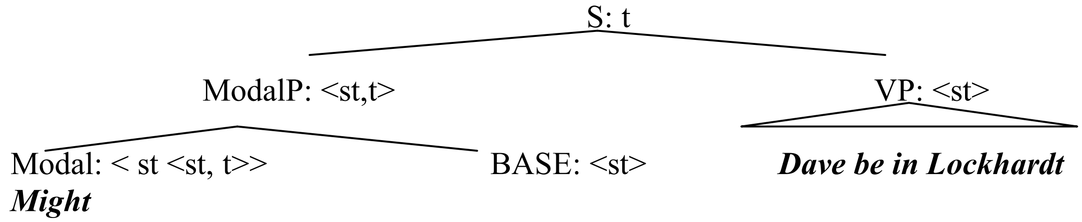
b. $[[must]]^{w,g,f}$ = $\lambda B_{<st>} . \lambda p_{<st>} . \forall w' \in B . p(w') = T$

[3] Throughout our informal discussion of modals here, we've been ignoring the 'Ordering Source' component. We will continue to do so in our formal treatment, though the Ordering Source will re-appear on a future homework.

(53) **Quick Illustration of the System**

a. <u>Sentence:</u> Dave might be in Lockhardt.

b. <u>Structure:</u>



c. <u>Background Assumpton</u>

f = λw. λw'. everything that is known in w is true in w'

d. <u>Derivation of Truth-Conditions</u>

i. [[Dave might be in Lockhardt]]^{w,f} = T *iff* (by FA, IFA)

ii. [[might]]^{w,f} ([[BASE]]^{w,f}) ([[Dave be in Lockhardt]]¢) = T *iff* (by FA, Lex.)

iii. [[might]]^{w,f} ([[BASE]]^{w,f}) (λw'. Dave is in Lockhardt in w') = T *iff* (by (51))

iv. [[might]]^{w,f} (f(w)) (λw'. Dave is in Lockhardt in w') = T *iff* (by (53c))

v. [[might]]^{w,f} (λw'. everything that is known in w is true in w')
(λw'. Dave is in Lockhardt in w') = T *iff* (by Lex.)

vi. [λB . λp . ∃w'' ∈ B . p(w'') = T] (λw'. everything that is known in w is true in w')
(λw'. Dave is in Lockhardt in w') = T *iff* (by LC)

vii. ∃w'' ∈ { w' : everything that is known in w is true in w' }
[λw'. Dave is in Lockhardt in w'](w'') = T *iff* (by LC)

viii. **∃w'' ∈ { w' : everything known in w is true in w' } : Dave is in Lockhardt in w''**

(54) **Main Features of the System**

- The pronoun "BASE", which provides the restriction of the modal, is of type <st>
- The modal auxiliary itself is a function of type <st, <st,t>>

3.2 Special Assumptions Regarding Conditionals

Let us make the following, crucial assumption regarding the semantics and syntax of the antecedent of a conditional

(55) **Antecedents are Propositions**

$[[\text{ if S }]]^{w,} \quad = \quad [[\text{ S }]]_{¢}$

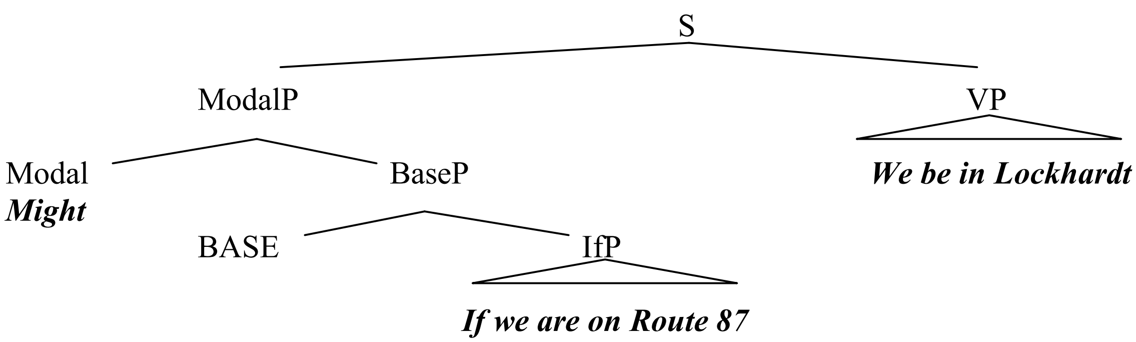
(56) **Antecedents are Adjuncts to 'BASE'**

a. The antecedent of a conditional – the clause prefixed by "if" – is initially merged as an adjunct to the pronoun 'BASE'.

b. By 'Spell Out', the antecedent can move from its base position (adjoined to BASE), to some left (or right) peripheral position in the clause.

c. However, by LF, the antecedent is *reconstructed* back into its base position.

d. Thus, at LF, the form of a conditional such as (38a) is the following:



> *But, if this is the structure at LF…*
> *<u>How</u> does the semantics interpret a phrase like 'BaseP'?*

Well, recall the following rule from our review lectures on extensional semantics?...

(57) **The Rule of 'Predicate Modification' (From 'Extensional Review; Part 1')**

If a phrase X is a structure consisting of two daughters – Y and Z – and if both Y and Z are of type $< \sigma\ \tau >$, then $[[X]] = [\ \lambda x_{\sigma} .\ [[Y]](x)\ \&\ [[Z]](x)\ ]$

Note, notice the following:

- $[[\text{ BASE }]]^{w,f,g}$ is of type <st>
- $[[\text{ if S }]]^{w,f,g}$ is of type <st>
- **Thus, we can interpret 'BaseP' using the rule in (57)**

(58) **The Semantics of 'BaseP'**

a. $[[\text{ BaseP }]]^{w,f}$ = (by (55))

b. [λw' . $[[\text{BASE}]]^{w,f}$ (w') & $[[\text{ If [we are on Rte 87] }]]^{w,f}$(w')] = (by (51))

c. [λw' . *f*(w)(w') & $[[\text{ If [we are on Rte 87] }]]^{w,f}$(w')] = (by (53c))

d. [λw' . [λw''. λw'''. everything that is known in w'' is true in w'''](w)(w')
& $[[\text{ If [we are on Rte 87] }]]^{w,f}$(w')] = (by LC)

e. [λw' . everything that is known in w is true in w'
& $[[\text{ If [we are on Rte 87] }]]^{w,f}$(w')] = (by (55))

f. [λw' . everything that is known in w is true in w'
& [λw''. we are on Rte 87 in w''](w')] = (by LC)

g. **[λw' . everything that is known in w is true in w' & we are on Rte 87 in w']**

(59) **What We've Just Done**

- We've analyzed the antecedent of the conditional – the *if clause* – as a *modifier* of the modal base.
- To put it differently, in this system, the antecedent of a conditional functions to *further restrict* the restriction of the modal!

We now have enough to compositionally derive the targeted truth conditions in (47)!

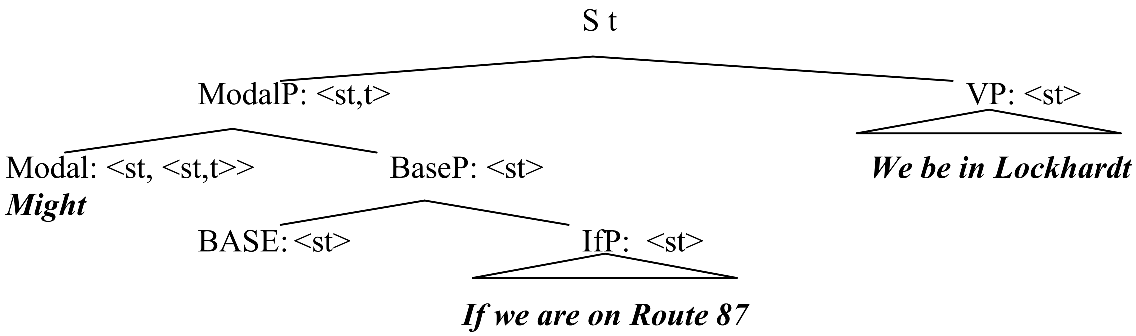
… this is shown below…

3.3 Deriving the Targeted Truth Conditions

(60) **Demonstration of the System**

a. Sentence: If we are on Route 87, we might be in Lockhardt.

b. Structure:



c. Background Assumpton

f = λw. λw'. everything that is known in w is true in w'

d. Derivation of Truth-Conditions

i. $[[\text{ S}]]^{w,f}$ = T *iff* (by FA, IFA)

ii. $[[\text{ might }]]^{w,f}$ ($[[\text{BaseP}]]^{w,f}$) ($[[\text{ we be in Lockhardt }]]_{¢}$) = T *iff* (by FA, Lex.)

iii. $[[\text{ might }]]^{w,f}$ ($[[\text{BaseP}]]^{w,f}$) (λw'. we are in Lockhardt in w') = T *iff* (by (58))

iv. $[[\text{ might }]]^{w,f}$ (λw' . everything known in w is true in w' & we are on Rte 87 in w')
(λw'. we are in Lockhardt in w') = T *iff* (by Lex.)

v. [λB . λp . ∃w'' ∈ B . p(w'') = T]
(λw' . everything known in w is true in w' & we are on Rte 87 in w')
(λw'. we are in Lockhardt in w') = T *iff* (by LC)

vi. ∃w'' ∈ { w' . everything known in w is true in w' & we are on Rte 87 in w' }
[λw'. we are in Lockhardt in w'](w'') = T *iff* (by LC)

viii. **∃w'' ∈ { w' : everything known in w is true in w' & we are on Rte 87 in w' } :**
we are in Lockhardt in w''

There is some world w' that is consistent with what we know *and* *where we are on Route 87, and we are in Lockhardt in w'.*

4. Discussion

The system developed in Section 3 – which analyzes conditional antecedents as *modifiers* (or *restrictors*) of the modal base – assigns the more accurate truth-conditions in (47) to the sentences in (38), and so correctly predicts:

- Sentence (38a) is true in context (37)
- Sentence (38b) is false in context (37)

It also makes accurate predictions regarding the sentences in (61) below.

(61) **Conditions that Contain *Strong* Modals in the Consequents**

a. If we are on Route 87, then we **must** be in Lockhardt. (False in context (37))
b. If we are on Route 91, then we **must** not be in Lockhardt. (True in context (37))

The system developed in Section 3 predicts that the sentences will have the T-conditions below:
(Proof left as an exercise to the reader…)

(62) **Truth-Conditions Predicted for the Sentences in (61)**

a. Truth Conditions for (61a)
∀w'' ∈ { w' : everything known in w is true in w' & we are on Rte 87 in w' } :
we are in Lockhardt in w''

In all worlds w' that are consistent with what we know <u>*and*</u> *where we are on Route 87, we are in Lockhardt in w'.*

b. Truth Conditions for (61b)
∀w'' ∈ { w' : everything known in w is true in w' & we are on Rte 91 in w' } :
we are not in Lockhardt in w''

In all worlds w' that are consistent with what we know <u>*and*</u> *where we are on Route 91, we are* <u>*not*</u> *in Lockhardt in w'*

(63) **Truth-Conditions in (62) Predict Judgements in (61)**

a. For Sentence (61a)
In context (37), there is a possible world which is consistent with the information on the map, and in which Sandy & Kim are on Route 87, but where they are in *Petersburg* – not Lockhardt.

b. For Sentence (61b)
Consider any possible world which is consistent with the information on the map, and in which Sandy & Kim are on Route 91. Clearly, since 91 doesn't go through Lockhardt, in any such possible world, they are not in Lockhardt.

(64) **An Important Question**

So, the system in Section 3 works very well for conditions whose consequents contain modals….

…But what about conditionals whose consequents <u>don't</u> contain modals?

(65) **One Possibility: Systematic Ambiguity**

Perhaps the analysis in Section 3 is correct for conditionals whose consequents contain modals, and the analysis in Section 2 is correct for conditionals whose consequents *don't* contain modals?...

(66) **Against Ambiguity**

- We saw in Section 2.1 that the analysis in (21) *incorrectly* predicts readings that don't exist for conditionals whose consequents contain modals.
- Thus, if the semantics in (21) is a grammatical possibility, it must be one that is systematically *unavailable* for conditionals that happen to have modals in their consequent.
- What could rule out the interpretation in (21) for such conditionals is unclear…
  (That is, how does one *prevent* conditionals with modal consequents from receiving the interpretation in (21), and thereby generate readings that don't exist?...)

(67) **Another Possibility: The Ubiquity of Modals**

Contrary to first appearances, whenever you have a conditional, there is *some* (possibly covert) modal operator whose 'BASE' the antecedent is modifying / restricting.

This isn't such a crazy idea…
Note that in all the other conditionals we've seen in this handout, there ***is*** *a modal-type element in the consequent… namely, the auxiliary 'will' ('would'):*

(68) **The Auxiliary 'Will' as a Modal Restricted by the Conditional Antecedent**

a. If the sun explodes tomorrow, then Alpha Centauri **will** explode tomorrow too.
b. If I wear the same sweater every day, then the sun **will** not explode next week.
c. If kangaroos had no tails, they **would** fall over.
d. (Even) if Goethe hadn't died in 1832, he **would** still be dead today

Future homework problem: analyze the sentences above using the system from Section 3

Of course, it *is* possible for conditional consequents not to contain any clear modal element at all.

(69) **Conditionals with No Overt Modals Elements in their Consequents**

a. If it is three o'clock, then Dave is in NYC.
b. If the red blotches don't go away, then you have eczema.
c. If the Red Sox trade Ortiz, there's no chance for the pennant.
d. If *Dave* is worried, then *I'm* definitely worried.

Note, however, that each of these conditionals are semantically equivalent to a conditional that *does* contain an overt modal in its consequent.

(70) **Semantically Equivalent Conditionals that *Do* Contains Modals**

a. If it is three o'clock, then Dave **must** be in NYC.
(*'epistemic' must*)

b. If the red blotches don't go away, then you **must** have eczema.
(*'epistemic' must)*

c. If the Red Sox trade Ortiz, there **will** be no chance for the pennant.

d. If Dave is worried, then I **will** definitely be worried.

(71) **Idea: Even When You Don't Hear a Modal, it's Actually There**

Perhaps the actual syntactic structure of the conditionals in (69) is directly akin to the structure of the equivalent sentences in (70).

However, in (69) the modal that the conditional antecedent restricts if phonologically empty…

a. If it is three o'clock, then Dave $\varnothing_{\mathbf{MUST}}$ [is in NYC].
b. If the red blotches don't go away, then you $\varnothing_{\mathbf{MUST}}$ [have eczema].
c. If the Red Sox trade Ortiz, there $\varnothing_{\mathbf{WILL}}$ [is no chance for the pennant].
d. If Dave is worried, then I $\varnothing_{\mathbf{WILL}}$ [am definitely worried].

(72) **Potential Problem**

How do we constrain these 'invisible' modals so that they appear only in conditionals?...

(73) **Possible Answer**

Maybe we don't! Consider discourses like the following…

a. Doctor and Patient (With Gloves On)

Patient: I don't want to take them off, but underneath my gloves are red blotchy patches that itch and won't go away!
Doctor: Well, then, you have eczema.

(i) Assessment:
The doctor isn't fully asserting that the patient has eczema.
Rather, it's more of a *conditional* assertion…
… 'Given what you say, you must have eczema'…

b. Two Sports Fans, One of Whom is a Persistent Liar

Fan 1 (Liar): Did you hear the news?
Fan 2: This isn't another lie, is it?
Fan 1: No, I swear! The Sox are gonna trade Ortiz!
Fan 2: Really? You swear?
Fan 1: Yes, I swear!

Fan 2: Well, then, there's no chance for the pennant.

(i) Assessment:
Fan 2 isn't really, fully asserting that there is no chance for the pennant.
Again, it's more of a conditional assertion…
… 'Given what you way, there will be no chance for the pennant'

(74) **Some Final Commentary**

Combining the analysis in Section 3 with the hypothesis that there are null modals will provide a theory of conditionals that resolves all of the problems in (1).

a. Even under the analysis in Section 3, conditionals involve quantification over possible worlds, and so we don't predict them to be false *only* when the antecedent is true and the consequent is false.

b. Even under the analysis in Section 3, the modal quantification in a conditional will be restricted by an *ordering source* (provided, ultimately, by the modal), and so the invalid inferences in (13) will not go through.

c. The analysis in Section 3 provides an accurate account of the meaning of conditionals whose consequents contain (overt) modals.

(75) **A Choice Quote from Angelika (Kratzer 1986)**

The history of the conditional is the story of a syntactic mistake. There is no two-place "if…then" connective in the logical forms of natural langaugs. "If"-clauses are devises for restricting the domains of various operators. ***Whenever there is no explicit operator, we have to posit one****. As shown above, epistemic modals are candidates for such hidden operators...*

…Irene Heim argues in her dissertation (1982) that the operators that have to positied in order to obtain a unified semantics for "if"-clauses might have an additional use: they might act as unselective binders for variables… ***Heim's work, then, gives further support to the hidden operator approach to bare indicative conditionals…***

(Kratzer 1986, p. 11-12; emphasis added)

Kratzer, Angelika (1986) "Conditonals" *Chicago Linguistics Socity* 22(2): 1-15. Available at: http://semanticsarchive.net/Archive/ThkMjYxN/Conditionals.pdf