Homework 11

1. (PMW 8.4) Prove: $\forall X \forall n \in \mathbf{N}$ [$|X| = n \rightarrow |\wp(X)| = 2^n$]

0. Basis: $|\wp(\emptyset)| = 2^0 = 1$ ☺

1. Induction step: For any X, let $|X| = n$ & $|\wp(X)| = 2^n$

Show that for any $x \notin X$, $|X \cup \{x\}| = n+1 \rightarrow |\wp(X \cup \{x\})| = 2^{n+1}$

Let $|X \cup \{x\}| = n+1$ and show $|\wp(X \cup \{x\})| = 2^{n+1}$.

$\wp(X) \subset (\wp(X \cup \{x\}) - \wp(X))$ and

$\forall Y \in \wp(X)\ \exists! Z \in \wp(X \cup \{x\}) : Z = Y \cup \{x\}$

$|\wp(X)| = 2^n$, so $|\wp(X \cup \{x\})| = 2^n + 2^n = 2^{n+1}$ ☺

3. Since (0) and (1) are proved By Mathematical Induction, $\forall X \forall n \in \mathbf{N}$ [$|X| = n \rightarrow |\wp(X)| = 2^n$] ☺

2. (PMW 8.5) Prove the generalized dsitributive law of multiplication over addition.

0. Basis: $a \times (b_1+b_2) = (a \times b_1) + (a \times b_2)$ [Distribution of multiplication over addition. Granted[1]]

1. Induction step: Let $a \times (b_1+ b_2 \ldots +b_n) = (a \times b_1) + (a \times b_2) \ldots + (a \times b_n)$

<u>Show</u>: $a \times (b_1+ b_2 \ldots +b_{n+1}) = (a \times b_1) + (a \times b_2) \ldots + (a \times b_{n+1})$

1.1 $a \times (b_1+ b_2 \ldots +b_{n+1}) = a \times ((b_1+ b_2 \ldots + b_n)+b_{n+1})$ [Add. is associative]

1.2 $a \times ((b_1+ b_2 \ldots + b_n) +b_{n+1}) = (a \times (b_1+ b_2 \ldots + b_n)) + (a \times b_{n+1})$ [from (0)]

1.3 $(a \times (b_1+ b_2 \ldots + b_n)) = (a \times b_1) + (a \times b_2) \ldots + (a \times b_n)$ [from prev.ass.]

1.4 $((a \times b_1) + (a \times b_2) \ldots + (a \times b_n)) + (a \times b_{n+1})$ [from 1.2, Leibniz Law]

---

[1] [I prove Q(2), since Q (0) is meaningless and Q(1) is trivial]

1.5 $(a \times b_1) + (a \times b_2) \ldots + (a \times b_n) + (a \times b_{n+1})$ [add. is associative] ☺

3. Since (0) and (1) are proved, it follows by Mathematical Induction that $\forall n \in \mathbf{N}$ [$a \times (b_1 + \ldots b_n) = (a \times b_1)\ldots+ (a \times b_n)$] ☺

3. HORSES, HORSES, HORSES…

0. Basis: {Nero's horse}. Nero's horse has just one color.☺

1. Induction: Assume all members of X ($|X| = n$) are of the same color.

Show that all members of X U {y} (where $y \notin X$) are of the same color.

My horse, unlike Nero's, is blue. Take X = {Nero's horse}. Then not all members of {Nero's horse} U {my horse} are of the same colour! ☹