Some Very Basic Questions on the Semantics of Plurals

(1) **Computing Sets of Pluralities**

Please list the members of the following set: $*\{ a, b, c, d, e \}$

(2) **A Key Property of Plural NPs**

First, note that our plurality forming operator "+" has the following property: for any two objects, a and b, the entity 'a+b' is defined.

Given this property of "+", please prove that the following claim is true: for any set S, MAX(*S) is defined.

(3) **Some Terms from Lattice Theory**

In the literature on plurals, it's common to encounter terms like 'lattice' or 'sub-semilattice', *etc.* These terms come from a field of mathematics known as 'lattice theory'. Such terms show up in the literature on plurals because they offer handy ways of describing and characterizing the domain of plural individuals.

This question will offer some key background and then a few comprehension questions. Let's start with our plurality forming operator "+". In class, we saw that this operator can define the following relation on our domain of (plural) entities:

a. $x \leq y$ *iff* $\exists z.\ x+z = y$

Now, given any ordering relation like "≤" above, we can define certain functions:

b. <u>'Supremum' of x and y</u>

$SUP(x,y) = z$ *iff* $x,y \leq z$ and $\neg\exists s\ .\ s \neq z\ \&\ x,y \leq s \leq z$
The smallest entity that contains both x and y

Example:
$SUP(\ a+b+d\ ,\ b+c\ ) = a+b+c+d$

c. <u>'Infimum' of x and y</u>

$INF(x,y) = z$ *iff* $z \leq x,y$ and $\neg\exists s\ .\ s \neq z\ \&\ z \leq s \leq x,y$
The biggest entity that is a part of both x and y

Example:
$INF(\ a+b+d\ ,\ b+c\ ) = b$

Now, using these concepts of 'supremum' and 'infimum', we can define some key concepts from lattice theory.

d. Some Key Terms from Lattice Theory

(i) *Lattice*
A pair consisting of a set S and an ordering relation $< S, \leq >$ is a **lattice** *iff* for every a,b ∈ S, both SUP(a,b) and INF(a,b) are defined (i.e., exist).

(ii) *Meet Semilattice*
A pair consisting of a set S and an ordering relation $< S, \leq >$ is a **meet semilattice** *iff* for every a,b ∈ S, INF(a,b) are defined (i.e., exist).

(iii) *Join Semilattice*
A pair consisting of a set S and an ordering relation $< S, \leq >$ is a **join semilattice** *iff* for every a,b ∈ S, SUP(a,b) are defined (i.e., exist).

e. THE QUESTION
Consider now the pair $<{*D}, \leq>$ consisting of our domain of plural entities *D and the relation defined in (3a) above. *Is this pair a lattice, a meet semilattice or a join semilattice?*

(4) **A Little Bit on Plurals and Mass Nouns**

We won't be explicitly discussing mass nouns in class, but Lasersohn (2008) lays out some of the key issues in the literature. This question reinforces some of the ideas regarding mass nouns discussed by Lasersohn.

Let the extension of the mass nouns "water" and "furniture" be as follows.

a. [[water]] = { x : x is water }
b. [[furniture]] = { x : x is furniture }

c. THE QUESTION
Which of the following properties do the mass nouns 'water' and 'furniture' share with the plural NP 'boys'?

(i) *Cumulative Reference:*
If x, y ∈ [[XP]], then x+y ∈ [[XP]]

(ii) *Distributive Reference:*
If x+y ∈ [[XP]], then x, y ∈ [[XP]]

(iii) *Non-Atomicity*
If x ∈ [[XP]], then ∃z. z ≠ x & z ≤ x & z ∈ [[XP]]

Given your answers, could any of these properties be what distinguishes mass nouns from count nouns?