Homework 6

I. From PtMW

Chapter 6, p.129

Exercise 3

(a)

$$((p \wedge q) \wedge s)$$
$$= ((1 \wedge 1) \wedge 0)$$
$$= (1 \wedge 0)$$
$$= 0$$

(b)

$$(p \wedge (q \wedge s))$$
$$= (1 \wedge (1 \wedge 0))$$
$$= (1 \wedge 0)$$
$$= 0$$

(e)

$$((p \wedge q) \leftrightarrow (r \wedge \neg s))$$
$$= ((1 \wedge 1) \leftrightarrow (1 \wedge \neg 0))$$
$$= (1 \leftrightarrow (1 \wedge 1))$$
$$= (1 \leftrightarrow 1)$$
$$= 1$$

Exercise 4

(a)

p	q	$\neg q$	$(p \vee \neg q)$
1	1	0	1
1	0	1	1
0	1	0	0
0	0	1	1

(b)

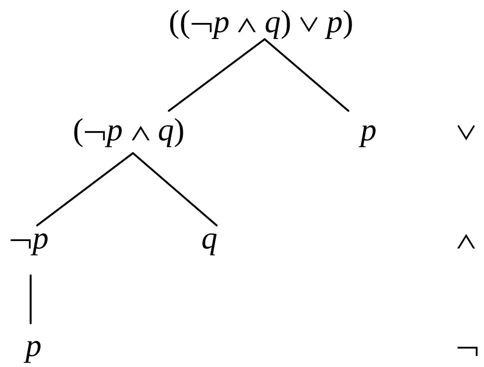
p	q	$\neg p$	$(\neg p \wedge q)$	$\neg(\neg p \wedge q)$
1	1	0	0	1
1	0	0	0	1
0	1	1	1	0
0	0	1	0	1

Since the last column in both truth tables agrees, these two expressions are logically equivalent.

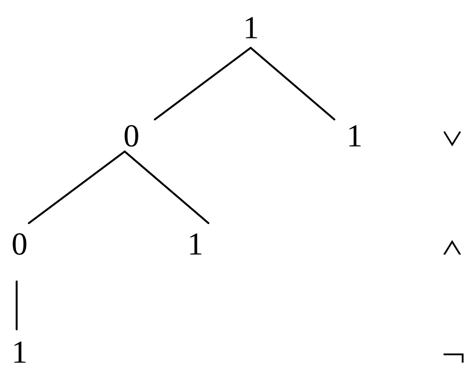
Chapter 13, p. 365

Exercise 1

(a) (i)



(b)(i)



II. About the algebra Form

I am not sure about the answer below. It seems too easy to be correct. But maybe it is indeed as simple as this?

It is easy to show that **Form** is not Boolean. We can choose any of the five properties of a Boolean algebra, and easily show that it doesn't hold of **Form**. Choose the Complementation Law, which *inter alia* says that the following statement must be true of a Boolean algebra:

$$(a \wedge (\neg a)) = 0$$

In algebra **Form**, the only way to get the formula $(a \wedge (\neg a))$, is to apply the operation $\wedge$ to the arguments a and $\neg a$. Stated more formally:

$$\wedge_{\mathbf{Form}}\ (a, \neg a) = (a \wedge (\neg a))$$

Similarly, the only way in which to get the atomic formula 0, is to apply to nullary function 0. Again stated more formally:

$$0_{\mathbf{Form}} = 0$$

And since $\wedge_{\mathbf{Form}}\ (a, \neg a) \neq 0_{\mathbf{Form}}$, it follows that $(a \wedge (\neg a)) = 0$ is not true in **Form**, and therefore that **Form** is not Boolean.