

**Towards an Intensional Semantics, Part 1:  
Sentential Complementation and the Formalization of Intensions<sup>1</sup>**

**1. The Inadequacies of a Purely Extensional Semantics**

**(1) Our Current System: A Purely Extensional Semantics**

- a. The interpretation function “[[ ]]” is (always) a function from natural language expressions to their *extensions* in the (actual) world.
- b. The extension of a complex phrase is (always) derived by computing the *extensions* (and *only* the extensions) of its component parts.

**(2) The Surprising Power of This System**

- As we have seen throughout this semester, a purely extensional system of this sort is *surprisingly powerful*.
- Such a system is able to correctly derive T-conditions for a wide variety of sentential structures in English.
- ...*However, powerful as it is, a purely extension system is not sufficient for the interpretation of natural language...*

**(3) An Empirical Problem: Counter-Factual Language**

Consider sentences like the following:

- a. Obama *might* smoke.
- b. *If* Joe smoked, then Obama *wouldn't* be alone.
- c. Michelle wants *Obama to quit smoking*.

The truth of these sentences doesn't depend upon whether Obama smokes in the *actual world/situation* – but whether Obama smokes in certain *purely hypothetical/imaginary worlds/situations*...

The Problem: There is nothing in our purely extensional ‘semantic toolkit’ that seems to offer a decent treatment of such ‘hypothetical / counterfactual’ language.

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<sup>1</sup> These notes are based upon material in Heim & Kratzer (1998: Chapter 12), as well as Chierchia & McConnell-Ginet (2000: Chapter 5).

(4) **Acute Empirical Problem: *Some Semantic Arguments Can't be Extensions***

Consider the verb “believe”; from sentences like the following, it seems to have a meaning that combines with the meaning of a sentence (its complement clause).

a. Lou believes [ that Obama smokes ].

In our (purely) extensional semantics, the ‘meaning’ of a sentence is a truth value.

b. [[ Obama smokes ]] = T

So, if we were to analyze the verb “believes” in our (purely) extensional semantics, we would have to view it as a function of type  $\langle t \langle e t \rangle \rangle$ .

But, now consider the fact that the extension of (4a) [in the actual world] is T

c. [[ Lou believes [that Obama smokes] ]] = T

Thus, the extension of “believes” must (qua function) contain the following ordered pair:  $\langle T, \langle \text{Lou}, T \rangle \rangle$ .

But, now consider that the extension of the following sentence is *also* T:

d. [[ Obama is a natural-born citizen ]] = T

Thus, our extensional semantics for “believe” would entail/predict that:

e. [[ Lou believes [that Obama is a natural-born citizen] ]] = T

**Epic Fail:**

Our putative extensional semantics for “believe” makes the *obviously false* prediction that **if X believes one true/false sentence, then X believes all true/false sentences!**

...But this obviously false prediction is a necessary consequence of two core assumptions of our purely extensional semantic system:

- (i) The semantic value of a structure is (always) its extension
- (ii) The extension of a sentence is its truth value.

(5) **The Key Conclusion: A Purely Extensional Semantics is Not Enough**

For words like “believe”, their meaning does *not* combine with the *extension* of their sentential complement (*cf.* purely extensional ‘boolean operators’ like *and, or, not*).

- Thus, in this structural context, our ‘semantic valuation’ function “[ ]” has to provide something *other* than the extension of the complement clause.
- Thus, for sentences containing the verb “believes”, their extension (T-value) is not determined purely by computing the *extensions* of their component parts.
- **Thus the core assumptions of our purely extensional semantics in (1) are wrong!**

2. **Towards a Solution: Intensions?**

*Interim Conclusion:*

The T-conditions of sentences containing *believes* suggests that *believes* has a meaning that doesn’t take the *extension* of its sentential complement as argument...

*Question:*

- What, then, does the meaning of *believes* take as its first argument?
- What is the semantic contribution of the sentential complement of *believes*, if not its *extension*?

(6) **Recap: The Distinction Between ‘Intension’ and ‘Extension’**

- a. The *extension* of a phrase is the thing ‘out in the world’ that it ‘picks out’.
  - (i) The *extension* of a definite description is the thing it refers to  
EXTENSION(the president) = Barack Obama
  - (ii) The *extension* of a sentence is its T-value  
EXTENSION(the president smokes) = TRUE
- b. The *intension* of a phrase is (vaguely put) the ‘general concept’ behind the phrase, which determines (for a given time/situation) what the *extension* of the phrase is.
  - (i) The *intension* of a definite description is (vaguely) the ‘conceptual content’ of the description.
  - (ii) The *intension* of a sentence is (vaguely) its T-Conditions

*Observation:* Given our formal theory of extensional semantics, we have a rather more precise idea of what the *extension* of a phrase is than what its *intension* is.

(7) **Key Observation**

Sentences with the same *extension* (truth value) can nevertheless have two different *intensions* (truth conditions)

- “Obama smokes” is T *iff* **Obama smokes**
- “Obama is a natural-born citizen” is T *iff* **Obama is a natural-born citizen.**

(8) **One Line of Thought...**

Given the observation in (7), if *believes* took the *intension* of its sentential complement as argument (rather than its *extension*), we could avoid the false prediction in (4)!

- Since the intension of “Obama smokes” is distinct from that of “Obama is a natural-born citizen”, Lou can stand in the ‘believes’-relation to the former, but not the latter!

(9) **Some Independent Motivation**

*Question:* What kind of relation does the verb “believes” represent?

- a. Not a Relation Between an Entity and Truth Value  
See reasoning above in (4)...
- b. Not a Relation Between an Entity and a Sentence
  - The following seems true: “Julius Caesar believed that Gaul surrendered.”
  - But, what kind of possible relation could Julius Caesar have had to the *Modern English sentence* “Gaul surrendered” ?  
... he never said it, he never thought it, he never assented to it...
- c. Relation Between an Entity and a Sentential Intension (T-Conditions, Proposition)
  - Even though Julius Caesar never uttered or assented to the *English sentence* “Gaul surrendered”, he *did* utter and assent to a *Latin sentence* that had the same T-conditions / intension.
  - So really, *believes* seems to denote a relation between an individual and some sentential *intension*, which we could label a ‘proposition’...

(10) **Conclusion**

- From the considerations in (8) and (9), it seems that the meaning of *believes* takes as argument the *intension* of its sentential complement.
- Thus, our semantic valuation function “[[ . ]]” must be able, in some contexts to deliver *intensions* rather than extensions as values...

### 3. Formalizing the Notion of an Intension

*Interim Conclusion:* The verb *believes* has an extension which takes as argument the *intension* of its sentential complement.

#### (11) Problem: How Do We Actually Model This?

- Thus far, our concept of an ‘intension’ has been quite vague (6b)...
- Indeed, it’s been allowed to stay vague because so far our formal semantic system hasn’t needed to operate over them...
- But, if we want a formal system that ‘manipulates’ intensions, we need some kind of a formal model of what an ‘intension’ is.

#### (12) Towards A Formal Model of ‘Intensions’: The Basic Idea

- a. The Core Property of an ‘Intension’ (6b)  
For any structure X, the ‘intension’ of X determines the extension of X (in the actual world/situation).
- b. The Formal Insight  
Thus, the ‘intension of X’ can be thought of as a *function!*
  - It takes a world/situation as argument/input...
  - ...and gives the extension of X at that world/situation as output!

#### (13) Problem: What’s a World/Situation?

If we want to treat intensions as functions in the way (12b) suggests, we need to get clear on what their argument/inputs are...

#### (14) Novel Terminology

Instead of talking loosely of ‘worlds’ and ‘situations’, we will employ the more precise (though still confusing) philosophical notion of a *possible world*.

#### (15) The Metaphysics of ‘Possible Worlds’ (Leibniz, Kripke, Lewis)

- The ‘real world’ is more-or-less the sum total of all the facts in the universe throughout time.
- However, the real world is only one of an infinite number of other, *possible* worlds.
- The facts in these other possible worlds may be different from those of the real world.
- For example, there are possible worlds where:
  - John McCain won the 2008 presidential election.
  - Human beings never evolved.
  - People can fly using telepathy.
  - Everything else is the same, except that the letter “s” is written backwards.

(16) **The World-Dependency of an Expression's Extension**

Clearly, if we buy into the metaphysics in (15), the extension of an expression will depend upon which possible world the expression is evaluated in.

Examples:

- a. In any possible world where McCain won the 2008 election, the extension of "the president" is John McCain.
- b. In any possible world where Obama never started smoking, the extension of "Obama smokes" is *False*.

So, under the metaphysics in (15), an expression will only have a given extension *relative* to some possible world...

... therefore, it no longer makes sense to talk about an expression's 'extension' in any absolute sense...

...So, let us adjust our notation for calculating extensions accordingly...

(17) **New Notation**       $[[ X ]]^w$       'the extension of X at world w.'

Observation: We are adding into our 'contextual parameters' (which already includes *g* and *C*) the possible world *w* that an expression is uttered in.

(18) **Relativized T-Conditional Statements**

For any world *w*,  $[[ \text{Obama smokes} ]]^w = T$  iff Obama smokes in *w*

- This generalization does seem to be intuitively true...

Consider a world where Obama doesn't smoke. "Obama smokes" is F there.

Consider a world where Obama does smoke. "Obama smokes" is T there.

- Knowing this generalization is functionally equivalent to knowing the T-conditions of a sentence.

Suppose we assume the speaker is being truthful in saying "Obama smokes".

Given the generalization above, we can conclude that the world we are in (whatever that world is) is a world where Obama smokes...

Key Question:

Can we augment our extensional semantics so that it is able to derive this generalization?

(19) **Initial Question**  
Relative to some world  $w$ , what is the extension of “smokes”?

(20) **Answer**  
It’s that function which takes an entity  $x$  and gives T *iff*  $x$  smokes **in that world  $w$**

(21) **Summary in Notation**      For any world  $w$ ,  $[[\text{smokes}]]^w = [\lambda x : x \text{ smokes in } w ]$

(22) **Initial Question**  
Relative to some world  $w$ , what is the extension of “Obama”?

(23) **Answer (Controversial)**  
In any possible world, the name “Obama” just picks out Obama, the same guy across all possible worlds.

(24) **Summary in Notation**      For any world  $w$ ,  $[[\text{Obama}]]^w = \text{Obama}$

(25) **A Note on Rigid Designation**

- Following the lexical entry in (24), the entity that “Obama” has as its extension is the *same* in all possible worlds.
- The term for a word that has the same extension in all possible worlds is a *rigid designator* (Kripke)
- For now, we can view this treatment of proper names as just a ‘simplifying assumption’...
- ...however, it is a rather serious (and widely-held) philosophical position, one that has some important argumentation in support of it...

*We almost have all we need in order to derive generalizations like (18)...*

*... all we need is one minor adjustment to our rule of Function Application*

(26) **Function Application**

If  $X$  is a branching node that has two daughters –  $Y$  and  $Z$  – and if  $[[Y]]^w$  is a function whose domain contains  $[[Z]]^w$ , then  $[[X]]^w = [[Y]]^w ( [[Z]]^w )$

(27) **Derivation of Relativized T-Conditions**

Let  $w$  be any possible world:

- a.  $[[ \text{Obama smokes} ]]^w = T$  iff (by FA)
- b.  $[[ \text{smokes} ]]^w ( [[ \text{Obama} ]]^w ) = T$  iff (by TN, (24))
- c.  $[[ \text{smokes} ]]^w ( \text{Obama} ) = T$  iff (by TN, (21))
- d.  $[\lambda x : x \text{ smokes in } w ] ( \text{Obama} ) = T$  iff (by notation)
- e. Obama smokes in  $w$ .

(28) **What Have We Done So Far?**

- We've introduced an ontology/metaphysics of possible worlds.
- We noted that, following this ontology/metaphysics, the extension of an expression depends upon the world relative to which the expression is being evaluated.
- We noted that, consequently, T-conditional statements for sentences of natural language should appear explicitly relativized to possible worlds, as in (18).
- We therefore augmented our semantic system so that it could:
  - a. Capture these new, relativized T-conditional statements, and
  - b. Explicitly reflect the fact that the extension of an expression crucially depends upon the possible world that the expression is being evaluated in.

*We actually now have all the machinery in place to deliver a formal theory of 'intensions' as semantic values....*

*... recalling the 'key idea' (29), consider the function in (30)...*

(29) **An 'Intension' as a Function (12b)**

The 'intension' of an expression  $X$  can be thought of as a *function!*

- It takes a possible world as argument/input...
- ...and gives as value/output the extension of  $X$  at that possible world.

(30) **The Intension of "Obama Smokes" (?)**

The following functional descriptions are all equivalent.

- a.  $[\lambda w : [[ \text{Obama smokes} ]]^w = T ]$
- b.  $[\lambda w : \text{Obama smokes in } w ]$
- c. *The function from possible worlds to T values, which when given a possible world  $w$  as an argument, yields T iff Obama smokes in  $w$*
- d. *The function from possible worlds to T values, which given a possible world  $w$  as argument, yields **the extension of "Obama smokes" in  $w$***

(31) **Key Observation**

Given that (30a,b) are just the functions in (30c,d), it follows that we can regard the formulae in (30a,b) as describing **the intension of the sentence “Obama smokes”**.

*This basic result generalizes to all structures of natural language...*

(32) **The Intension of “X”**

Recall the definition of the following notation from (17):

a.  $[[ X ]]^w$  = the extension of X at world w.

Thus, the function in (32b) clearly can be characterized by the prose in (32c)

b.  $[ \lambda w : [[ X ]]^w ]$

c. *The function whose domain is the set of possible worlds, and when given a possible world w as argument, yields **the extension of X at w** as its value.*

**Thus, following our ‘targeted formal insight’, the function in (32b) is identifiable as ‘the intension of X’**

(33) **General Conclusion**

For any structure X, the function ‘  $[ \lambda w : [[X]]^w ]$  ’ is the *intension of X*.

(34) **Quick Amendment**

Due to the vagaries of our idiosyncratic lambda notation, if X is of type t, then its intension is more perspicuously written as ‘  $[ \lambda w : [[X]]^w = T ]$  ’

(35) **Some New Terminology for Certain Kinds of Intensions**

a. Proposition: *function from worlds to truth values (the intension of a sentence)*

b. Property: *function from worlds to <et> functions (the intension of a VP, NP)*

c. Individual Concept: *function from worlds to entities (the intension of a DP)*

(36) **‘Intensional-izing’ our Theory of Types**

- The reason why we’ve developed this formal treatment of ‘intensions’ is that we want these objects to sometimes serve as the *semantic value* of an expression.
- Therefore, our system of semantic types will necessarily have to incorporate such ‘intensional types’.
- The following is the simplest, most appropriate way to accordingly expand our system of semantic types:

a. The Set of Possible Worlds

Besides  $D_t$  and  $D_e$ , there is also a set  $W$ , taken to be the set of all possible worlds.

b. Inductive Statement for Intensional Domains

If  $D_X$  is a domain, then  $D_{\langle s, X \rangle}$  is a domain, and is the set of all functions from  $W$  into  $D_X$ .

c. Inductive Statement for Intensional Types

If  $X$  is a type, then  $\langle s, X \rangle$  is a type.

*Thus, for every semantic type (extension)  $X$  of our earlier extensional semantics, we also have the ‘intensional type’  $\langle s, X \rangle$  of functions from possible worlds to things of type  $X$ .*

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**4. The Truth-Conditions of ‘Belief’-Sentences**

(37) **What We Have Thus Far**

- An informal hypothesis that the meaning of the verb “believes” takes as argument the *intension* of its complement clause.
- A formalized theory of what *intensions* are, which allows us to compute the intension of a complex phrase from the ‘meanings’ of its component parts.

(38) **What We Need to Do Now**

Develop a lexical entry for the verb “believes” which accomplishes the following:

- a. Takes as argument the (formalized) *intension* of its sentential complement.
- b. Captures the intuitive, logical content of the verb “believes” in a perspicuous way.

(40) **Problem**

*Before we can hypothesize a lexical entry for “believes”, we need to have a more refined theory of the T-conditions of sentences containing it...*

*So... what are the truth conditions of sentences containing the verb “believes”?...*

(41) **Core Observation 1**

A person’s beliefs seem to determine a particular set of possible worlds: those possible worlds which are *compatible* with the person’s beliefs.

a. Compatibility With ‘the Beliefs of X’

For any entity X, the *worlds compatible with the beliefs of X* are those worlds which X would recognize as possibly being the actual world.

b. Illustration:

- Let’s suppose I believe only one thing: the Earth is flat.
- Let’s suppose we’re playing a game where:
  - (i) You submit to me various possible worlds  $w$  from  $W$ , and
  - (ii) I have to say whether or not I think that  $w$  could be the actual world
- a. If you were to submit to me a world where the Earth is *round*, I would say “No; according to my beliefs, this could not be the actual world”
- b. If you were to submit to me a world where the Earth is *flat*, I would say “Yes; according to my beliefs, this could be the actual world”.
- c. Since I only believe a finite number of things (namely, that the Earth is flat), there will be *many* worlds that I say ‘yes’ to.
  - Suppose we have two worlds  $w'$  and  $w''$ :
  - In  $w'$ , the Earth is flat, and there are three million tree frogs.
  - In  $w''$ , the Earth is flat, and there are two million tree frogs.
  - Since I don’t have an opinion about how many tree frogs there are, I would accept both worlds as possibly being the actual world
- Now let’s suppose we gather up all those worlds that I said ‘yes’ to.  
**This set of worlds is ‘the set of worlds compatible with my beliefs’.**

(42) **Core Observation 2**

- We've just seen that, intuitively, 'the beliefs of X' determine a unique set of worlds.
- Consequently, for purposes of formalization, we could simply identify 'the beliefs of X' with that set of possible worlds.

Illustration:

- a. 'the beliefs of Seth' = { w : the world is flat in w }
- b. 'the beliefs of Seth's cat' = { w : tuna juice is tasty in w, and  
the vacuum cleaner is a monster in w, and  
the sofa is a fancy scratching post in w, and  
babies are a nuisance in w, and... }

(43) **New Notation**  $w_0 = \text{the actual world}$

(44) **New Notation** 'Beliefs(X, w)' = the beliefs of X *in world w*  
(the worlds compatible with the beliefs of X in w)

*Illustration:* **Beliefs(Seth, w<sub>0</sub>)** = { w : tuna juice is repulsive in w, and  
the vacuum cleaner is broken in w, and  
the sofa is scratched up in w, and  
cats are a nuisance in w, and... }

(45) **Core Observation 3**

- Intuitively, X 'believes S' in w *iff* 'S' is true in all the worlds that are compatible with X's beliefs in w.  
(to see this, just imagine playing the game described in (41b)...)
- Therefore, using the vocabulary and notation introduced in (42)-(44), it follows that X 'believes S' in w *iff* 'S' is true in all the worlds in **Beliefs(X, w)**

*From the 'Core Observation' in (45), the following key conclusion follows....*

(46) **The Truth Conditions of Belief Sentences**

[[ Lou believes that Obama smokes ]]<sup>w</sup> = T  
*iff* For all w' ∈ **Beliefs(Lou, w)**, Obama smokes in w'

## 5. The Lexical Entry for *Believes*

If we assume the T-conditional statement in (46), we can now begin to work backwards towards a lexical entry for the verb *believes*.

### (47) Deducing the Extension of the VP “*believes that Obama smokes*”

- a.  $[[ \text{Lou believes that Obama smokes} ]]^w = T \text{ iff}$   
For all  $w' \in \text{Beliefs}(\text{Lou}, w)$ , Obama smokes in  $w'$
- b.  $[[ \text{believes that Obama smokes} ]]^w ( [[\text{Lou}]]^w ) = T \text{ iff}$   
For all  $w' \in \text{Beliefs}( [[\text{Lou}]]^w, w)$ , Obama smokes in  $w'$
- c.  $[[ \text{believes that Obama smokes} ]]^w =$   
 $[ \lambda x : \text{For all } w' \in \text{Beliefs}(x, w), \text{Obama smokes in } w' ]$

Now recall the following core assumption of this entire approach to the semantics of ‘believes’:

### (48) Core Assumption

The meaning of “believes” takes as argument the *intension* of its sentential complement.

$$[[ \text{believes } S ]]^w = [[ \text{believes} ]]^w ( [ \lambda w' : [[ S ]]^w = T ] )$$

### (49) Consequence of (47c) and (48)

$$[[ \text{believes} ]]^w ( [ \lambda w' : [[ \text{Obama smokes} ]]^w = T ] ) =$$

$$[ \lambda x : \text{For all } w' \in \text{Beliefs}(x, w), \text{Obama smokes in } w' ]$$

From this equation in (49), we can now reason our way towards a lexical entry for ‘believes’:

### (50) Deducing the Lexical Entry for the V “*believes*”

- a.  $[[ \text{believes} ]]^w ( [ \lambda w' : [[ \text{Obama smokes} ]]^w = T ] ) =$   
 $[ \lambda x : \text{For all } w' \in \text{Beliefs}(x, w), \text{Obama smokes in } w' ]$
- b.  $[[ \text{believes} ]]^w ( [ \lambda w' : [[ \text{Obama smokes} ]]^w = T ] ) =$   
 $[ \lambda x : \text{For all } w' \in \text{Beliefs}(x, w), [ \lambda w'' : [[ \text{Obama smokes} ]]^w = T ](w') = T$
- c.  $[[ \text{believes} ]]^w = [ \lambda p_{\langle st \rangle} : [ \lambda x : \text{For all } w' \in \text{Beliefs}(x, w), p(w') = T ] ]$

(51) **Hypothesized Lexical Entry for *Believes***

$$[[ \text{believes} ]]^w = [ \lambda p_{\langle st \rangle} : [ \lambda x : \text{For all } w' \in \text{Beliefs}(x,w), p(w') = T ] ]$$

Now, to confirm that (51) will indeed work to derive our targeted T-conditions in (46), let us attempt to directly compute those T-conditions...

(52) **Derivation of the Truth-Conditions in (46)**

a.  $[[ \text{Lou believes that Obama smokes} ]]^w = T \quad \textit{iff} \quad (\text{by FA})$

b.  $[[ \text{believes that Obama smokes} ]]^w ([[ \text{Lou} ]]^w) = T \quad \textit{iff} \quad \dots$

**We're stuck!**

We can't continue the 'proof' in (52) past the line in (52b)!

- According to our lexical entry in (51),  $[[\text{believes}]]$  must take a *proposition* as argument.
- **As of yet, our semantic system *still* doesn't ever yield propositions (intensions) as the values of "[[ ]]" (see our rule of FA in (26))**

(53) **New Rule: Intentional Function Application (IFA)**

If X is a structure consisting of two daughters – Y and Z – and if  $[[Y]]^w$  is a function whose domain contains  $[\lambda w' : [[Z]]^{w'}]$ , then  $[[X]]^w = [[Y]]^w ([\lambda w' : [[Z]]^{w'}])$

(54) **Derivation of the Truth-Conditions in (46)**

a.  $[[ \text{believes that Obama smokes} ]]^w ([[ \text{Lou} ]]^w) = T \quad \textit{iff} \quad (\text{by IFA})$

b.  $[[\text{believes}]]^w ([\lambda w' : [[\text{that Obama smokes}]]^{w'} = T]) ([[ \text{Lou} ]]^w) = T \quad \textit{iff} \quad (\text{by Rules})$

c.  $[[\text{believes}]]^w ([\lambda w' : \text{Obama smokes in } w'])(\text{Lou}) = T \quad \textit{iff} \quad (\text{by TN})$

d.  $[ \lambda p_{\langle st \rangle} : [ \lambda x : \text{For all } w'' \in \text{Beliefs}(x,w), p(w'') = T ] ]$   
 $( [\lambda w' : \text{Obama smokes in } w'])(\text{Lou}) = T \quad \textit{iff} \quad (\text{by LC})$

e.  $[ \lambda x : \text{For all } w'' \in \text{Beliefs}(x,w),$   
 $[ \lambda w' : \text{Obama smokes in } w'](w'') = T ](\text{Lou}) = T \quad \textit{iff} \quad (\text{by notation})$

f. **For all  $w'' \in \text{Beliefs}(\text{Lou},w)$ , Obama smokes in  $w''$**

(55) **The Empirical Adequacy of Our Semantics in More Detail.**

Let's see in finer detail *how* our semantics avoids the 'epically false' prediction in (4):

- Given our lexical entry for "believes", our semantics predicts that:

$[[ \text{Lou believes that Obama smokes} ]]^w = T$

**For all  $w' \in \text{Beliefs}(\text{Lou}, w)$ : Obama smokes in  $w'$**

$[[ \text{Lou believes that Obama is a natural-born citizen} ]]^w = T$  iff

**For all  $w' \in \text{Beliefs}(\text{Lou}, w)$ : Obama is a nat.-born citizen in  $w'$**

- However, consider that there are possible worlds  $w$  such that:

Obama smokes in  $w$ , but Obama is *not* a natural born citizen in  $w$ .

Obama *doesn't* smoke in  $w$ , but Obama *is* a natural born citizen in  $w$ .

- It follows that:

The worlds  $w$  such that Obama smokes in  $w$  are not all such that Obama is natural-born citizen in  $w$ .

The worlds  $w$  such that Obama is a natural-born citizen in  $w$  are not all such that Obama smokes in  $w$ .

- Consequently:

If it's the case that **for all  $w' \in \text{Beliefs}(\text{Lou}, w)$ : Obama smokes in  $w'$**  ...

...it doesn't necessarily follow that:

**for all  $w' \in \text{Beliefs}(\text{Lou}, w)$ : Obama is a nat.-born citizen in  $w'$**

- And therefore:

If it's the case that "Lou believes that Obama smokes" is T ...

... it doesn't necessarily follow that:

"Lou believes that Obama is a natural born citizen" is T

**6 Some Evidence that Intensions are *Still* Not Enough**

(56) **Key Problem for Our 'Intensional' Treatment of *Believes***

*Even for two sentences  $S1$  and  $S2$  sharing the same intension, it is possible for " $X$  believes  $S1$ " to be true while " $X$  believes  $S2$ " is false.*

(57) **Illustrative Example**

- a. Two Sentences Sharing the Same Intension  
(i)  $2 + 2 = 4$  (true in *all* possible worlds)  
(ii)  $14589 - 658 = 13931$  (true in *all* possible worlds)
- b. Two Sentences With Different Truth Values  
(i) My daughter believes that  $2 + 2 = 4$  (True)  
(ii) My daughter believes that  $14589 - 658 = 13931$  (False)

(58) **Another Illustrative Example**

The problem in (57) doesn't just hold for sentences that are necessarily true/false. It also applies to *any* two sentences that are logically equivalent.

- a. Two Sentences Sharing the Same Intension (Due to Logical Equivalence)  
(i) Muno is orange.  
(ii) Everything that is not orange has a property that Muno lacks.
- b. Two Sentences With Different Truth Values  
(i) My daughter believes that Muno is orange. (True)  
(ii) My daughter believes that everything that is not orange has a property that Muno lacks. (False?)

(59) **One Possible Response: Impossible Worlds**

Suppose we allow the existence of worlds where  $14589 - 658 \neq 13931$ ....

...After all, the fact that I don't know the answer to '14589 - 658' (or get it wrong) means that I believe that  $w_0$  could be a *mathematically impossible* world where '14589 - 658' is some *other* number.

...If we allow such worlds, then it's no longer obvious that the two equations in (57a) have the same intension!

(60) **Another Possible Response: Ambiguity of *Believe***

Some people have noted that there *does* seem to be a reading of (58bii) where it *does* follow from (58bi).

... Thus, maybe our intensional semantics accurately correctly captures one 'reading' of *believes S*...

... but there is also another reading of *believes S* where it means something more 'behavioral', like 'X is disposed to assent to the English sentence S'...

(61) **A Third Possible Response / Conclusion**

- Our intensional semantics for *believe* is just wrong.
- The verb *believe* is *not* a relation between an entity and an *intension*.

*So what kind of a relation does the verb “believe” describe?...*

(62) **One Line of Thought (“Structured Intensions/Meanings”)**

- Looking to the examples in (57) and (58), we notice that two different *formulas/sentences* can have the same intension.
- Intuitively, though, we obtain that intension in two separate ways for each of these sentences, *by combining different constituent meanings in different orders*.
- Let’s suppose, then, that the object of belief is (in some sense) a kind of representation of the way we *obtain/compute* the intension. For purposes of discussion, let’s call this kind of an object a ‘structured intension (SI)’:
  - SI(“Muno is orange”) =  
< [ $\lambda x. x$  is orange in  $w$ ], Muno >  
*The meaning of ‘orange’ applied to the meaning of ‘Muno’*
  - SI(“Everything that is not orange has a property Muno lacks”) =  
< [ $\lambda P. \lambda Q. \text{for all } x, \text{ if } P(x) = T \dots\dots$ ] >  
*The meaning of ‘everything’ applied to....*
- Consequently, two sentences with the same intension might nevertheless have two different ‘structured intensions’.
- So, if we assume that “believe” is a relation between entities and *structured intensions*, we would accurately capture the possibility of (57b) and (58b).

This ‘structured intensions’ approach is (to my knowledge) the oldest and best-worked-out solution to the problems raised by (57) and (58)  
[it is particularly well-developed in the work of Max Cresswell...]