Answers to Homework # 4

1. (a) Write down the translation into the λ-calculus of "A student walks and talks". This handout already shows the translations of "a student" (p.8) and "walks and talks" (p.5). Put them together by "function-argument application".

(b) Apply λ-conversion to simplify the formula. There will be two applications, and the resulting formula should have no λ's.

(c) Write down the translation of "A student walks and a student talks"; simplify by λ-conversion.

(d) The two formulas (if you did parts (a-c) correctly) are not equivalent. Describe a situation (a model) in which one of them is true and the other one is false.

Answer

(a) *a student* : $\lambda \boldsymbol{P}[\exists \boldsymbol{x}\ (\ \mathbf{student}(\boldsymbol{x})\ \&\ \boldsymbol{P}(\boldsymbol{x})\ )]$

walks and talks: $\lambda \boldsymbol{y}[(\mathbf{walk}\ (\boldsymbol{y})\ \&\ \mathbf{talk}(\boldsymbol{y}))]$

a student walks and talks: $\lambda \boldsymbol{P}[\exists \boldsymbol{x}\ (\ \mathbf{student}(\boldsymbol{x})\ \&\ \boldsymbol{P}(\boldsymbol{x})\ )](\lambda \boldsymbol{y}[(\mathbf{walk}\ (\boldsymbol{y})\ \&\ \mathbf{talk}(\boldsymbol{y}))])$

(b) Simplify the expression by two applications of λ-conversion.

Step 1: $\lambda \boldsymbol{P}[\exists \boldsymbol{x}\ (\ \mathbf{student}(\boldsymbol{x})\ \&\ \boldsymbol{P}(\boldsymbol{x})\ )](\lambda \boldsymbol{y}[(\mathbf{walk}\ (\boldsymbol{y})\ \&\ \mathbf{talk}(\boldsymbol{y}))]) \equiv$

$\exists \boldsymbol{x}\ (\ \mathbf{student}(\boldsymbol{x})\ \&\ \lambda \mathbf{y}[(\mathbf{walk}\ (\mathbf{y})\ \&\ \mathbf{talk}(\mathbf{y}))]\ (\boldsymbol{x}))$

Step 2: $\exists \boldsymbol{x}\ (\ \mathbf{student}(\boldsymbol{x})\ \&\ \lambda \mathbf{y}[(\mathbf{walk}\ (\mathbf{y})\ \&\ \mathbf{talk}(\mathbf{y}))]\ (\boldsymbol{x})) \equiv$

$\exists \boldsymbol{x}\ (\mathbf{student}(\boldsymbol{x})\ \&\ (\mathbf{walk}\ (\boldsymbol{x})\ \&\ \mathbf{talk}(\boldsymbol{x})))$

(c) *Every student walks and every student talks*:

$\lambda \boldsymbol{P}[\exists \boldsymbol{x}(\ \mathbf{student}(\boldsymbol{x})\ \&\ \boldsymbol{P}(\boldsymbol{x}))](\mathbf{walk})\ \&\ \lambda \boldsymbol{P}[\exists \boldsymbol{x}(\ \mathbf{student}(\boldsymbol{x})\ \&\ \boldsymbol{P}(\boldsymbol{x}))](\mathbf{talk})$

$\equiv\ \exists \boldsymbol{x}\ (\ \mathbf{student}(\boldsymbol{x})\ \&\ \mathbf{walk}(\boldsymbol{x}))\ \&\ \exists \boldsymbol{x}\ (\ \mathbf{student}(\boldsymbol{x})\ \&\ \mathbf{talk}(\boldsymbol{x})\ )$

(d) For formula (b) to be true, there must be at least one student who both walks and talks. (There may be additional students who walk and don't talk and/or vice versa – that's not specified one way or the other by (b).)

For formula (c) to be true, there must be at least one student who walks and at least one student who talks, but there may or may not be any students who do both.

There are models in which both formulas are true (those are models where at least one student does both) and models in which both formulas are false (those are models in which either no student walks or no student talks (including models where no student does either one). But what question (d) asks for is a model in which one formula is true and the other is false (recall Cresswell's 'most certain principle'): these can't be models in which (b) is true and (c) is false, because there aren't any: (b) *entails* (c).

Answer to (d): Here's a model in which (c) is true but (b) is false:

Let D = {John, Bill}. Let I(**student**) = {John, Bill}, I(**walk**) = {John}, I(**talk**) = {Bill}.

NOTE: (This came up when many of you answered part (c) above.) The following two expressions are equivalent: why? (i) **walk** (ii) $\lambda \boldsymbol{y}[(\mathbf{walk}\ (\boldsymbol{y})]$

What are their types? What are their denotations? What do you get when you apply each of them to an argument **j**, of type e?

2. In the predicate calculus, the sentence "No student talks" can be represented as follows:

$\neg\exists x$ (**student**(x) & **talk**(x)) or equivalently as $\forall x$(**student**(x) $\rightarrow \neg$**talk**(x))
But in the predicate calculus, there is no way to represent the meaning of the NP "no student". Using the λ-calculus in the way illustrated above for the NPs "every student", "a student", "the king", write down a translation for the NP "no student". (There are two logically equivalent correct answers; write down either or both.)

Answer:
(i) $\lambda P[\neg\exists x$ (**student**(x) & $P(x)$)] (ii) $\lambda P[\forall x$(**student**(x) $\rightarrow \neg P(x)$)]

==========

Additional questions, optional.

3. Write out the semantic derivation of *John walks* two ways, once using Montague's generalized quantifier interpretation of *John*, once using the type-e interpretation of *John*.

Answer:
(i) Montague's way. *John:* $\lambda P[P(\mathbf{j})]$ type: $(e \rightarrow t) \rightarrow t$
walks: **walk** type: $e \rightarrow t$
John walks: $\lambda P[P(\mathbf{j})](\mathbf{walk})$ (the subject NP is the function, the verb is its argument) By λ-conversion, this is equivalent to: **walk**(**j**) .

(ii) With e-type interpretation of *John:*
John: **j** type: e
walks: **walk** type: $e \rightarrow t$
John walks: **walk**(**j**) (the verb is the function, the subject NP is its argument.)

4. Write the translation for *every*, by abstracting on the CNP in the given translation of *every man*. Hint: the translation of *every*, like that of every DET, should begin with λQ, where Q is a variable of type $e \rightarrow t$, the type of the CNP with which the DET "wants to" combine.

Answer: $\lambda Q[\lambda P[\forall x\ (\ Q(x) \rightarrow P(x)\ )]]$

Note: Heidi Quinn did this one right and then commented, "Now it makes sense why Larson defined determiners as binary relations between sets of entities!" I agree! (There are some notes about that on pp 8 and 9 of the Lecture 7 notes; we didn't have time to really discuss that.)

5. Work out the translation, using lambda-conversion for simplification of results, of the following. Always apply lambda-conversion as soon as it is applicable, so that the formulas do not become more complex than necessary.
Every violinist who loves Prokofiev is happy.

Helpful strategy: Draw a syntactic tree first to guide yourself through the relevant part-whole structure. Remember our earlier discussion about where restrictive relative clauses attach and how they combine semantically with noun. Review further examples in this handout.

Semi-answer: I won't write out the whole answer, but I'll note that you should first combine the relative clause with the noun *violinist*, following the example of *man who loves Mary* on p. 6 of Lecture 7. Then combine that with *every*, using the translation of *every* in the answer to 4 above, and doing λ-conversion to simplify. From there, follow example (3) on p. 9, using your translation of *every violinist who loves Prokofiev* in place of the simple *every violinist.*