

First sample exercise: Work out the compositional semantics of the formula

$$\forall x \text{ happy}(x)$$

with respect to the model **M2** and an assignment g that assigns j to every variable. (We'll see at the end that the choice of g to start with in this case doesn't matter, since this is a *closed formula* and (the second) x is a *bound variable*.)

The model M2:

$$D = \{j, m\}$$

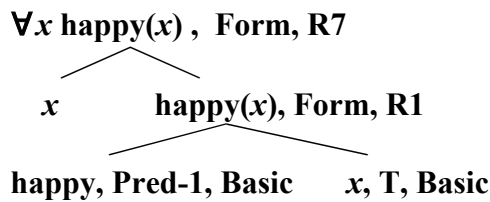
$$I_2(\text{John}) = j$$

$$I_2(\text{Mary}) = m$$

$$I_2(\text{love}) = \{ \langle j, j \rangle, \langle m, j \rangle \}$$

$$I_2(\text{happy}) = \{m\}$$

First step: construct the syntactic "derivation tree" for the formula.



Second step: That is followed by a derivation of the truth-conditions of the formula according to the compositional semantic rules of the predicate calculus. Each line is annotated to identify what semantic rule was applied in the derivation of that line, and what node of the syntactic derivation tree it corresponds to.

Annotated semantic derivation of truth conditions:

$$1. \|\forall x \text{ happy}(x)\|^{M_2, g} = 1 \text{ iff for each } d \text{ in } D, \|\text{happy}(x)\|^{M_2, g[d/x]} = 1. \\ \text{By rule S7 at the "R7" node.}$$

$$2. \text{That will hold iff for each } d \text{ in } D, \|x\|^{M_2, g[d/x]} \in \|\text{happy}\|^{M_2, g[d/x]}. \\ \text{By S1 at the R1 node.}$$

$$3. \text{That will hold iff for each } d \text{ in } D, g[d/x](x) \in \|\text{happy}\|^{M_2, g[d/x]}. \\ \text{By rule A (for variables) at the } x \text{ node.}$$

What does that say? What is $g[d/x](x)$?

$$4. \text{That will hold iff for each } d \text{ in } D, d \in I_2(\text{happy}). \text{ -- What is } I_2(\text{happy})?$$

By rule B (for constants) at the node for happy.

And does that hold, or not?

Do you see why it didn't matter what choice of g we start with? Find a formula for which it DOES make a difference what choice of g we start with.