Question 4

*Two models for A2-A6 which are not isomorphic to the models of L nor to each other. Lets call the system **L4***

Model 1

D = {Joni M., Judy C.}
Joni =1, Judy = 2

R = recorded-a-song-written-by

a. Joni recorded a song written by Judy
b. Judy recorded a song written by Joni

Not isomorphic to models of L since there are only two members, and without the same number of elements in the domain as models of L (which had 3) an isomorphism couldn't possibly hold.

Model 2 **This one is very interestingly wrong. I'll write a separate paragraph about it at the end. Volodja and I started discussing it over the phone just now, and we have partly same and partly different ideas about where the confusion may be coming from, so we'll both write comments. I think it might be very useful to put this on the website together with our commentary (anonymously, of course!).**

D = {Joni, Robert Zimmerman, Bob Dylan}
Joni = 1, Zimmerman = 2, Dylan = 3 and 2 = 3

This does not verify A1, since 2 and 3 are the same person.

R = recorded a song written by

a. Joni recorded a song written by Zimmerman
b. Zimmerman recorded a song written by Dylan
c. Dylan recorded a song written by Joni

This has the pattern R12, R23, R31 but since 2 = 3 I think this cannot be isomorphic to models verifying L. Also, it is not isomorphic to Model 1 above since the cardinality of the domain is different.

The lesson about A1: A1 has to be independent, since the language that results from deleting it, L', has models that are not isomorphic to each other, i.e. creates a system **L4** that is non-categorical.

Discussion of your Model 2 of Question 4.

Identity as expressed by the = sign is understood very strictly in logic and set theory. so if Zimmermann = Dylan, that must mean that those are two different names for one and the same person. And in that case your set has only 2 members, not 3.

Now I can proceed in one of two ways to show what's going wrong.

(i) Keeping all 3 names, I can say not only is it true that R12 & R23 &R31, but also R13 & R22 & R33 & R21. And then you can see that various axioms are violated, of which the most obvious is probably A2, the irreflexivity axiom. (Also A3 and A4.)

(ii) Or we might just as well forget your "3" and your "Dylan" since they are just extra names. And just thinking in terms of 1 and 2, it's a 2-element model (that's true no matter whether we hang on to the name '3' or not), but it's not isomorphic to Model 1, and it's not even a model of L, because we have R12 & R21 & R22. So again it violates A2 (and also A3 and A4).

VB. I am not American and don't know are B. Dilan and R. Zimmerman different names for the same person (Dilon is a pseudonym Zimmerman took) ? I suppose they are.

I think that may be it is partly our fault that in our lectures about Predicate Logic we didn't stress enough the difference between constants and elements of a carrier in models.

In logical language we have a signature, in every concrete first order language, it is a set of constants and a set of predicate symbols.

In every model in this signature we have a carrier (or domain D) and the interpretation function I which maps constants into D and predicate symbols to relations on D. For constants and "normal" predicate symbol we are free in choice of interpretation function, to define model we choose D and I. But in the case of equality symbol "=" we have no freedom in choice of relation I(=), in all models we map it to identity relation.

In this example 1, 2 and 3 are constants, R and = are binary predicate symbols. And to build a model you should choose elements of domain. In this example you should decide – what are they, names or people. Both cases are possible, just in the first one you need to define mapping I: {1,2,3} $\rightarrow$ D1 from constants to names and the relation I(R) on domain of names. And in the second case to do the same for domain D2 of people, i.e. to define . I: {1,2,3} $\rightarrow$ D2 and the relation I® on D2.

So, if you deal with the domain of names you have
D1 = {Joni, Zimmerman, Dylan} and, for example, I(1) = Joni, I(2) = Zimmerman and I(3) = Dilan. And you have to define the relation I(R) on D1.

If you deal with people, then Zimmerman and Dilan are two names for the same person, so you have:
D2 = {Joni, Zimmerman/Dilan} and you just use these name to denote people. In this case I(1) = Joni, I(2) = I(3) = Zimmerman/Dilan. And you should define the relation I(R) on D2.

And in language we deal only with constants, predicate symbols but not with elements of models. So R12, R2,3, etc, are atomic formulas which are true or false in a concrete model (i.e. on concrete domain D with a concrete interpretation function I which you should define.