Steindlian models of growth and stagnation

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Abstract

This paper examines Steindl’s original 1952 model and relates it to subsequent stagnationist models. The model is then extended by introducing endogenous changes in the markup and a reformulation of the investment function. These extensions address weaknesses of the simpler models, find support in Steindl’s writing and leave intact some of Steindl’s key results. In a further extension, we add a labour market and analyse the stabilizing influence of a Marxian reserve-army mechanism. The implications of this model for the effects of increased monopolization are largely in line with Steindl’s predictions.

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1 Introduction

Steindl explained the depression in the interwar period by the inability of the economy "to adjust to low growth rates because its saving propensity is adapted to a high one" (Steindl 1979, p.1) The argument was laid out in Steindl (1952). In the process of capitalist development, he argued, previously competitive industries become oligopolized. This change in competitive conditions puts upward pressure on the profit margin and makes the profit margin less responsive to changes in demand conditions (as reflected in the utilisation of capital). An increase in the profit margin may provide the trigger for reduced demand and a reduction in growth rates; the unresponsiveness of the margin to lower demand and increasing amounts of excess capacity potentially turn the downturn into secular depression or stagnation. The economy, in his terminology, becomes 'mature', where maturity is defined "as the state in which the economy and its profit function are adjusted to the high growth rates of earlier stages of capitalist development, while those high growth rates no longer obtain" (1979, p. 7).

The postwar economy was revitalized and experienced a golden age with near full employment and high growth rates from the 1950s to around 1970. This golden age, in Steindl’s view, was explained by a combination of expansionary policy (large increases in the government sector in all OECD countries), an acceleration of R&D stimulated by the cold war, increased cooperation between western countries, and the potential for technological catch-up in both Europe and Japan. The stimulus from these factors, however, was temporary, and other influences also contributed to a re-assertion of stagnationist tendencies in the 1970s. Steindl singles out, in particular, an increasing trend of personal saving and "a changed attitude of governments towards full employment and growth" (1979, p. 12). This latter influence, which is seen as "the most striking feature of the new economic climate"(1979, p. 12) is explained in terms of a Kaleckian political cycle "as a reaction against the long period of full employment and growth which has strengthened the economic position of workers and the power of the trade unions, and has led to demands for workers' participation" (p. 12-13). Writing in 1979, Steindl therefore expected "low growth for some time to come".

It is beyond the scope of the present paper to evaluate Steindl’s explanations of trends in capitalist development in relation to the empirical evidence. We have reservations with respect to some of his views; indeed the very notion of increasing monopolization may be questioned (e.g. Auerbach (1988), Auerbach and Skott (1988)). Quite independently of any such reservations, however, Steindl has made very significant theoretical contributions, and his work raises a host of interesting questions. It is our purpose in this paper to address some of these theoretical questions.

Most existing formalizations of Steindl’s argument focus on the product market and treat the markup as exogenous. This is unfortunate. An exogenous markup clashes with Steindl’s analysis of how "elastic profit margins" tend to eliminate undesired excess capacity in com-
petitive industries and how the "growth of the monopolistic type of industry may lead to a fundamental change in the working of the economy: bringing about greater inelasticity of profit margins’ (1952, p. ix). Moreover, both financial and labour markets would seem to be important for the analysis - financial markets because of Steindl’s emphasis on internal finance and changes in household saving; labour markets because Steindl regarded prolonged full employment in the 1950’s and 1960s as a key factor behind the subsequent stagnation.

In this paper, we first consider Steindlarian models of the product market. We examine Steindl’s original specification and relate it to subsequent stagnationist models in section 2. The important weaknesses of these formalizations, we argue, are the exogeneity of the markup and an inappropriate specification of the long-run investment function. Section 3 therefore extends the analysis by introducing endogenous changes in the markup as well as a reformulation of the investment function. These extensions, which find support in Steindl’s writing, significantly influence the properties of the system but leave intact some of Steindl’s key results. In section 4, we add a labour market and analyse the stabilizing influence of a Marxian reserve-army mechanism. The implications of this model for the effects of increased monopolization are largely in line with Steindl’s predictions. The paper closes, in section 5, with some conclusions and remarks on future work.

2 Stagnationist models

2.1 The 1952 argument

Steindl’s formal 1952 model (pp. 211-228) is cast in terms of mixed difference-differential equations. The key investment equation (equation (39), p. 213) can be written

\[
I(t + \theta) = \gamma \dot{C}(t) + q (C(t) - g_0 K(t)) + m(kY(t) - u_0 K(t))
\]

(1)

where \( \theta \) is the discrete investment lag; \( I, Y \) and \( K \) denote investment, output and the capital stock; the impact of financing conditions are captured by the retained earnings \( \dot{C} \) and the stock of "entrepreneurs’ capital" \( C \); \( k \) is the ratio of the stock of capital to productive capacity and \( u_0 \) the desired utilisation rate; \( g_0 \) is the inverse of the desired gearing ratio; a dot over a variable is used to denote rate of change (i.e. \( \dot{x} = \frac{dx}{dt} \)); and the parameters \( \gamma, q \) and \( m \) are all positive.\(^1\) Using straightforward assumptions concerning the determination of retained earnings and personal saving, Steindl derives a dynamic equation for the evolution of the capital stock,

\[
\dot{K}(t + \theta) - L\dot{K}(t) + M\dot{K}(t) + NK(t) = 0
\]

(2)

\(^1\)Steindl (1952, p. 213) uses \( Z \) rather than the standard notation \( K \) to denote the capital stock.
where the composite parameters $L, M$ and $N$ can be expressed in terms of the underlying parameters from the functions describing investment and saving.

To solve equation (2), Steindl assumes that the equation represents "a long-run model of moving averages" (p. 227) and that long-run movements may plausibly be described by exponential trends. Thus, implicitly it is assumed that the initial conditions (i.e. the trajectory of the system over a time interval corresponding to the discrete lag $\theta$) can be written

$$K(t) = \sum c_i \exp \rho_i t$$  \hspace{1cm} (3)

where the $c_i$ 's are constants and the $\rho_i$ 's represent the real roots of the characteristic equation

$$\rho^2 \exp(\theta \rho) - L \rho^2 + M \rho + N = 0$$  \hspace{1cm} (4)

Given these initial conditions, the solution to equation (2) for $t$-values beyond the initial interval also takes the form (3).

The next step is to find the real roots of (4). It turns out that in order to get any positive real roots, additional restrictions on the parameter values have to be introduced: basically the ratio of $mk$ to the average saving rate in the economy has to be sufficiently high. When this condition is satisfied, there are three real roots, and the movements of the capital stock can be described by

$$K(t) = c_1 e^{\rho_1 t} + c_2 e^{\rho_2 t} + c_3 e^{\rho_3 t}$$

where $\rho_1 < 0 < \rho_2 < \rho_3$. Asymptotically, the largest of the three roots dominates the movements in $K$ and, Steindl concludes, the capital stock must therefore grow asymptotically at the high rate $\rho_3$.2

The comparative statics of the steady growth path associated with $\rho_3$ can now be examined. From a Steindlian perspective, the effects of changes in the degree of monopoly are particularly interesting. Increasing monopoly is associated with an upward shift of the profit function and this rise, Steindl finds, produces a decline in the rate of growth as long as the expansionary financial effects on investment (represented by the parameters $\gamma$ and $q$ in equation (1)) are weak relative to the effect of utilisation (represented by $mk$ in equation (1)). This condition seems plausible. Moreover, the results are strengthened if the rise in the degree of monopoly also leads to increased fears of excess capacity in the industry and

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2This analysis (p. 220) is slightly flawed by a failure to realize that the capital stock will be declining from some point onwards (and reach zero in finite time) if the coefficient $c_3$ associated with the dominant root is negative. Meaningful non-negative solutions for the long-run capital stock require that the initial conditions are such that $c_3 > 0$ (or, alternatively, such that either $c_2 > c_3 = 0$ or $c_1 > c_2 = c_3 = 0$). Implicitly, Steindl's analysis presumes that $c_3 > 0$.

Note also that although the stability analysis is conditional on very restrictive assumptions concerning the initial conditions, it is not quite correct, as suggested by Dutt (1995, p. 17), that Steindl "does not discuss the dynamic properties of his model" but "only the limiting (or equilibrium) state of the economy".
a corresponding increase in the desired utilisation rate $u_0$. Thus, the model appears to support Steindl’s central conclusion:

On the basis of the present model it is thus possible to demonstrate that the development of monopoly may bring about a decline in the rate of growth of capital. I believe that this is, in fact, the main explanation of the decline in the rate of growth which has been going on in the United States from the end of the last century. (p. 225)

Unfortunately, the empirical application of the model raises difficulties, and Steindl is refreshingly forthright and clear about these difficulties. He points out that "if plausible values are given to the structural coefficients ... then it appears that the limiting rate of growth thus obtained is very big" (p. 226). This problem is serious since it implies that it "is difficult to explain, on the basis of my model, moderate rates of growth, such as have been observed in the history of capitalism" and "either the model requires modifications in important respects in order to be realistic, or else, it follows that an exponential trend in the strict mathematical sense is not a proper description of long-run growth" (p. 226).

2.2 A simplified 1952 model

The complex nature of mixed systems of differential equations with discrete lags makes it difficult to ascertain the reasons for this empirical anomaly in the model. These reasons become clearer if one considers a simplified version of the model in a discrete-time setting. Thus, let

$$I_{t+1} = m(kY_t - K_t)$$
$$S_t = s_f\pi Y_t + s(1 - \pi + (1 - s_f)\pi)Y_t = s(\pi)Y_t$$
$$I_t = S_t$$

where the desired utilisation rate has been normalized to unity and $k$ is the capital-output ratio at the desired rate of capital utilisation; $\pi$ is the share of profits in income; firms retain a proportion $s_f$ of profits and distribute the rest to households in the form of

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3 This second mechanism - introduced partly, perhaps, to get around the ambiguity of the direct effect of changes in the profit share - seems doubtful. If anything, one might expect a decline in desired utilisation following an increase in oligolization: excess capacity may serve as a deterrent to new entry and the higher the mark-up, the more excess capacity may be required to deter entry. This type of argument is used in a formal model of growth and cycles by Skott (1989a).

In the rest of this paper we shall ignore these possible complications and assume a constant desired rate of utilisation.
interest payments and dividends; there is a uniform saving rate $s$ out of distributed incomes, including wages, and the average saving propensity in the economy therefore is

$$s(\pi) = sf\pi + s(1-\pi + (1-sf)\pi) = s + sf(1-s)\pi.$$ 4

Aside from the switch to a pure discrete-time system, equation (5) differs from (1) by leaving out the effects of retained earnings and the gearing ratio on accumulation. These effects, it may be recalled, were assumed small relative to the effects of the utilisation rate, and it simplifies matters to leave them out altogether.

Equations (5)-(7) imply that

$$\frac{I_t}{K_t} = m\frac{K_{t-1}}{K_t} (u_{t-1} - 1) = m\frac{1}{1 + \frac{s(\pi)}{k}u_{t-1}}(u_{t-1} - 1) = s(\pi)\frac{Y_t}{K_t} = \frac{s(\pi)}{k}u_t = \frac{S_t}{K_t}$$

or

$$u_t = \frac{mk}{s(\pi)}\left(\frac{u_{t-1} - 1}{1 + \frac{s(\pi)}{k}u_{t-1}}\right)$$

(8)

where $u_t = kY_t/K_t$ is the actual rate of utilisation. It is readily seen that generically this difference equation has either no stationary point or two stationary points. 5 Furthermore, the existence of stationary points requires (as a necessary condition), that

$$\frac{mk}{s(\pi)} > 1$$

In the case with two stationary points, the high equilibrium is locally stable; the low is unstable. Qualitatively, these conclusions mirror Steindl’s results: positive steady growth rates require parameter restrictions with respect to the ratio of $mk$ to the average saving rate.

The outcome is illustrated in figure 1 which uses the parameter values $m = 0.2, k = 2, s(\pi) = s + sf(1-s)\pi = 0.1$. The right hand side of equation (8) is decreasing in $s(\pi)$

4 A uniform saving rate out of distributed incomes is in line with Steindl’s specification (1952, p. 214, equation (40 vii)). In the presence of retained earnings, the aggregate saving rate depends positively on the profit share. Thus, the introduction of differential saving rates $s_w$ and $s_p$ for household saving out of wage income and distributed profits would leave the structure of the model substantively unchanged.

5 Let $f(u) = \frac{mk}{s} \frac{u - 1}{1 + \frac{s}{k}u}; u \geq 0$

The function $f(u)$ is increasing and strictly concave: $f'(u) > 0, f'' < 0$. Furthermore,

$$f(u) \geq 0 \text{ for } u \geq 1$$

$$f(u) \rightarrow m\left(\frac{k}{s}\right)^2 \text{ for } u \rightarrow \infty$$

Hence, $u > f(u)$ both when $u$ is small and when $u$ is sufficiently large. The inequality, however, may be reversed for intermediate values of $u$. Since $f(u) < \frac{mk}{s}u$ for all $u$, however, the parameter restriction $\frac{mk}{s} > 1$ is a necessary condition for this to happen.
and, using figure 1, it is therefore readily seen that a rise in the saving rate \( s(\pi) \) (associated with an increase in profit share) generates a decline of the stable solution for \( u \). The growth rate \( su/k \) also suffers. To see this, note that the growth rate can be written

\[
g = \frac{s(\pi)u}{k} = m\left(\frac{s(\pi)}{s(\pi)} - \frac{1}{1 + \frac{s(\pi)}{k}u}\right) = m\left(\frac{\frac{gk}{s(\pi)} - 1}{1 + g}\right)
\]

The existence of two solutions for the utilisation rate implies that this equation in \( g \) will also have two solutions; graphically the picture is similar to figure 1. The expression on the extreme right hand side of the equation is decreasing in \( s(\pi) \), and it follows that a rise in \( s(\pi) \) leads to a decline in the high solution for \( g \).

The stability of the high solution for \( u \) and \( g \) may suggest that these, rather than the low and unstable solutions, are the relevant ones. This indeed is the reasoning that guided Steindl’s analysis. But consider the special case where the sensitivity \( m \) of investment to changes in utilisation goes to infinity. The stable solution goes to infinity as \( m \to \infty \) and we get a unique, unstable \( u \) solution: \( u^* = 1 \). For finite values of \( m \), a high and locally stable solution may exist, but Steindl’s problem re-emerges in this simplified setup: for plausible parameter values the high value of \( u^* \) becomes unreasonably high and, as a corollary, the growth rate also becomes too high.\(^6\)

The reason for this problem is transparent in the simplified version. The stable equilibrium owes its existence to the non-linearity on the right hand side of (8). This non-linearity is quite weak, especially for realistic, small values of \( s(\pi) \). Hence, the high equilibrium value necessarily becomes large. In figure 1, for instance, the high equilibrium yields a utilisation rate of over 58, with desired utilisation normalized at unity. Since it is hard

\(^6\)Dutt (1995) also obtains two steady-state equilibria for some parameter values in his formalization of Steindl’s theory. Again, the low equilibrium is unstable while the high is stable. Dutt does not comment explicitly on the plausibility of the high equilibrium but notes (p. 28, n.7) that ”the model will cease to apply” if the economy hits the full capacity constraint.
to envisage an economy that experiences steady growth with utilisation significantly above the desired rate, these observations indicate the empirical and theoretical irrelevance of the high solution.

In support of this conclusion, it should be noted that the economic logic behind the specific non-linearity in equation (8) is difficult to justify. The non-linearity arises because the investment function (5) imposes a lag: it is investment at time $t+1$ rather than at time $t$ that is determined in period $t$. The existence of this lag may be reasonable, but it would seem plausible to suppose that firms take into account expected changes in output as well as the changes in the capital stock that are already in the pipeline when they form their investment plans. Thus, we may want to respecify the investment function as

$$I_{t+1} = g_{t+1}e K_{t+1}e + m(kY_{t+1}e - K_{t+1}e)$$

or

$$\frac{I_{t+1}}{K_{t+1}e} = g_{t+1}e + m\left(\frac{kY_{t+1}e}{K_{t+1}e} - 1\right)$$

where $g_{t+1}e$ is the expected growth rate of demand between periods $t+1$ and $t+2$ (when period-$(t+1)$ investment enters service as part of the productive capital stock) and where $K_{t+1}e$ and $Y_{t+1}e$ denote the expected values of the capital stock and the level of output in period $t+1$.

The specification in (5) is obtained as a special case when firms expect both output and the capital stock to remain unchanged so that $K_{t+1}e = K_t, Y_{t+1}e = Y_t, g^e = 0$. Changes in the capital stock, however, have already been planned by past - and known - investment decisions. Thus, the capital stock at time $t+1$ should also be known and $K_{t+1}e = K_{t+1} = K_t(1 + \frac{I_t}{K_t}) = K_t(1 + \frac{s(\pi)}{k}u_t)$. The assumption of static output expectations seems questionable, too, in a long-run model with positive growth rates. It would seem more reasonable to suppose that the expected output growth between period $t$ and period $t+1$ is positively related to actual growth in output between periods $t-1$ and $t$. Since output at period $t$ is proportional to investment in period $t$ ($I_t = s(\pi)Y_t$), the growth rate between periods $t-1$ and $t$, in turn, will be positively dependent on the accumulation rate $I_t/K_t$. Combining these observations, the rate of accumulation may be determined by

$$\frac{I_{t+1}}{K_{t+1}} = g_{t+1}e + m(u_t \frac{1 + g_t^e}{1 + s(\pi)u_t} - 1)$$

$$g_t^e = f\left(\frac{s(\pi)}{k}u_t, z\right)$$

where $z$ captures other influences on expected output growth. If, as a simple benchmark, the expected growth rate in the near future - between $t$ and $t+1$ - is related to recent

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7 Other factors, including output movements before period $t-1$ may influence expectations, too. These complications are irrelevant for present purposes.
experience while growth rates further into the future - between \( t + 1 \) and \( t + 2 \) - are treated as a constant, we get \( g_t^e = \frac{s(\pi)}{k} u_t \) and \( g_{t+1}^e = g \), and

\[
\frac{I_{t+1}}{K_{t+1}} = g + m(u_t - 1)
\]

In this case the difference equation becomes

\[
u_t = \frac{k}{s(\pi)} [g + m(u_{t-1} - 1)] \tag{9}\]

and the non-linearity is gone. Instead, there is a unique, unstable equilibrium when \( mk > s(\pi) \), and an increase in the saving rate raises the equilibrium solutions for both utilisation and the growth rate. Note also that if \( g = s(\pi)/k \), the equilibrium solution for the utilisation rate satisfies \( u^* = 1 \), that is, utilisation is at the desired rate. Furthermore, in this case both the growth rate of output and the accumulation rate will be equal to \( g \). Thus, expectations are being met along this Harrodian warranted growth path.

The assumptions underlying (9) are, we would argue, as plausible as the ones underlying (8). Of course, one may reject both sets of simplifying assumptions. In a more general specification, however, \( u_t \) may become convex or concave in \( u_{t-1} \). Convexity - which may arise when \( (1+g_t^e)/(1+I_t/K_t) \) is increasing in \( u_t \) - seems as likely as concavity, and convexity rules out a high and locally stable solution.

\subsection*{2.3 A stagnationist version in continuous time}

Steindl (1952, p. 135) explicitly acknowledges the possibility of instability and unbounded divergence. Thus, the cumulative process arising from the interaction between investment and utilisation "may tend to a finite limit ... but it is not certain whether this will be the case". Similar statements can be found elsewhere (e.g. p. 115 and p. 137) but, although cumulative divergence may be possible, this outcome does not fit the general tenor of Steindl’s argument. His vision of long-term stagnation would seem better described by a stable equilibrium with slow growth and/or high unemployment.

To achieve stability of the equilibrium, the sensitivity of investment to variations in utilisation needs to be reduced. Indeed, this is what happens - through the back door - at the high equilibrium in the non-linear case depicted in figure 1. But we do not need non-linearity. A simple linear specification can be used to get a unique and stable solution, while avoiding the anomaly of excessive utilisation and growth rates. Thus, removing the discrete lag and using a setting with continuous time, let investment per unit of capital be given by

\[
\frac{I}{K} = a + m(u - 1) + b^s f \frac{s_f}{k} \pi u \tag{10}\]

\footnote{A continuous-time formulation is more convenient analytically, especially when we get to consider extensions of the model in sections 3-4. The argument in this section could be recast using a discrete-time model with or without a one-period lag in the investment function.}
where, following Steindl, we have allowed investment to depend positively on retained earnings \((b \geq 0)\) and assumed that firms retain the proportion \(s_f\) of profits (before interest on debt). All variables are contemporaneous and, as in section 2, \(k\) is the capital-output ratio at the desired utilisation rate (normalized to one), \(u\) the utilisation rate and \(u/k\) the actual output-capital ratio.

Using the saving function (6), the equilibrium condition now becomes

\[
a + m(u - 1) + b\frac{s_f}{k} \pi u = \frac{u}{k} [s_f(1 - s)\pi + s] = \frac{s(\pi)}{k} u
\]  

(11)

This equation determines the rate of capacity utilisation \(u\) as a function of the profit share \(\pi\).

The ‘Keynesian stability’ of the adjustment in output (for given profit share) requires \(mk + bs_f\pi < s(\pi)\), and, given this stability condition, an increase in \(s(\pi)\) leads to a decline in both \(u\) and \(g\). Note that when the Keynesian stability condition is imposed, a positive constant \(a > m\) is needed to get an equilibrium with positive utilisation. Steindl’s own analysis leaves out the constant (cf. equation (1) above). In his analysis, however, the Keynesian stability condition is satisfied at the high equilibrium, and a Taylor approximation to the non-linear accumulation function around the high equilibrium has a positive constant. Thus, the imposition of the Keynesian stability condition in linear models with a constant term in the investment function might appear to capture the spirit of Steindl’s argument. This approach, indeed, has become standard in formalizations of the Kalecki-Steindl theory, e.g. Rowthorn (1981), Dutt (1984), Taylor (1985), Sawyer (1985), Marglin and Bhaduri (1990).

Solving equation (11) for \(u\) we get

\[
u = \frac{k(a - m)}{s + s_f(1 - s - b)\pi - mk}
\]  

(12)

The Keynesian stability condition implies that the denominator of this expression will be positive, \(s + [s_f(1 - s - b)]\pi - mk > 0\), and the parameter restriction \(a > m\) ensures a positive solution for the utilisation rate. Suitable choices of the value of \(a\), moreover, will give utilisation rates in the neighbourhood of one, thus overcoming the anomaly of Steindl’s original model.

Equation (12) gives rise to Marglin-Bhaduri (1990) possibilities of exhilarationist or stagnationist outcomes. Assuming the Keynesian stability condition, an increase in \(\pi\) will lead to a decline in \(u\) if the ‘Robinsonian stability condition’ \(0 < s_f(1 - s - b)\) is satisfied; a reversal of this latter stability condition implies that \(u\) will rise with \(\pi\). The effects on growth are ambiguous. Differentiating \(g = \frac{s(\pi)}{k} u(\pi)\) with respect to \(\pi\), we get

\[
\frac{\partial g}{\partial \pi} = \frac{u}{k} \left\{ s(\pi) \frac{-s_f(1 - s - b)}{s + s_f(1 - s - b)\pi - mk} + s_f(1 - s) \right\}
\]
Hence, if both the Keynesian short-run stability condition and the Robinsonian stability condition are met, an increase in the profit share will have a negative impact on growth if \( mk \) is large and/or \( b \) is small and/or \( s \) is small. These ambiguous outcomes mirror the results in Steindl (1952) as well as in Dutt (1995, p.32). Thus, Steindl (1952, p. 224) finds that a rise in the markup will only depress growth if the direct effect of the profit share on investment (corresponding to \( bs_f \)) is small relative to the effect of utilisation on investment (measured by \( mk \)), a condition which is similar to the condition above.

3 A core model for the product market

3.1 Two shortcomings

The stagnationist version of Steindl’s theory suffers from two shortcomings. The first of these is shared by Steindl’s own formal model and indeed is fully acknowledged by Steindl:

The profit function which I assumed constant in my long run model should not really be so. In reality there will be a certain elasticity of the profit margins, that is the profit function will depend on the degree of utilisation (a high utilisation shifting it upwards, and a low utilisation downwards). My mathematical model does not include this complication, and it is in this respect poorer than the verbal exposition of the theory in the earlier chapters. (p.228)

In competitive regimes the movements in the markup (which in our simple version is identical to the profit share) may be strong. The transition to oligopolistic regimes weakens the mechanism, but a tendency remains for the markup to rise (resp. fall) when utilisation is above (resp. below) the desired level.\(^\text{10}\)

\(^9\)Since \( s + s_f(1-s-b)\pi - mk > 0 \), we have

\[
s(\pi)\frac{-s_f(1-s-b)}{s + s_f(1-s-b)\pi - mk} + s_f(1-s) < 0
\]

iff

\[
(s + s_f(1-s)\pi)s_f(1-s-b) > s_f(1-s)[s + s_f(1-s-b)\pi - mk]
\]

or, equivalently,

\[
sbs_f < (1-s)s_fmk.
\]

\(^{10}\)The adjustment of price margins to maintain a 'normal' or desired rate of utilisation can also be found in Joan Robinson’s writings. Robinson (1962, p. 46), for instance, assumes that "competition (in the short-period sense) is sufficiently keen to keep prices at the level at which normal capacity output can be sold".
The second shortcoming relates to the investment function. The stagnationist model assumes that the sensitivity of investment to changes in utilisation is small. Using the specification in section 2.3, the Keynesian stability condition could be expressed \( mk + bs_f \pi < s(\pi) \).

This relative insensitivity of investment is perfectly plausible in the short run. But a weak impact effect of a change in utilisation on investment does not guarantee that the long-term effects of a sustained increase in utilisation will be weak, too. In fact, it is hard to conceive of a steady growth path where firms are content to accumulate at a constant rate if, along this path, they experience utilisation rates that differ significantly (and systematically) from the rate they would like to achieve. Putting it differently, the long-run accumulation function should be infinitely elastic at the desired rate of utilisation. Managerial constraints or other bottlenecks may make it costly to expand at high rates and modify this conclusion. It seems implausible, however, to assume that the long-run accumulation function is anything but highly elastic within the relevant range of observed growth rates. Thus, problems arise with the stagnationist model in section 2.3 because of the implausible extension to the long run of a restriction - the insensitivity of investment to fluctuations in the utilisation rate - that is perfectly reasonable for the short run.

Steindl implicitly agreed with this view. The short-run Keynesian stability condition reverses Steindl’s (1952, p. 219) own assumption that \( L > 1 \). Using the definition of the composite parameter \( L \), this latter assumption translates into the condition \( (mk + bs_f \pi) / s(\pi) > 1 \) in our simplified version of the model. In Steindl’s formalization, investment at time \( t \) is determined by utilisation at time \( t - \theta \). Thus, investment is predetermined at any moment, and the impact effect of changes in utilisation is zero. As a result, his theory can allow investment to be highly sensitive to changes in utilisation in the long run without jeopardizing the stability of the short-run Keynesian equilibrium in each period. The dynamic properties of the sequence of these short-run Keynesian equilibria are a different matter. Unfortunately, as we argued above, Steindl assumed convergence of this sequence to a long-run equilibrium characterized by unrealistically high utilisation and growth rates.

The rest of this section introduces modifications of the model to overcome these two sets of shortcomings. The modifications involve dynamic equations to describe induced shifts in both the markup and the accumulation function. The implications of these shifts and their interactions will be examined in subsection 3.4. First, however, the two types of dynamic adjustment are considered separately in subsections 3.2-3.3.

### 3.2 Markup dynamics

The endogenous movements in the profit share can be captured by an expectations-augmented ‘price Phillips curve’ which relates price inflation to deviations of actual from
desired utilisation (rather than to conditions in the labour market).\textsuperscript{11} Thus, let

\[ \hat{p} = f(u, \pi) + (\hat{w}^e - \alpha); \]

where \( \hat{p} = \dot{p}/p \) is the rate of inflation. Utilisation \( u \) reflects current demand conditions in the product market and has a positive effect on price inflation. It is assumed that firms always aim for a profit share between zero and one (that is, \( f(u, 0) > 0, f(u, 1) < 0 \) for any value of the utilisation rate), and a non-positive feedback from the current profit share is included to ensure this property. The term \( \hat{w}^e - \alpha \), finally, is the expected growth rate of nominal unit wage cost: \( \hat{w}^e \) denotes expected wage inflation and we assume a fixed-coefficient production function with Harrod-neutral technical progress at the rate \( \alpha \).

If firms correctly anticipate wage inflation \((\hat{w}^e = \hat{w})\), this price Phillips curve implies that

\[ \dot{\pi} = \frac{d}{dt} \left( \frac{w}{p} \frac{L}{Y} \right) = \frac{w}{p} \frac{L}{Y} (\hat{w} - \alpha - \hat{p}) = (1 - \pi) f(u, \pi) \]

(13)

Combining equation (13) with the stagnationist equilibrium condition for the product market, equation (12), the result is a first-order differential equation in \( \pi \),

\[ \dot{\pi} = (1 - \pi) f(u(\pi), \pi) = \phi(\pi); \quad \phi(0) > 0; \phi(\pi) < 0 \text{ for } \bar{\pi} < \pi < 1 \]

(14)

This equation has at least one locally stable stationary solution between zero and one. Uniqueness is ensured if the Robinsonian stability condition is met since, in this case, \( \phi'(\pi) < 0 \). If the Robinsonian stability condition fails to be satisfied, the derivative of \( \phi \) cannot be unambiguously signed, and there may be multiple solutions. Even with multiple solutions, we still get convergence of the profit share to a stationary point but initial conditions will determine which one. Using (12), it follows that \( u \to u(\pi^*) \) if \( \pi \to \pi^* \).

The comparative statics are straightforward. At a locally stable stationary point

- a marginal upward shift in the investment function (a rise in \( a \)) leads to an increase in both \( u^* \) and \( \pi^* \). The growth rate also increases.
- a marginal increase in the saving propensity \( s \) leads to a decline in both \( u^* \) and \( \pi^* \), and the growth rate also suffers.
- a marginal weakening of competition (an upward shift in the \( f \)-function) leads to an increase in \( \pi^* \). Utilisation falls if the Robinsonian stability condition is met, and the growth effects of weaker competition and higher profit margins are ambiguous, as in section 2.3.

\textsuperscript{11}See Flaschel and Krolzig (2002) for a general analysis of the specification and interaction of wage and price Phillips curves.

\textsuperscript{12}If \( f(u, \pi) \) is continuous and \( f(u, 1) < 0 \), the inequality \( \phi(\pi) = (1 - \pi) f(u, \pi) < 0 \) must hold for values of \( \pi \) above some threshold \( \bar{\pi} \) (that is, for \( \bar{\pi} < \pi < 1 \)).
3.3 Accumulation dynamics

The combination of a low short-run but high long-run sensitivity of investment to changes in utilisation can be captured by introducing dynamic adjustments in the constant term $a$ in the investment function. Thus, let

$$\dot{a} = h(u, \pi, g); \quad h_1 > 0, h_2 \geq 0, h_3 \leq 0$$

(15)

where $g$ is the current rate of accumulation. This formulation generalizes the approach used by, among others, Dutt (1995).

Dutt takes actual accumulation $g$ as predetermined at each moment while the desired accumulation rate is determined by utilisation and profitability (as well as the gearing ratio, a variable that we have left out so far). The change in $g$ is assumed proportional to the difference between desired and actual accumulation. Thus,

$$\dot{g} = \theta(g^d(u, \pi) - g)$$

Setting $m = b = 0$ in the accumulation function (11), Dutt’s specification emerges as a special case of equations (11) and (15).

Combining (10), (12) and (15) - and treating $\pi$ as constant - we get a one-dimensional differential equation for the movements in ‘animal spirits’,

$$\dot{a} = h(u(a), \pi, g(a, u(a), \pi)) = \psi(a)$$

The sign of $\psi'$ is ambiguous: both utilisation and accumulation depend positively on the value of $a$, and the net feedback from $a$ to the rate of change in $a$ therefore depends on the partials $h_u$ and $h_g$ that describe the relative strength of the effects of $u$ and $g$. In principle there could be multiple stationary points (or no stationary points), and even in the case of a unique stationary point, the stability properties are undetermined. But since, as argued above, the effects of utilisation are likely to be strong and the negative feedback effects from changes in the growth rate are likely to be weak within the relevant range, the most likely outcome is one with a unique, unstable stationary point.

As a simple example, consider the Harrodian case where the change in $a$ is proportional to the difference between actual and desired utilisation, that is $h_2 \equiv h_3 \equiv 0$ and

$$\dot{a} = \lambda(u - 1); \quad \lambda > 0$$

Substituting for $u$, we get

$$\dot{a} = \lambda\left(\frac{k(a - m)}{s + sf(1 - s - b)\pi - mk} - 1\right)$$

This equation has a unique, unstable stationary solution

$$a = \frac{s + sf(1 - s - b)\pi - mk}{k} - m$$
and the warranted growth rate at the associated (unstable) growth path is given by the standard Harrodian expression \( g = \frac{s(\pi)}{k} \). This growth rate is a continuous-time analogue to the unstable solution in Steindl’s model.

Comparative statics can be readily derived (and in fact are well-known from the Harrodian literature). These comparative statics are interesting insofar as one has some indication that forces outside the model keep the economy near the steady growth path. Policy intervention, for instance, or feedback effects from the labour market may play this role. As an alternative Steindlian mechanism, the next subsection considers the stabilizing influence induced changes of the markup.

### 3.4 Combining markup and accumulation dynamics

Using equations (10), (12) and (14)-(15), we get a two-dimensional system of differential equations in \( a \) and \( \pi \):

\[
\begin{align*}
\dot{a} &= h(u(a, \pi), \pi, g(a, \pi)) = H(a, \pi); H_a > 0 \\
\dot{\pi} &= (1 - \pi)f(u(a, \pi), \pi) = G(a, \pi); G_a > 0, G_\pi < 0
\end{align*}
\]

The properties of this system depend on the functions \( H \) and \( G \). We assume that both the Keynesian and the Robinsonian stability conditions are met so that \( u_\pi < 0 \). Hence,

\[
\begin{align*}
G_a &= (1 - \pi)f_u u_a > 0 \\
G_\pi &= (1 - \pi)(f_a u_\pi + f_\pi) - f = (1 - \pi)(f_a u_\pi + f_\pi) < 0 \text{ at a stationary point with } \dot{\pi} = (1 - \pi)f(u, \pi) = 0
\end{align*}
\]

Turning to the partials of \( H \), we have

\[
\begin{align*}
H_a &= h_a u_a + h_g g_a \\
H_\pi &= h_a u_\pi + h_\pi + h_g g_\pi
\end{align*}
\]

The first term in the expression for \( H_a \) is positive and the second negative. In line with the discussion in subsection 3.3, however, we expect that the first term will dominate and that the pure accumulation dynamics is destabilizing; that is, we consider the case in which \( H_a > 0 \). The partial \( H_\pi \), on the other hand, is difficult to sign on either theoretical or empirical grounds. As a result, qualitatively diverse dynamic scenarios are possible.

**Case 1:** Assume that

- current accumulation depends non-negatively on profitability, that the change in \( a \) depends negatively on the current growth rate, and that there are no direct effects of profitability on the change in \( a \); that is \( h_g < 0, h_\pi \equiv 0 \) and \( g_\pi \geq 0 \) in (16).
• price inflation is completely insensitive to small variations in the profit share in the neighborhood of the stationary point; that is in (17) we have \( f_\pi \equiv 0 \) in this neighborhood.

• market conditions are competitive in the Steindlian sense that price adjustment is sensitive to deviations of actual utilisation from desired utilisation. Moreover, the speed of price adjustment is fast relative to shifts in the accumulation function \( (f_u >> h_u) \).\(^\text{13}\)

Using the first two assumptions, we get

\[
\frac{d\pi}{da}|_{\dot{\pi}=0} = -\frac{G_a}{G_\pi} = \frac{u_a}{-u_\pi} > \frac{u_a + \frac{h_u g_a}{h_\pi g_\pi}}{-u_\pi - \frac{h_u g_\pi}{h_\pi g_\pi}} = \frac{H_a}{H_\pi} = \frac{d\pi}{da}|_{\dot{a}=0} > 0
\]

It follows that the \( \dot{\pi} = 0 \) locus is steeper than the \( \dot{a} = 0 \) locus. This implies, assuming existence of a stationary solution, that the stationary point is a node or a focus. Furthermore, the third assumption on the relative adjustment speeds of \( a \) and \( \pi \) ensures the local stability of the stationary solution. To see this, consider the Jacobian of the system

\[
J(a, \pi) = \left( \begin{array}{cc} h_u u_a + h_g g_a & h_u u_\pi + h_g g_\pi \\ (1 - \pi) f_u u_a & (1 - \pi) f_u u_\pi \end{array} \right)
\]

We have

\[
\text{Det} = (1 - \pi) f_u h_g [g_a u_\pi - g_\pi u_a] > 0
\]

\[
Tr = h_u u_a + [h_g g_a + (1 - \pi) f_u u_\pi]
\]

The term in square brackets in the expression for the trace is negative while the first term is positive. Stability - quite intuitively - can be undermined if the destabilizing adjustments of the investment function are fast relative to the speed of the stabilizing markup adjustments. Or putting it differently, the analysis of this case demonstrates that a system which has an unstable equilibrium when the markup is exogenous may be stabilized by endogenous changes in the markup, provided the adjustments of the markup are sufficiently fast (the third assumption above). These results are illustrated in figure 2.

**Case 2:** Our second case shows that fast price adjustments will not always suffice to stabilize the system. Assume that

\(^{13}\)The term "price adjustment" may be misleading. Price adjustments are used to change the markup (cf. the inclusion of expected wage inflation on the right hand side of the price Phillips curve). It is fast adjustments of the markup that matters. Proportional changes of money wages and prices have no real effects in the present model. Extended Steindlian models may break this dichotomy between nominal and real magnitudes, but there is no presumption that fast adjustments in nominal wages and prices would be stabilizing in these extended models.
Figure 2: Stabilizing markup dynamics

- there is no negative feedback from the current accumulation rate to the change in $a$ (that is, $h_g = 0$
- the profit share exerts a positive effect on the change in $a$ and/or a direct negative effect on price inflation ($h_\pi > 0$ and/or $f_\pi < 0$

It is readily seen that with these assumptions the determinant of the Jacobian becomes negative and the stationary point is a saddlepoint.\textsuperscript{14} Figures 3a and 3b illustrate the outcome. In figure 3a, $H_\pi > 0$ and the $\dot{a} = 0$ locus is negatively sloped. In figure 3b, $H_\pi < 0$, and both loci are positively sloped; the slope of the $\dot{a} = 0$ locus, however, is steeper than that of the $\dot{\pi} = 0$ locus. In both figures the stationary point exhibits saddlepoint instability, and global analysis is needed to decide what will happen over the longer run.

\textsuperscript{14}We get

\[
\det = h_u a_u (f_u u_\pi + f_\pi) - (1 - \pi) f_u a_a (h_u u_\pi + h_\pi) \\
= h_u a_a f_\pi - (1 - \pi) h_\pi u_a f_u < 0
\]
3.5 Comparative statics

The comparative statics are of interest in case 1 above, in which the stationary solution may be stable. Consider, for instance, changes in the saving rate. A rise in $s$ leads to a downward shift in the $u(.,.)$-function but - using (17) and $f_\pi = 0$ - the stationary solution for $u$ is unchanged. Using (16) and $h_\pi = 0$, it then follows that the equilibrium value of $g$ is also unaffected by the rise in $s$. Since $g = s(1-s)^{\pi+s}/k$ we therefore must have a fall in the stationary value of $\pi$.

More interesting, from a Steindlian perspective, is the analysis of changes in competition. According the Steindl, the degree of competition has a dual effect. A decline in competition puts upward pressure on the markup and, secondly, leads to a decline in the adjustment speed of the markup. The first effect corresponds to an upward shift in the Phillips curve (the $f$-function) while the second can be parameterized by introducing a multiplicative constant $\mu$ in the equation for the change in the profit share. Thus, the effects of changes in the degree of competition can be captured by re-writing equation (17) as

$$\dot{\pi} = (1 - \pi)\mu [f(u(a, \pi), \pi) + \nu] = G(a, \pi; \mu, \nu)$$

The benchmark degree of competition in (17) is associated with $\mu = 1$ and $\nu = 0$; increased monopolization leads to a reduction in $\mu$ (slower adjustment speeds) and a rise in $\nu$ (upward pressure on the markup).

Consider now the effects of a rise in monopolization. The increase in $\nu$ implies an upward shift in the $\dot{\pi} = 0$ locus while changes in $\mu$ have no effects on the slope or position of either of the two loci. The upward shift in the $\dot{\pi} = 0$ locus entails a decline in the stationary
solutions of both $a$ and $\pi$ (see figure 2), and the rates of utilisation and accumulation must fall too. To see this, note that by assumption $f_\pi \equiv 0$, and the rise in $\nu$ must therefore be associated with a decline in $u$ if $f(u, \pi) + \nu$ is to remain equal to zero. The fall in the rate of accumulation now follows from the decline in both $u$ and $\pi$ (since $g = \frac{s(\pi)}{k} u$).

Although changes in the parameter $\mu$ have no effects on the shapes and positions of the $\dot{\pi} = 0$ and $\dot{a} = 0$ loci, these changes may still be of critical importance. By assumption the pure accumulation dynamics is unstable ($H_a > 0$) while the feedback from $\pi$ to $\dot{\pi}$ is negative ($G_\pi < 0$). Since $Tr = H_a + G_\pi = H_a + \mu(1 - \pi)f_u u_\pi$ there is therefore a critical value $\bar{\mu}$ such that $Tr \geq 0$ for $\mu \leq \bar{\mu}$. It follows that a decline in competition may destabilize a previously stable equilibrium.

Overall then, the Steindlian system (16)-(17) with the Case-1 restrictions implies that increased monopolization will (i) produce a decline in the equilibrium values of utilisation and the rate of accumulation, and (ii) endanger the local stability of the equilibrium. These implications are consistent with Steindl’s conclusions. But somewhat surprisingly - and contrary to Steindl’s analysis - an upward shift in the price Phillips curve in this model ultimately generates a fall in the profit share. This counter-intuitive rise in the profit share following an increase in monopolization is reversed when we add reserve-army effects.

4 Adding a labour market

4.1 The reserve army of labour

The dynamic systems developed so far have focused on the product market and the interaction between investment, saving, finance and pricing decisions. The neglect of the labour market is striking but not entirely un-Steindlian. Steindl (1952, p. 168) for instance points out that, since it is strongly influenced by immigration, the growth of the working population is as much an effect as a cause of the trend in accumulation. The same conclusion is reached in his discussion of Marx on pp. 233-34. According to this position, the growth of the labour force is endogenous and does not constrain accumulation.

It is hard to square this dismissal of any role of the labour supply with Steindl’s (1979, p. 12) argument that "the most striking feature of the new economic climate" is the way prolonged near-full employment "has strengthened the economic position of workers and the power of trade unions, and has led to demands for workers’ participation", and that, as a result of these demands, the attitudes of governments and big business alike have changed:

Formerly there was a general conviction in most countries that the government would intervene to prevent a prolonged depression; this reduced uncertainty and
therefore made for higher and more stable private investment. This confidence has been shattered. Here is another reason why the function $\phi$ [the investment function] has shifted downwards (1979, p. 13)

Steindl’s seemingly contradictory suggestions with respect to labour market conditions may be reconciled by noting that although there have been periods of low official unemployment both before the big depression and, especially, in the 1950s and 1960s, the supply of labour to the modern, capitalist part of the economy was quite elastic. Up until the 1960s most OECD countries had hidden reserves of unemployment in agriculture, in parts of the service sector and among women, and, as pointed out by Steindl, immigration also helped alleviate any shortages of labour. The hidden reserve army gradually became depleted, and immigration was hampered by growing political resistance. As a result, the economy became mature in Kaldor’s (1966) sense of the word: its growth rate became constrained by the growth in the labour force.

We formalize this argument (which may or may not be a reasonable representation of Steindl’s thinking) by including an effect of labour market conditions on the shifts in the investment function. Thus, let

$$\dot{a} = h(u(a, \pi), \pi, g, e); \quad h_1 > 0, h_2 \geq 0, h_3 \leq 0, h_4 < 0$$

$$= H(a, \pi, e)$$

where $e$ is the measure of labour market conditions. We shall refer to this variable simply as the employment rate. In any empirical application, however, the role of hidden unemployment as well as the possibility of obtaining workers through immigration must be taken into account. The equation describes how ‘animal spirits’ suffer under full employment, leading to a gradual, downward shift in the investment function. This equation captures, we believe, Steindl’s main point - a point which is closely related to Kalecki’s (1943) insights that persistent high employment undermines "the social position of the boss" and "the self assurance and class consciousness of the working class" grows (Kalecki (1971, p. 140-1). It should be noted, perhaps, that the employment effects on animal spirits are likely to be non-linear: the effects of changes in employment are likely to be negligible at high levels of unemployment but very substantial when the economy approaches full employment.

The movements in the employment rate depend on changes in the labour force, output and technology. Assuming Harrod-neutral technical progress, we have

$$\dot{e} = e(\dot{u} + g - n)$$

$$= e \left[ \frac{u_a(a, \pi)}{u(a, \pi)} \dot{a} + \frac{u_a(a, \pi)}{u(a, \pi)} \dot{\pi} + g(a, \pi) - n \right]$$

where $n$ is the growth rate of the labour force in efficiency units. For simplicity we shall ignore the possibility of induced changes in the growth of the labour supply and take $n$ to be constant.
4.2 Employment and accumulation dynamics

Equations (18)-(19) and (14) constitute a three-dimensional system in $a, \pi, e$. First, however, we shall consider the special case with a constant markup, that is, the case in which $\dot{\pi} \equiv 0$. We then have the following two-dimensional system

\[
\dot{a} = H(a,e); H_a > 0, H_e < 0 \\
\dot{e} = e \left[ \frac{u_a(a)}{u(a)} \dot{a} + g(a) - n \right] = F(a,e)
\]

where it is assumed, as in section 3, that the pure accumulation dynamics is unstable ($H_a > 0$).

At a stationary point we get

\[
F_a = e \left[ \frac{u_a(a)}{u(a)} H_a + g'(a) \right] > 0 \text{ since } g'(a) > 0, \frac{u_a(a)}{u(a)} > 0, H_a > 0 \\
F_e = e \frac{u_a(a)}{u(a)} H_e < 0
\]

Hence, evaluated at a stationary point, the Jacobian takes the following form

\[
J(a,e) = \begin{pmatrix}
H_a & H_e \\
\left[ e \left( \frac{u_a(a)}{u(a)} H_a + g'(a) \right) \right] & e \frac{u_a(a)}{u(a)} H_e
\end{pmatrix}
\]

and

\[
Det(J) = -eg'(a)H_e > 0 \\
Tr(J) = H_a + e \frac{u_a(a)}{u(a)} H_e
\]

It follows that the stationary solution represents a node or a focus. Unlike the system with markup and accumulation dynamics, saddlepoint instability can be excluded. But analogously to case 1 above, stability is ensured if animal spirits adjust slowly relative to the adjustment in the stabilizing variable, in this case the employment rate. A Marxian reserve army effect, in other words, may help to stabilize the economy.
4.3 Employment, accumulation and markup dynamics

Now consider the full three dimensional system consisting of (18)-(19) and (14). If \( g_\pi \geq 0 \), there is (at most) one stationary solution.\(^{15}\) To see this, note that stationarity requires

\[
\begin{align*}
    g(a, \pi) &= n; \quad g_a > 0, g_\pi \geq 0 \\
    G(a, \pi) &= 0; \quad G_a > 0, G_\pi < 0
\end{align*}
\]

These two equations cannot have more than one solution for \( a^* \) and \( \pi^* \). Having found \( a^*, \pi^* \), the equilibrium solution for the employment rate can be derived from

\[
\dot{a} = H(a^*, \pi^*, e) = 0; \quad H_e < 0
\]

We shall not attempt a full analysis of this three-dimensional system but focus, instead, on the implications of adding a reserve-army mechanism to in the two cases that we considered in section 3. In case 1, the markup-cum-accumulation dynamics yields a node or focus. The stationary point, however, is unstable if the markup adjustments are slow, and the interesting question now is whether - when this happens - the reserve-army mechanism can stabilize the equilibrium. In case 2, the markup-cum-accumulation dynamics produced a saddlepoint, and the question again concerns the stabilizing effects of adding the reserve-army effect: will the two-dimensional saddlepoint turn into a locally stable equilibrium in the three-dimensional system if the reserve-army mechanism is sufficiently strong. As we shall see, the answer to both of these questions is affirmative.

Consider first the three-dimensional version of Case 1; that is, assume that

\[ h_\pi = f_\pi = 0; g_\pi \geq 0, h_y < 0 \]

and, to simplify, let\(^{16}\)

\[ g_\pi = 0 \]

Evaluated at a stationary point the Jacobian now takes the following form

\[
J(a, \pi, e) = \begin{pmatrix}
\alpha_1 - \varepsilon & -\alpha_2 & -\alpha_3 \\
\mu & -\mu & 0 \\
\beta_1(\alpha_1 - \varepsilon) - \beta_2\mu \alpha_1 + \gamma & -\beta_1 \alpha_2 + \beta_2 \mu \alpha_2 & -\beta_1 \alpha_3
\end{pmatrix}
\]

where the positive parameters are defined by \( \alpha_1 = h_u u_a, \alpha_2 = h_u u_\pi, \alpha_3 = -h_e, \beta_1 = e \frac{u_a}{u}, \beta_2 = -e \frac{u_\pi}{u}, \gamma = e g_a, \varepsilon = -h_g g_a, \mu = (1 - \pi) \frac{f_u}{h_a} \). The necessary and sufficient Routh-Hurwitz conditions for local stability of this system are that

\(^{15}\)Without the restriction \( g_\pi \geq 0 \) there may (but need not) be multiple solutions. The restriction is satisfied in the special case analyzed by Dutt (1995) - who assumes \( g = a \) and \( g_\pi = 0 \) - as well as by all exhilarationist cases.

\(^{16}\)The partial \( g_\pi \) will be identically equal to zero if \( m = b = 0 \); that is, in the specification of accumulation dynamics used by Dutt (1995).
• \( Tr(J) = \alpha_1 - \varepsilon - \mu \alpha_2 - \beta_1 \alpha_3 < 0 \)
• \(|J_1| + |J_2| + |J_3| = \alpha_3[\mu(\beta_1 \alpha_2 - \beta_2 \alpha_1) + \gamma] + \mu \varepsilon \alpha_2 = \alpha_3 \gamma + \mu \varepsilon \alpha_2 > 0 \)
• \( Det(J) = -\alpha_3 \mu \alpha_2 \gamma < 0 \)
• \(-Tr(J)[|J_1| + |J_2| + |J_3|] + |J| > 0 \)

The second and third conditions are always satisfied, and the first and fourth will be met if the stabilizing effect of the reserve army - as measured by \( \alpha_3 \) - is sufficiently strong. To see that the fourth condition will be met for high values of \( \alpha_3 \), note that 
\(-Tr(J)[|J_1| + |J_2| + |J_3|] \) is quadratic in \( \alpha_3 \) while \( Det(J) \) is linear.

Turning to a three-dimensional version of Case 2, in which the pure markup-accumulation dynamics produces a saddlepoint, the restrictions are

• \( h_g = 0, h_\pi > 0 \) and/or \( f_\pi < 0 \)

To simplify, we assume

• \( h_\pi = g_\pi = 0, f_\pi < 0 \)

The Jacobian now can be written

\[
J(a, \pi, \varepsilon) = \begin{pmatrix}
\alpha_1 & -\alpha_2 & -\alpha_3 \\
\mu \alpha_1 & -\mu(\alpha_2 + \varepsilon) & 0 \\
\beta_1 \alpha_1 - \beta_2 \mu \alpha_1 + \gamma & -\beta_1 \alpha_2 + \beta_2 \mu(\alpha_2 + \varepsilon) & -\beta_1 \alpha_3
\end{pmatrix}
\]

where \( \varepsilon = \frac{h_\pi}{f_\pi} \) and the definition of the remaining parameters is as in case 1 above. The Routh-Hurwitz conditions in this case become:

• \( Tr(J) = \alpha_1 - \mu(\alpha_2 + \varepsilon) - \beta_1 \alpha_3 < 0 \)
• \(|J_1| + |J_2| + |J_3| = \alpha_3[\mu \varepsilon \beta_1 + \gamma] - \mu \varepsilon \alpha_1 > 0 \)
• \( Det(J) = -\gamma \mu(\alpha_2 + \varepsilon) \alpha_3 < 0 \)
• \(-Tr(J)[|J_1| + |J_2| + |J_3|] + |J| > 0 \)

\(^{17}\)The second equality follows from the definitions of the composite parameters which imply that \( \beta_1 \alpha_2 - \beta_2 \alpha_1 = 0 \).
The third condition is always satisfied and it is readily seen that the other three conditions will be met for large values of $\alpha_3$.

It may be interesting to look briefly at the comparative statics of increasing monopolization for the three-dimensional system. Increasing monopolization generates an upward shift in the $G(a, \pi)$-equation that describes mark-up dynamics. Since $G_a > 0$ and $G_\pi < 0$, this upward shift has to be offset by an increase in $\pi$ and/or a decline in $a$. The value of $a$ cannot change, however: the long-run rate of growth $g(a, \pi)$ must remain equal to $n$ and $g_\pi$ is set equal to zero in both case 1 and 2. It follows that $\pi$ must increase and, as a result, utilisation must fall (given the Keynesian and Robinsonian stability conditions). The effect on employment can be found from the stationarity condition for $a$:

$$h(u, \pi, g, e) = 0.$$ Both cases 1 and 2 assume $h_\pi = 0$ and, since $g = n$ is unchanged and $u$ has declined, the value of $e$ must also fall. Intuitively, a larger reserve army is needed to boost animal spirits in order to make up for the depressing effects of lower utilisation. These effects, it should be noted, are consistent with Steindl’s predictions: increased monopolization raises the profit share but generates stagnation in the form of lower employment and capital utilisation.

The long-term effects of a transition, finally, from a stage of large hidden unemployment (in which $h_y = 0$) to one of Kaldorian maturity may be analysed by comparing a stationary point of the two-dimensional accumulation-mark-up dynamics (with $g > n$) to a stationary point of the three-dimensional system. But for the comparison to be meaningful, it must be assumed that the stationary points are stable and the pattern of the two-dimensional system in case 2 is therefore excluded. Assuming that we are in case 1, the negative effect of the employment rate on the rate of change of animal spirits as the economy reaches a mature stage can be depicted as a rightward shift of the $\dot{a} = 0$ locus in figure 2. The result is a decline in both $a$ and $\pi$. The rate of utilisation is unaffected, however, since in case 1 we assumed that $f_\pi = 0$, and this assumption implies that utilisation is constant along the $\dot{\pi} = 0$ locus. Thus, the transition to a new stationary point associated with a constant employment rate implies a fall in the rate of accumulation to bring it into line with the growth of the labour force ($g = n$), an unchanged rate of utilisation and a drop in the profit share.

## 5 Conclusions and extensions

We opened this paper by looking at Steindl’s (1952) model. Steindl himself noted a puzzling and unsatisfactory aspect of this model: it generates unreasonably high values of the profit share.  

These results mirror the effects obtained by Skott (1989a, pp. 151-153) in a related model which, unlike the present setup, treats output as a gradually adjusting state variable and output price as instantaneously adjusting. In this Marshallian setting, changes in output are related to the difference between realized and target profit margins and, by raising the target, increased monopolization therefore depresses output for any given realized profit margin.
(locally) stable steady-state solutions for utilisation and the rate of growth. One contribution of this paper is to demonstrate that weak and questionable non-linearities lie behind this problematic feature of the model. The main contribution, however, lies in the analysis of extended Steindlian models.

Using a continuous-time framework, we first incorporated the interaction between markup dynamics and accumulation dynamics. Steindl, more than any other contributor to the post Keynesian tradition, has emphasized the influence of competitive conditions on the sensitivity of the markup to changes utilisation, and he has consistently combined this emphasis with a keen awareness of the possibilities of Harrodian instability arising from strong, lagged effects of utilisation on the rate of accumulation. Hence, in our view the dynamic interaction between accumulation and the markup constitutes the core of a Steindlian model. We have formalized this interaction in the form of a two-dimensional system of differential equations, one for shifts in the markup and one for shifts in 'animal spirits'. Consistent with Steindl’s vision, we find that fast adjustment of the markup may (but need not) contribute to a stabilization of the steady growth path. The model also supports Steindl’s position on the stagnationist effects of increased monopolization: an upward shift in the dynamic equation for the markup generates a decline in both utilisation and growth. Paradoxically, however, in the stable case it also leads to a decline in the stationary solution for the markup.

The core model - developed in section 3 - can be extended in various ways. Our extension in section 4 focuses on the Marxian influence of the reserve army. There is a tension in Steindl’s views on this issue. Steindl (1952) largely dismisses the idea that accumulation could be constrained by a declining reserve army. In Steindl (1979), however, his position on this issue appears to be reversed since in this paper the effects of prolonged near-full employment on accumulation plays a key role. In any case, the inclusion of a reserve army effect tends to stabilize the economy, as indeed it does in Skott’s (1989a, 1989b) integration of Keynesian and Marxian ideas, and the effects of increasing monopolization are quite Steindlian: increasing monopolization must leave the growth rate unchanged - since it is tied to the growth of the labour force in this model - but monopolization has stagnationist effects in the form of a fall in both the employment rate and the rate of capital utilisation. It should be noted also that this three dimensional system - incorporating employment dynamics as well as markup and accumulation dynamics - reverses the paradoxical effect that characterizes the two-dimensional system without a labour market: in this system an upward shift of the dynamic equation for the markup raises the stationary value of the markup.

The model, we believe, captures important Steindlian insights and overcomes some shortcomings of earlier formalizations. But clearly it has weaknesses. Financial factors, for instance, play a very limited role. We have allowed for retained earnings to stimulate investment. The 'principle of increasing risk' - the cost and riskiness of high degrees of external finance - may provide a rationale for the role of retained earnings. But with a
constant retention rate, retained earnings might also appear in the investment function simply because high current profitability signals the profitability of additions to the capital stock. Furthermore, the principle of increasing risk suggests that in terms of financial constraints, the gearing ratio rather than the flow of retained profits may be the more important variable. Thus, following Steindl (1952), one may extend the model by including the gearing ratio in the investment function. The gearing ratio, indeed, is a key variable in Dutt’s (1995) examination of the interaction between the product market and financial aspects. His analysis, which leaves out markup dynamics and labor market effects, can be seen as complementary to the one presented in this paper.

Other prominent aspects of Steindl’s verbal analysis also suggest further extension of the model. We have taken all saving rates as well as firms’ financial environment as constant. These assumptions clearly could be relaxed to allow for the presence of stock markets, capital gains and endogenous changes in saving behaviour (emphasized by Steindl in several contributions, e.g. Steindl (1982)) or in acceptable standards of financial behaviour by firms (along the lines suggested by Minsky; e.g. Steindl (1990 [1989], p. 173)). From an applied perspective, however, the most severe shortcomings probably relate to the closed-economy assumption and the neglect of policy, both fiscal and monetary. We leave extensions in these and other directions for future research.

References


19 The financial aspects are left out in Steindl (1979). The 1979 paper instead emphasizes labour market effects and markup dynamics, although neither of these factors are included in the formal equations.


