

4. Given a set A that is not empty and R that is an empty set,

R is symmetric, because for every ordered pair  $\langle x, y \rangle$ ,  $\langle x, y \rangle \notin R$ .

Also, R is transitive, because for every ordered pairs  $\langle x, y \rangle$  and  $\langle y, z \rangle$ ,  $\langle x, y \rangle \notin R$  and  $\langle y, z \rangle \notin R$ .

However, R is not reflexive, because it is not the case that for every  $x \in A$ ,  $\langle x, x \rangle \in R$ .

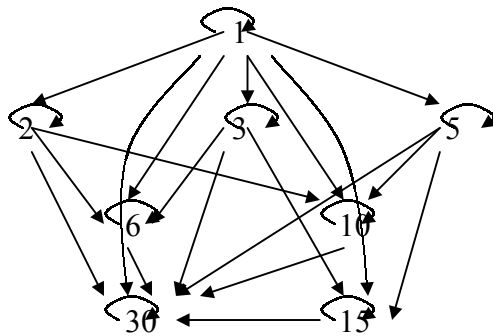
Therefore, reflexivity is not a consequence of symmetry and reflexivity.

5.  $A = \{1, 2, 3, 5, 6, 10, 15, 30\}$ ,  $R = \{\langle x, y \rangle \mid x \text{ divides } y \text{ without remainder}\}$

(a)  $R = \{\langle 1, 1 \rangle, \langle 1, 2 \rangle, \langle 2, 2 \rangle, \langle 1, 3 \rangle, \langle 3, 3 \rangle, \langle 1, 5 \rangle, \langle 5, 5 \rangle, \langle 1, 6 \rangle, \langle 2, 6 \rangle, \langle 3, 6 \rangle, \langle 6, 6 \rangle, \langle 1, 10 \rangle, \langle 2, 10 \rangle, \langle 5, 10 \rangle, \langle 10, 10 \rangle, \langle 1, 15 \rangle, \langle 3, 15 \rangle, \langle 5, 15 \rangle, \langle 15, 15 \rangle, \langle 1, 30 \rangle, \langle 2, 30 \rangle, \langle 3, 30 \rangle, \langle 5, 30 \rangle, \langle 6, 30 \rangle, \langle 10, 30 \rangle, \langle 15, 30 \rangle, \langle 30, 30 \rangle\}$

R is a weak order because it is transitive, reflexive, and antisymmetric, but it is a partial order because it is non-connected.

(b)



The element 30 is maximal and greatest.

The element 1 is minimal and least.

(c)  $\wp(B) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}$

$R = \{\langle \emptyset, \emptyset \rangle, \langle \emptyset, \{a\} \rangle, \langle \emptyset, \{b\} \rangle, \langle \emptyset, \{c\} \rangle, \langle \emptyset, \{a, b\} \rangle, \langle \emptyset, \{b, c\} \rangle, \langle \emptyset, \{a, c\} \rangle, \langle \emptyset, \{a, b, c\} \rangle, \langle \{a\}, \{a\} \rangle, \langle \{a\}, \{a, b\} \rangle, \langle \{a\}, \{a, c\} \rangle, \langle \{a\}, \{a, b, c\} \rangle, \langle \{b\}, \{b\} \rangle, \langle \{b\}, \{a, b\} \rangle, \langle \{b\}, \{b, c\} \rangle, \langle \{b\}, \{a, b, c\} \rangle, \langle \{c\}, \{c\} \rangle, \langle \{c\}, \{a, c\} \rangle, \langle \{c\}, \{b, c\} \rangle, \langle \{c\}, \{a, b, c\} \rangle, \langle \{a, b\}, \{a, b\} \rangle, \langle \{a, b\}, \{a, b, c\} \rangle, \langle \{b, c\}, \{b, c\} \rangle, \langle \{b, c\}, \{a, b, c\} \rangle, \langle \{a, c\}, \{a, c\} \rangle, \langle \{a, c\}, \{a, b, c\} \rangle, \langle \{a, b, c\}, \{a, b, c\} \rangle\}$

The element  $\{a, b, c\}$  is maximal and greatest.

The element  $\emptyset$  is minimal and least.