<u>Partee, ter Meulen & Wall, Chapter 3</u>

Exercise 1

(a) (i) *is a child of*

Irreflexive – No-one can be a child of him/herself.

Asymmetric – No-one can be both the child and parent of someone else.

Intransitive – No-one can be the child of his/her parents and grandparents.

Non-connected – There are many people who are not related.

(ii) *is a brother of*

Irreflexive – You cannot be your own brother.

Non-symmetric – Whenever you have two brothers x and y, both $<x,y>$ and $<y,x>$ will be in R. However, since y can also be a sister, it is possible that $<x,y>$ is in R, while $<y,x>$ is not.

Non-transitive – If x and y are brothers of each other, then both $<x,y>$ and $<y,x>$ are in R. However, $<x,x>$ are not in R. But there are indeed some transitive cases – consider a family with three boys a, b, c. It is true of such situation that $<a,b>$, $<b,c>$ and $<a,c>$.

Non-connected – Not all people are related.

(iii) *is a descendant of*

Irreflexive – No-one can be a descendant of himself/herself.

Asymmetric – If x is a descendant of y, the y cannot also be a descendent of x. This also holds true of the case where x and y are identical, that is x cannot be a descendant of x. This is why this is not simply anti-symmetric.

Transitive.

Non-connected – Not all people are related.

(iv) *is an uncle of*

Non-reflexive – First, to show that it is not irreflexive. Suppose that *John* is the uncle of *Peter,* i.e. <*John*, *Peter*> is in *R*. Suppose further that *Peter* marries the wife the *John* (after *John* and his wife divorced or *John* has passed away). Then *Peter* will be his own uncle, i.e. <*Peter*, *Peter*> will be in *R*. However, not all men will marry their aunts and there will be many people of whom it will not be true that they will be their own uncles. There are, of course, also women who can never be uncles but who can have uncles.

Non-symmetric. First, to show that it is not asymmetric. Suppose that *John* is the uncle of *Peter*, i.e. <*John*, *Peter*> is in *R*. Suppose further that *John* has an uncle *Joe*, therefore <*Joe, John*>. Now, since you can marry your uncle or aunt, I suppose that you can also marry your great-aunt or great-uncle. It is therefore possible that *Peter* can marry the wife of *Joe* (under the assumption that *Joe* and his wife got divorced or *Joe* has passed away). Then *Peter* will be married to the aunt of *John*, and therefore *Peter* will be an uncle of *John*, i.e. <*Peter*, *John*>. Therefore, symmetric pairs are possible. However, not all men will marry their great-aunts, and there will therefore be many pairs of people for whom this symmetric relation will not hold. And again, there are also many women who can never be the uncle of anybody but who can have uncles.

Non-transitive. First, to show that it is not intransitive. Suppose that *John* is the uncle of *Peter*, *Mary* and *Susan*. Suppose further that *Susan* has a child by the name of *Harry*. *Peter* will therefore be the uncle of Susan's child *Harry*. Therefore, we have <*John*, *Peter*>, and <*Peter*, *Harry*> in *R*. Since you are allowed to marry your uncle, *Mary* can marry *John*. *Mary*'s husband will then be the uncle to her sister's children, i.e. *John* will be uncle of *Harry*, i.e. <*John*, *Harry*>. We then have the transitive relation <*John*, *Peter*>, <*Peter*, *Harry*> and <*John*, *Harry*>. But, again not all women will marry their uncles, and therefore this relation will not always be transitive.

Non-connected. There are many unrelated people.

(b) (ii) This relation will now be *symmetric*. Since it is no longer the case that one of the members in a related pair can be a sister, it follows that whenever x is a brother y, then y is also a brother of x.

Exercise 2

(a) Definition of minimal pair: Two words form a minimal pair iff: (i) they consist of the same number of segments, (ii) they differ in only one segment, (iii) the segments in which they agree have the same precedence relation with regard to each other, and (iv) the segments in which they differ occur in the same positions in the two words relative to the other segments.

Irreflexive – No word can form a minimal pair with itself.

Symmetric – The order between the words in a minimal pair doesn't matter.

Non-transitive – The pairs <*pet*, *pot*>, <*pot*, *pat*> and <*pet*, *pat*> are in *A*. However, the pairs <*pet*, *pot*> and <*pot*, *lot*> are in *A*, but not the pair <*pet*, *lot*>.

Non-connected.

(b) Definition of complementary distribution: Two sounds are in complementary distribution if the contexts in which they can occur are mutually exclusive.

Irreflexive – A sound cannot be in complementary distribution with itself, that is a sound cannot occur only in the contexts where it cannot occur.

Symmetric – The order between a pair of sounds in complementary distribution doesn't matter. That is, if [t] and [t^h] are in complementary distribution, then so are [t^h] and [t].

Non-transitive – If a phoneme has three allophones in complementary distribution, then a transitive relationship holds between them. For instance the phoneme /t/ has three such allophones [t, t^h, |]. From this it follows that <t,t^h>, <t^h,|> and <t,|> are all in *C*. However, since a phone can never be in complementary distribution with itself, this relation is not transitive. We have <t,t^h> and <t^h,t>, but we don't therefore have <t,t>.

Non-connected. There are many sounds that are on overlapping distribution.

(c) Definition of free variation: Two sounds are in free variation if each can occur in all contexts where the other can occur.

Reflexive – Every sound can occur in all contexts where itself can occur.

Symmetric – The order between two sounds in free variation doesn't matter.

Transitive – If sound *x* can occur in all contexts where sound *y* can and *vice versa*, then we have <*x*, *y*> and also <*y*, *x*> in *F*. And since a sound is in free variation with itself, we also have <*x*, *x*>. If sound *x* can occur in all contexts where sound *y* can and *vice versa*, then we have <*x*, *y*>. If sound *y* can occur in all contexts where sound *z* can and *vice versa*, then sound *x* can of course also occur in all contexts where sound *z* can and *vice versa*. Therefore, we have both <*y*, *z*> and <*x*, *z*>.

Non-connected – Not all sounds are in free variation.

This is an equivalence relation that partitions the set of all sounds of English into the subsets consisting of all sounds that have the same distributional properties. Exactly how these sets will be determined, depends on how we define the context in which sounds can occur – in terms of position in a word, in terms of some prosodic theory, in terms of syllable structure, etc.

(d) Some allophones are in free variation (think of instances of variation), while others in complementary distribution.

Reflexive – If a phoneme has an allophone *x*, the <*x*,*x*> is a pair consisting of only allophones of this phoneme.

Symmetric – The order between allophones doesn't matter.

Non-transitive – First, to show that it can be transitive. If a phoneme has three allophones, like /t/ has [t, t^h, |], we have <t,t^h>, <t^h,|> and <t,|> in *A*. Now, to show that it is not always transitive. The phoneme /i/ has at least two allophones, [i] and [«] – as in *compete* but *competition*. Therefore, we have <i,«> in *A*. The phoneme /«U/ also has at least two allophones, [«U] and [«] – as in *compose* and *composition*. We therefore have <«,«U> in *A*. However, it is not true that [i] and [«U] are allophones of one phoneme, i.e. we don't have <i,«U> in *A*.

Non-connected. There are many sounds that are not allophones of the same phoneme.

(e) Reflexive – A set always has the same number of members as itself.

Symmetric – If set *A* has the same number of members as set *B*, then set *B* also has the same number of members as set *A*.

Transitive – If set *A* has same number of members as set *B*, and set *B* has the same number of members as set *C*, then it follows that set *A* also has the same number of members as set *C*.

Non-connected – On the assumption that the set of sets over which this relation is defined contains at least two sets that differ from each other in the cardinality. Or stated differently: Only in the special case where the cardinality of all the sets in the collection of sets is equal, will *Q* be connected.

This is equivalence relation that partitions the set of sets into subsets consisting of all the sets of equal size.

Exercise 3

(a) R_1 Reflexive.

Anti-symmetric. We have pairs like <2,2> which is symmetric with themselves. But we also have <2,1> but not <1,2>.

Non-transitive. We have <3,4> and <4,1>, but not <3,1>. However, we also have <2,1>, <1,1>, and then trivially <2,1> again.

Non-connected. Values 3 and 1 are not related at all.

R_1^{-1} = {<1,1>, <1,2>, <4,3>, <2,2>, <3,3>, <4,4>, <1,4>}

Reflexive.

Anti-symmetric. We have pairs like <2,2>. But we also have <1,2> but not <2,1>.

Non-transitive. We have <1,4> and <4,3>, but not <1,3>. However, we also have <1,2>, <2,2>, and then trivially <1,2> again.

Non-connected. Values 3 and 1 are not related at all.

R_1' ={<1,2>, <1,3>, <1,4>, <2,3>, <2,4>, <3,1>, <3,2>, <4,2>, <4,3>}

Irreflexive.

Non-symmetric. We have <1,3> and <3,1>. But we also have <1,2> without <2,1>.

Non-transitive. We do have <1,2>, <2,3> and <1,3>. However, we also have <1,3>, <3,1> but not <1,1>.

Connected. The book says non-connected, but as far as I can determine, there is a relation defined between every two members.

R_2 Irreflexive. There is no x such that $<x,x> \in R_2$.

Asymmetric. There are no x and y such that $<x,y> \in R_2$ and $<y,x> \in R_2$.

Transitive.

Connected.

R_2^{-1} ={<4,3>, <2,1>, <4,1>, <3,2>, <4,2>, <3,1>}

Irreflexive.

Asymmetric.

Transitive.

Connected.

R_2' ={<1,1>, <2,1>, <2,2>, <3,1>, <3,2>, <3,3>, <4,1>, <4,2>, <4,3>, <4,4>}

Reflexive.

Anti-symmetric. The only symmetric pairs of the for $<x,x>$.

Transitive.

Connected.

R_3 Non-reflexive. We have <2,2>, but not <1,1>, <3,3> or <4,4>.

Symmetric.

Non-transitive. We have <2,2> and <2,4>, and then trivially <2,4> again. But we also have <3,1> and <1,3>, but not <3,3>.

Non-connected. Values 1 and 2 are not related.

R_3^{-1} = {<4,2>, <1,3>, <4,3>, <2,2>, <3,1>, <3,4>, <2,4>}

Non-relfexive.

Symmetric.

Non-transitive.

Non-connected.

R_3' = {<1,1>, <1,2>, <1,4>, <2,1>, <2,3>, <3,2>, <3,3>, <4,1>, <4,4>}

Non-reflexive. We have <1,1>, <3,3> and <4,4>, but not <2,2>.

Symmetric.

Non-transitive. We have <1,1>, <1,2> and then trivially <1,2> again. But we also have <1,2>, <2,3> and not <1,3>.

Non-connected. Values 1 and 3 are not related.

R_4 Reflexive.

Symmetric.

Transitive.

Non-connected. Values 1 and 2 are not related.

R_4^{-1} = {<1,1>, <4,2>, <3,1>, <2,2>, <1,3>, <4,4>, <3,3>, <2,4>}

Reflexive.

Symmetric.

Transitive.

Non-connected.

R_4' = {<1,2>, <1,4>, <2,1>, <2,3>, <3,2>, <3,4>, <4,1>, <4,3>}

Irreflexive.

Symmetric.

Intransitive.

Non-connected. Values 1 and 3 are not related.

R_4 and R_4^{-1} are equivalence relations.

$P_{R4/R4\text{-}1}$ = {{1,3}, {2,4}}

(b) {<1,1>, <2,2>, <2,3>, <3,2>, <3,3>, <4,4>}

(c) 1: {{1}, {2}, {3}, {4}}

2: {{1,2,3,4}}

3: {{1}, {2,3,4}}

4: {{2}, {1,3,4}}

5: {{3}, {1,2,4}}

6: {{4}, {1,2,3}}

7: {{1}, {2}, {3,4}}

8: {{1}, {3}, {2,4}}

9: {{1}, {4}, {2,3}}

10: {{2}, {3}, {1,4}}

11: {{2}, {4}, {1,3}}

12: {{3}, {4}, {1,2}}

13: {{1,2}, {3,4}}

14: {{1,3}, {2,4}}

15: {{1,4}, {2,3}}

Exercise 4

I couldn't figure this out for myself. However, once I've read the answer in the back of the book, I did get it. The basic problem is, that this "proof" assumes that $\forall x \exists y$ such that $\langle x,y \rangle \in R$. And this is not necessarily true of all relations that are symmetric and transitive. The example that the book uses to illustrate this is the relation R = "have the same parents and have parents who have at least two children". This relation is transitive – if John has the same parents as Mary, and Mary has the same parents as Peter, then John has the same parents as Peter. It is also symmetric – if John has the same parents as Mary, then Mary also has the same parents as John. However, suppose that Susan were an only child – this relation doesn't hold of her. Susan does have the same parents as herself, but she is not the child of parents with at least two children. Therefore it is not the case that $\langle \text{Susan, Susan} \rangle \in R$. Therefore, even though this relation is both symmetric and transitive, it is not reflexive.

Exercise 5

(a) *R* = {<1,1>, <1,2>, <1,3>, <1,5>, <1,6>, <1,10>, <1,15>, <1,30>, <2,2>, <2,6>, <2,10>, <2,30>, <3,3>, <3,6>, <3,15>, <3,30>, <5,5>, <5,10>, <5,30>, <10,10>, <10,30>, <15,15>, <15,30>, <30,30>}

R is transitive, reflexive, and anti-symmetric – i.e. it qualifies as a weak partial ordering.

(b) (Will hand in separately.)

1 is a minimal and the least element.

30 is a maximal and the greatest element.

(c) $\wp(\{a, b, c\}) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$

(Will hand in separately.)

The empty is a minimal and the least element.

The set $\{a, b, c\}$ is a maximal and the greatest element.