

The Semantics of Quantificational DPs¹

1. Introduction

Presently, our system can interpret sentences where the following sorts of DPs occupy subject position:

- Proper Names: *Barack, Joe, Michelle*
- Definite DPs: *the man, the president*

(1) **Observation 1:** All of the above DPs are of type e

(2) **Observation 2:**

- These aren't the only kinds of DPs that can occupy subject position
- All the following DPs, for which we don't yet have a semantics, can *also* occupy subject position.

(3) **Some Quantificational DPs**

- | | | |
|----|---------------------|-----------------------------|
| a. | <i>A / some man</i> | A / some man smokes. |
| b. | <i>No man</i> | No man smokes. |
| c. | <i>Every man</i> | Every man smokes. |
| d. | <i>Three men</i> | Three men smoke. |
| e. | <i>Many men</i> | Many men smoke. |
| f. | <i>Few men</i> | Few men smoke. |
| g. | <i>Most men</i> | Most men smoke. |

(4) **Observation 3:**

If we extend our system to provide a treatment of the DPs above, then we'll have the tools to interpret just about any DP in the English language!

... So let's try to develop a semantics for these DPs...

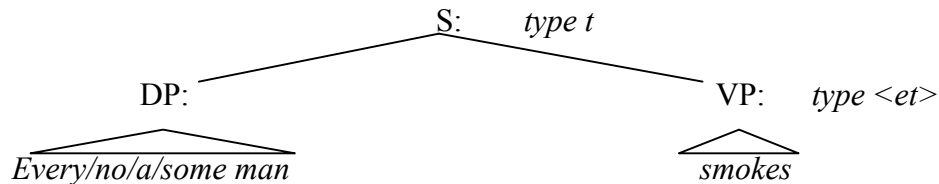
... We'll start with the first three (3a,b,c), and then work our way out...

¹ These notes are based upon the material in Heim & Kratzer (1998: 131-135, 140-147).

2. The Type of ‘Quantificational DPs’

Looking at the structure of sentences (3a,b,c), there are two logical possibilities regarding the semantic type of these DPs:

(5) The Possible Semantic Types of these DPs



- Possibility 1: The DPs are type e
- Possibility 2: The DPs are type $\langle et, t \rangle$

(6) Question:

- Thus far, we’ve treated DPs in subject position as expressions of *type e*.
- Can we continue to do so here?

(7) The Classic Answer

NO! For the following reasons:

2.1 Evidence Against a ‘Type e’ Analysis: Argument 1

(8) Generalization

- Suppose that we have two VPs – *VP1* and *VP2* – such that for all x , if $[[VP1]](x) = T$ then $[[VP2]](x) = T$. Moreover, suppose that we have a DP of type e .
- It follows that if $[[DP VP1]] = T$, then $[[DP VP2]] = T$.

(9) Illustration

- Clearly, for all x , if $[[smokes Marlboros]](x) = T$, then $[[smokes]](x) = T$
- And, intuitively, if $[[Barack smokes Marlboros]] = T$ then $[[Barack smokes]] = T$.

(10) Key Observation

The following DOESN’T hold:
if $[[no man smokes Marlboros]] = T$, then $[[no man smokes]] = T$

- (11) **Key Conclusion:** The DP *no man* is **not of type e**.

2.2 Evidence Against a ‘Type *e*’ Analysis: Argument 2

(12) Generalization

- Suppose that we have two VPs – *VP1* and *VP2* – such that for all *x*, $[[VP1]](x) = T$ iff $[[VP2]](x) = F$. Moreover, suppose that we have a DP of type *e*.
- It follows that $[[DP VP1]] = T$ iff $[[DP VP2]] = F$.

(13) Illustration

- Clearly, for all *x*, $[[smokes]](x) = T$ iff $[[doesn't smoke]](x) = F$
- And, intuitively, $[[Barack smokes]] = T$ iff $[[Barack doesn't smoke]] = F$

(14) Key Observation

The following DOESN'T hold:

$$[[a/some man smokes]] = T \text{ iff } [[a/some man doesn't smoke]] = F$$

That is, the following CAN hold:

$$[[a/some man smokes]] = T \text{ and } [[a/some man doesn't smoke]] = T$$

- (15) **Key Conclusion:** The DP *a / some man* is **not of type *e***.

2.3 Evidence Against a ‘Type *e*’ Analysis: Argument 3

(16) Generalization

- Suppose that we have two VPs – *VP1* and *VP2* – such that for all *x*, $[[VP1]](x) = T$ iff $[[VP2]](x) = F$. Moreover, suppose that we have a DP of type *e*.
- It follows that $[[[DP VP1] \text{ or } [DP VP2]]]$ is necessarily true.

(17) Illustration

Intuitively, the following is necessarily T: “Barack smokes or Barack doesn’t smoke.”

(18) Key Observation

The following doesn’t seem to be necessarily T:

“**Every man** smokes or **every man** doesn’t smoke.”

- (19) **Key Conclusion:** The DP *every man* is **not of type *e***.

3. The Semantics of Quantificational DPs and Ds

From the preceding arguments, we come to the following general conclusion:

(20) General Conclusion

- None of the following DPs are of *type e*: “a/some man”, “no man”, “every man”
- Therefore, given (5), it follows that these DPs are of type $\langle et, t \rangle$

Side-Note: These arguments can also be used to show that DPs in (3d-g) are also *type* $\langle et, t \rangle$

OK... so we know their abstract type...

But exactly what kind of type $\langle et, t \rangle$ function do these DPs denote???

(21) Core Intuition

- As an $\langle et, t \rangle$ function, the extension of *no man / some man / every man* takes an $\langle et \rangle$ property and returns a T-value.
- Thus, the extensions of these DPs are *predicates of predicates* (so-called ‘second order predicates/properties’)... they ‘say things about’ their $\langle et \rangle$ arguments.

(22) Illustration of Core Intuition

a. “No man” says that its VP argument is true of no man

- $[[\text{no man}]][[\text{VP}]] = T$ iff there is no man x such that $[[\text{VP}]](x) = T$
- $[[\text{No man}]][[\text{smokes}]] = T$ iff there is no man x s.t. $[[\text{smokes}]](x) = T$

b. “A / Some man” says that its VP argument is true of some man

- $[[\text{a/some man}]][[\text{VP}]] = T$ iff there is some man x such that $[[\text{VP}]](x) = T$
- $[[\text{A man}]][[\text{smokes}]] = T$ iff there is some man x such that $[[\text{smokes}]](x) = T$

c. “Every man” says that its VP argument is true of every man

- $[[\text{every man}]][[\text{VP}]] = T$ iff for all x , if x is a man, then $[[\text{VP}]](x) = T$
- $[[\text{Every man}]][[\text{smokes}]] = T$ iff for all x , if x is a man,
then $[[\text{smokes}]](x) = T$

Given the logical formulations above of our ‘core intuition’, it’s clear how we can represent these hypotheses using our lambda notation.

(23) **Hypothesized Extensions of Our Quantificational DPs**

- a. [[no man]] = [$\lambda f_{\langle et \rangle} : \text{there is no man } x \text{ such that } f(x) = T$]
- b. [[a/some man]] = [$\lambda f_{\langle et \rangle} : \text{there is some man } x \text{ such that } f(x) = T$]
- c. [[every man]] = [$\lambda f_{\langle et \rangle} : \text{for all } x, \text{ if } x \text{ is a man, then } f(x) = T$]

BUT WAIT!!!

Before we go any further with the hypothesized extensions in (23)...

... let’s confirm that they do indeed avoid the problems noted for a *type e* analysis in Sections 2.1 – 2.3!!

3.1 Evidence Against a ‘Type e’ Analysis: Argument 1

(24) **Earlier Observation**

Contrary to the predictions of a *type e* analysis, the following can hold:

- [[no man eats fish]] = T and [[no man eats]] = F

(25) **Question:** Does our *type* $\langle et, t \rangle$ analysis in (23a) predict that (24) can hold?

(26) **Answer: Yes!**

- a. Predicted T-conditions for “no man eats fish”
- [[no man eats fish]] = T *iff* there is no man x such that x eats fish.
- b. Predicted T-conditions for “no man eats”
- [[no man eats]] = T *iff* there is no man x such that x eats.
- c. *Conclusion:*
 Clearly, under these T-conditions, “no man eats fish” can be T at the same time that “no man eats” is F.
 (as the world can be such that no man eats fish, but that every man eats *something else other than fish*).

3.2 Evidence Against a ‘Type *e*’ Analysis: Argument 2

(27) Earlier Observation

Contrary to the predictions of a *type e* analysis, the following can hold:

- $[[\mathbf{a/some\ man\ eats\ fish}]] = T$ and $[[\mathbf{a/some\ man\ doesn't\ eat\ fish}]] = T$

(28) **Question:** Does our *type* $\langle et, t \rangle$ analysis in (23b) predict that (27) can hold?

(29) Answer: Yes!

a. Predicted T-conditions for “some man eats fish”

$[[\text{some man eats fish}]] = T$ iff there is some man x such that x eats fish.

b. Predicted T-conditions for “some man doesn’t eat fish”

$[[\text{some man doesn't eat fish}]] = T$ iff there’s some man x such that x don’t eat fish.

c. *Conclusion:*

Clearly, under these T-conditions, “some man eats fish” can be T at the same time that “some man doesn’t eat fish” is T.

(...the world can be such that there are two men, one who eats fish and one who doesn’t!)

3.3 Evidence Against a ‘Type *e*’ Analysis: Argument 3

(30) Earlier Observation

Contrary to the predictions of a *type e* analysis, the following can hold:

- $[[[\mathbf{Every\ man\ eats\ fish}] \text{ or } [\mathbf{every\ man\ doesn't\ eat\ fish.}]]] = F$

(31) **Question:** Does our *type* $\langle et, t \rangle$ analysis in (23c) predict that (30) can hold?

(32) Answer: Yes!

a. Predicted T-conditions for “every man eats fish or every man doesn’t eat fish”

$[[S]] = T$ iff for all x , if x is a man, then x eats fish, or
for all x , if x is a man, then x doesn’t eat fish.

b. *Conclusion:*

Clearly, under these T-conditions, the sentence in question *can* be false.

(...the world can be such that some men eat fish while others don’t)

4. The Semantics of Quantificational Determiners

So far, it seems like the equations in (33) are something our theory should capture...

(33) Hypothesized Extensions of Our Quantificational DPs

- a. $[[\text{no man}]]$ = $[\lambda f_{\langle et \rangle} : \text{there is no man } x \text{ such that } f(x) = T]$
 b. $[[\text{a/some man}]]$ = $[\lambda f_{\langle et \rangle} : \text{there is some man } x \text{ such that } f(x) = T]$
 c. $[[\text{every man}]]$ = $[\lambda f_{\langle et \rangle} : \text{for all } x, \text{ if } x \text{ is a man, then } f(x) = T]$

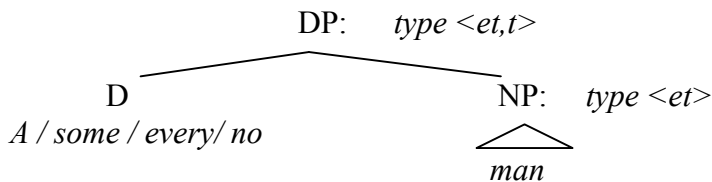
...However, it's also clear that our theory should be able to **derive** these equations...
 ... as there seems to be an **infinite** number of such quantificational DPs:

(34) Some Other Quantificational DPs

- $[[\text{no woman}]]$ = $[\lambda f_{\langle et \rangle} : \text{there is no woman } x \text{ such that } f(x) = T]$
 $[[\text{no man who smokes}]]$ = $[\lambda f_{\langle et \rangle} : \text{there is no man who smokes } x \text{ such that } f(x) = T]$
 $[[\text{no dog who barks}]]$ = $[\lambda f_{\langle et \rangle} : \text{there is no dog who barks } x \text{ such that } f(x) = T]$
 (etc., etc.)

(35) **Goal:** Develop lexical entries for the Ds *no*, *some*, *every* that will derive (33).

(36) The Type of Quantificational Determiners



CONCLUSION: The semantic type of *a*, *some*, *every* and *no* is $\langle et, \langle et, t \rangle \rangle$

OK... but what type $\langle et \langle et, t \rangle \rangle$ functions are these?

(37) First Step: Adjusting Our Equations

If we restate the equations in (33) to the following equivalent forms, it's a little bit clearer how they might be derived via FA.

- a. $[[\text{no man}]]$ = $[\lambda f_{\langle et \rangle} : \text{there is no } x \text{ such that } x \text{ is a man and } f(x) = T]$
 b. $[[\text{a/some man}]]$ = $[\lambda f_{\langle et \rangle} : \text{there is an } x \text{ such that } x \text{ is a man and } f(x) = T]$
 c. $[[\text{every man}]]$ = $[\lambda f_{\langle et \rangle} : \text{for all } x, \text{ if } x \text{ is a man, then } f(x) = T]$

(38) **Second Step: Applying Our Knowledge of Types**

Given that *no*, *some* and *every* are of type $\langle et, \langle et, t \rangle \rangle$, it follows that the equations below are a consequence of our rule of FA:

- a. $[[\text{no man}]]$ = $[[\text{no}]]([[\text{man}]])$
 b. $[[\text{a/some man}]]$ = $[[\text{a/some}]]([[\text{man}]])$
 c. $[[\text{every man}]]$ = $[[\text{every}]]([[\text{man}]])$

Thus, we can rewrite the equations in (37) to the following:

- c. $[[\text{no}]]([[\text{man}]])$ = $[\lambda f_{\langle et \rangle} : \text{there is no } x \text{ such that } x \text{ is a man and } f(x) = T]$
 d. $[[\text{a/some}]]([[\text{man}]])$ = $[\lambda f_{\langle et \rangle} : \text{there is an } x \text{ such that } x \text{ is a man and } f(x) = T]$
 e. $[[\text{every}]]([[\text{man}]])$ = $[\lambda f_{\langle et \rangle} : \text{for all } x, \text{ if } x \text{ is a man, then } f(x) = T]$

Moreover, given that we know what the extension of “man” is, we can rewrite each of these equations to the following:

- f. $[[\text{no}]]([\lambda y_e : y \text{ is a man }])$ = $[\lambda f_{\langle et \rangle} : \text{there is no } x \text{ such that } x \text{ is a man and } f(x) = T]$
 g. $[[\text{a/some}]]([\lambda y_e : y \text{ is a man }])$ = $[\lambda f_{\langle et \rangle} : \text{there is an } x \text{ such that } x \text{ is a man and } f(x) = T]$
 h. $[[\text{every}]]([\lambda y_e : y \text{ is a man }])$ = $[\lambda f_{\langle et \rangle} : \text{for all } x, \text{ if } x \text{ is a man, then } f(x) = T]$

(39) **Third Step: Rewriting the Extension to Contain the Argument**

We can rewrite the targeted extensions so that they explicitly contain the $\langle et \rangle$ argument of the determiners. This will help us see what the extensions of the determiners should be.

- a. $[[\text{no}]]([\lambda y_e : y \text{ is a man }])$ = $[\lambda f_{\langle et \rangle} : \text{there is no } x \text{ such that } [\lambda y_e : y \text{ is a man }](x) = T \text{ and } f(x) = T]$
 b. $[[\text{a/some}]]([\lambda y_e : y \text{ is a man }])$ = $[\lambda f_{\langle et \rangle} : \text{there is an } x \text{ such that } [\lambda y_e : y \text{ is a man }](x) = T \text{ and } f(x) = T]$
 c. $[[\text{every}]]([\lambda y_e : y \text{ is a man }])$ = $[\lambda f_{\langle et \rangle} : \text{for all } x, \text{ if } [\lambda y_e : y \text{ is a man }](x) = T, \text{ then } f(x) = T]$

(40) **Fourth Step: Stating the Generalizations**

Given the equations in (39), the following generalizations are apparent:

- a. [[no]] takes as argument some $\langle et \rangle$ function g and returns the following function:

[$\lambda f_{\langle et \rangle} : \underline{\text{there is no } x \text{ such that } g(x) = T \text{ and } f(x) = T}$]

- b. [[a/some]] takes as argument some $\langle et \rangle$ function g and returns the following function:

[$\lambda f_{\langle et \rangle} : \underline{\text{there is an } x \text{ such that } g(x) = T \text{ and } f(x) = T}$]

- c. [[every]] takes as argument some $\langle et \rangle$ function g and returns the following function:

[$\lambda f_{\langle et \rangle} : \underline{\text{for all } x, \text{ if } g(x) = T, \text{ then } f(x) = T}$]

(41) **Fifth Step: Writing it Out as a Lambda Expression**

Given the generalizations in (40), it's clear what the extensions of the Ds should be:

- a. [[no]] = [$\lambda g_{\langle et \rangle} : [\lambda f_{\langle et \rangle} : \underline{\text{there is no } x \text{ such that } g(x) = T \text{ and } f(x) = T}$]

- b. [[a/some]] = [$\lambda g_{\langle et \rangle} : [\lambda f_{\langle et \rangle} : \underline{\text{there is an } x \text{ such that } g(x) = T \text{ and } f(x) = T}$]

- c. [[every]] = [$\lambda g_{\langle et \rangle} : [\lambda f_{\langle et \rangle} : \underline{\text{for all } x, \text{ if } g(x) = T, \text{ then } f(x) = T}$]

(42) **Predicted T-Conditions**

The following T-conditional statements follow from the lexical entries in (41):

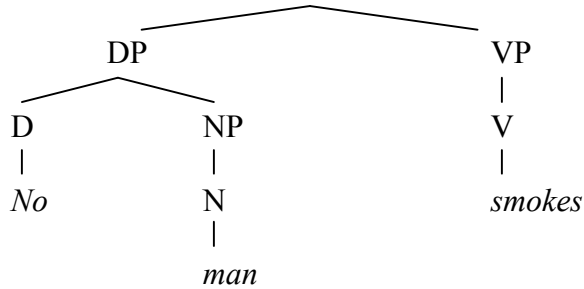
- a. “No man smokes” is T *iff* there is no x such that x is a man and x smokes.
b. “A/some man smokes” is T *iff* there is some x such that x is a man and x smokes.
c. “Every man smokes” is T *iff* for all x , if x is a man, then x smokes.

The following derivation shows how the T-conditions in (42a) are derived...

...the reader is encouraged to also write out their own derivations for (42b) and (42c), to confirm that those T-conditions are indeed derived as stated...

(43) **Sample Derivation**

a. “ S ” is T *iff* (by notation)



b. $[[S]] = T$

c. **Subproof:**

(i) $[[NP]] =$ (by NNx2, TN)

(ii) $[\lambda y_e : y \text{ is a man }]$

d. **Subproof:**

(i) $[[D]] =$ (by NN, TN)

(ii) $[\lambda g_{\langle et \rangle} : [\lambda f_{\langle et \rangle} : \text{there is no } x \text{ such that } g(x) = T \text{ and } f(x) = T]$

e. **Subproof:**

(i) $[[DP]] =$ (by FA, c, d)

(ii) $[[D]] ([[NP]]) =$ (by c, d)

(iii) $[\lambda g_{\langle et \rangle} : [\lambda f_{\langle et \rangle} : \text{there is no } x \text{ such that } g(x) = T \text{ and } f(x) = T]$
 $([\lambda y_e : y \text{ is a man }]) =$ (by LC)

(iv) $[\lambda f_{\langle et \rangle} : \text{there is no } x \text{ such that } [\lambda y_e : y \text{ is a man }](x) = T \text{ and } f(x) = T]$
 $=$ (by LC)

(v) $[\lambda f_{\langle et \rangle} : \text{there is no } x \text{ such that } x \text{ is a man and } f(x) = T]$

f. **Subproof:**

(i) $[[VP]] =$ (by NNx2, TN)

(ii) $[\lambda y_e : y \text{ smokes }]$

g. $[[S]] = T$ *iff* (by FA, e, f)

h. $[[DP]] ([[VP]]) = T$ *iff* (by e, f)

i. $[\lambda f_{\langle et \rangle} : \text{there is no } x \text{ such that } x \text{ is a man and } f(x) = T] ([\lambda y_e : y \text{ smokes }]) = T$ *iff*
 (by LC)

- j. there is no x such that x is a man and $[\lambda y_e : \underline{y \text{ smokes}}](x) = T$ iff (by LC)
- k. there is no x such that x is a man and x smokes.

(44) **Interim Summary**

- a. Quantificational DPs (*no man, some man, every man*) are of type $\langle et, t \rangle$
- b. Quantificational Ds (*no, some, every*) are of type $\langle et, \langle et, t \rangle \rangle$

(45) **Some (Confusing) Terminology**

- a. *Generalized Quantifiers*: Functions of type $\langle et, t \rangle$
- b. *Quantificational Determiners*: Functions of type $\langle et, \langle et, t \rangle \rangle$

4. Presuppositions and Quantificational Determiners

It seems that some quantificational Ds in natural language carry presuppositions regarding the extensions of their NP arguments.

(46) **First Case Study: *Neither***

- a. Observation 1:
Suppose that there are two cats. Then “Neither cat is hungry” seems T iff “no cat is hungry” is T, iff there is no x such that x is a cat and x is hungry.
- b. Observation 2:
- “Neither cat is hungry” implies that there are two cats.
 - “It’s not the case that neither cat is hungry” *still* implies that there’s two cats. (it still feels ‘wrong’ if there is either only one cat or more than two cats)
 - So, “Neither cat is hungry” seems to *presuppose* that there are (exactly) two cats...

(47) **Some Notation: The ‘Cardinality’ of a Set**

The *cardinality* of a set A – which we can represent with the notation ‘ $|A|$ ’ – is the number of members that A has.

- a. Examples
- (i) $|\{a, b, c\}| = 3$
- (ii) $|\{a, b, c, d, a\}| = 4$
- (iii) $|\{x : x \text{ is a first year linguistics graduate student}\}| = 5$

(48) **Lexical Entry for *Neither***

$$[[\text{neither}]] = [\lambda g : g \in D_{\langle \text{et} \rangle} \text{ and } | \{ x : g(x) = \mathbf{T} \} | = 2 . \\ [\lambda f_{\langle \text{et} \rangle} : \text{there is no } x \text{ such that } g(x) = \mathbf{T} \text{ and } f(x) = \mathbf{T}]]$$

(49) **More Controversial Example: *Every***

a. Fact 1:

- “Every unicorn is beautiful” implies that there are unicorns.
- “It’s not the case that every unicorn is beautiful” *still* implies that there are unicorns.
- *So*. “Every unicorn is beautiful” seems (via this test) to *presuppose* that there do exist unicorns.

b. Fact 2:

There is a significant population who find the following to be a valid inference:
If “Every NP VP” is T, then “Some NP VP” is T.
(e.g., Aristotle thought this was valid.)

(50) **Common Solution: Write in the Presupposition**

The determiner *every* has as its extension a function whose domain is restricted to those $\langle \text{et} \rangle$ functions that are true of *at least one entity*.

$$[[\text{every}]] = [\lambda g : g \in D_{\langle \text{et} \rangle} \text{ and } | \{ x : g(x) = \mathbf{T} \} | \geq 1 . \\ [\lambda f_{\langle \text{et} \rangle} : \text{for all } x, \text{ if } g(x) = \mathbf{T}, \text{ then } f(x) = \mathbf{T}]]$$

a. *First Consequence*

“Every NP VP” will now presuppose that there exists at least one x such that $[[\text{NP}]](x) = \mathbf{T}$

b. *Second Consequence*

Given its new presupposition, the T-conditions of “Every NP VP” now entail “Some NP VP”.

This difference between “every” and “no/some” also extends to other quantificational Ds...

(51) **The Determiner *Most* Seems to Behave like *Every***

“Most unicorns live at UMass.”

Implies there are unicorns.

“It isn’t the case that most unicorns live at UMass.” *Still implies that there are unicorns.*

(52) **Some Other Determiners that Seem to Behave like *Some* and *No***

- a. Numerals
“It isn’t the case that three unicorns live at UMass.” *Straightforwardly true;*
Does not imply there are unicorns
- b. Many
“It isn’t the case that many unicorns live at UMass.” *Straightforwardly true;*
Does not imply there are unicorns

We can write this difference in presuppositional behavior into the lexical items of these Ds:

(53) **Some Lexical Entries:**

- a. $[[\text{most}]] = [\lambda g : g \in D_{\langle \text{et} \rangle} \text{ and } | \{ x : g(x) = T \} | \geq 1 .$
 $[\lambda f_{\langle \text{et} \rangle} : | \{ x : g(x) = T \text{ and } f(x) = T \} | >$
 $| \{ y : g(y) = T \text{ and } f(y) = F \} |]]$
‘There are more NPs that VP than NPs that don’t VP.’
- b. $[[\text{three}]] = [\lambda g_{\langle \text{et} \rangle} : [\lambda f_{\langle \text{et} \rangle} : | \{ x : g(x) = T \text{ and } f(x) = T \} | \geq 3]]$
‘There are at least three entities for which the NP and the VP is T.’

(54) **Key Observation**

If a determiner D requires its first argument to be true of some entities, then the following existential sentence is ill-formed: “there is/are D NP (Location)”.

Evidence

- | | |
|--|--|
| (i) There is a/some cat (in my yard). | (v) * There is every cat (in my yard). |
| (ii) There is no cat (in my yard). | (vi) * There are most cats (in my yard). |
| (iii) There are three cats (in my yard). | (vii) * There is neither cat (in my yard). |
| (iv) There are many cats (in my yard). | (viii) *There is the cat (in my yard). |

(54) **Syntactic Terminology**

- a. Strong Determiners (Syntactic Property):
A determiner D is ‘strong’ if the following existential sentence is ill-formed: “there is/are D NP (Location)”.
- b. Weak Determiners (Syntactic Property):
A determiner D is ‘weak’ if the following existential sentence is well-formed: “there is/are D NP (Location)”.

(63) **Empirical Generalization (Falsifiable)**

A determiner is ‘strong’ *iff* it presupposes that its first argument is true of something