Homework 11

PtMW Chapter 8: Exercise 5

I will assume as premises the following:

(P1) **Distributivity**: $a \times (b_1 + b_2) = (a \times b_1) + (a \times b_2)$
(P2) **Associativity**: $b_1 + (b_2 + b_3) = (b_1 + b_2) + b_3$
(P2) **Adding equals to equals, give equals**: $\forall x \forall y \forall z\ ((x = y) \rightarrow (x + z = y + z))$

[1] Let $n = 0$.

Then $a = a$ is trivially true.

[2] Let $n = 1$.

Then, trivially, we have $a \times b_1 = a \times b_1$

[3] Let $n = 2$.

Then $a \times (b_1 + b_2) = (a \times b_1) + (a \times b_2)$ (P1)

[4] Assume that for some arbitrary k, we have:

$a\times(b_1+b_2+ \dots +b_k) = (a\times b_1)+(a\times b_2)+ \dots +(a\times b_k)$

[5] Then:

$[a\times(b_1+b_2+ \dots +b_k)]+(a + b_{k+1}) = (a\times b_1)+(a\times b_2)+ \dots +(a\times b_k)+(a + b_{k+1})$ (P3)

[6] But the left side can be rewritten as: (P1) reversed

$a\times ((b_1+b_2+ \dots +b_k) + b_{k+1})$

[7] Which is equal to:

$a\times (b_1+b_2+ \dots +b_k + b_{k+1})$ (P2)

[7] Therefore we have:

$a\times(b_1+b_2+ \dots +b_k+b_{k+1}) = (a\times b_1)+(a\times b_2)+ \dots +(a\times b_k)+(a\times b_{k+1})$ [5], [7]

[8] Since we have chosen an arbitrary k, we can conclude that whenever it is true that multiplication is distributive over k additive terms, then it is true that multiplication is distributive over $k + 1$ additive terms. [4], [8]

[9] And we have shown that multiplication is distributive over k terms when $k = 0$, $k = 1$ and $k = 2$. [1], [2], [3]

[10] Therefore, for all n, $a\times(b_1+b_2+ \dots +b_n) = (a\times b_1)+(a\times b_2)+ \dots +(a\times b_n)$.