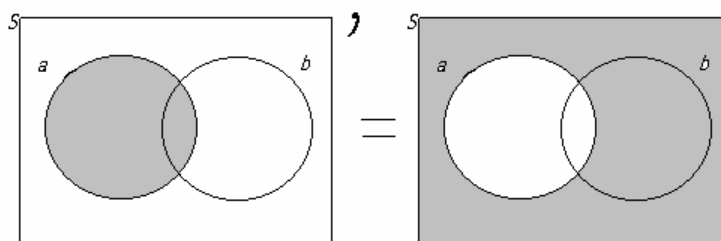
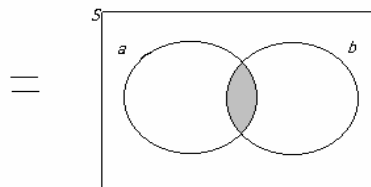
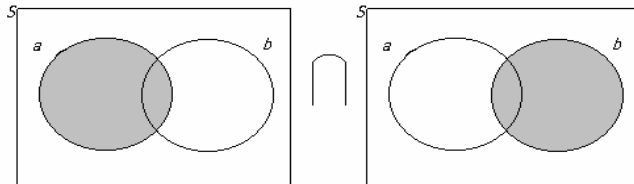
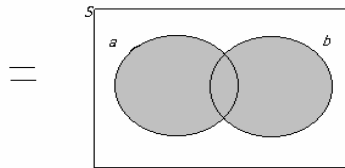
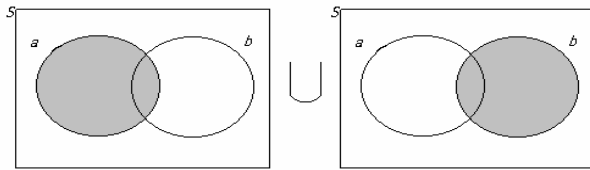


III. The algebra **Venn 16**.

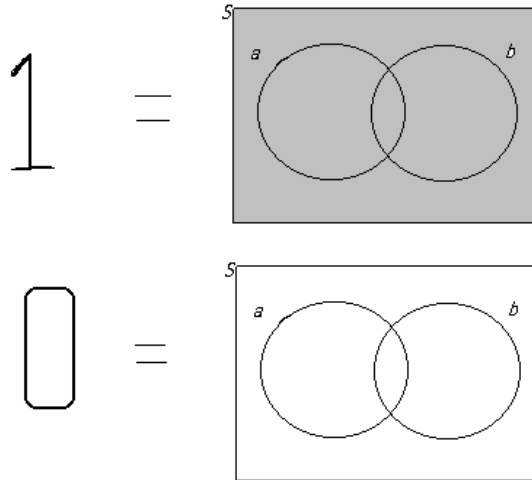
0. The signature of **Venn 16** is the set $\{\cap, \cup, ', 1, 0\}$.

1. The carrier **Venn16** is the set consisting of two-circle Venn diagrams (with distinct shadable areas). Its cardinality is 16, which corresponds to the cardinality of a power set for a four-member set.¹

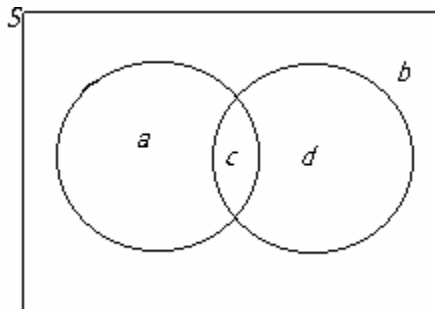
The operations of **Venn16** work as illustrated below:



¹ One way to make sense of this is as follows: There are four distinct shadable areas: one outside both circles, two inside each circle excluding the overlapping area, and the overlapping area.



2. Algebras with carrier sets consisting of “one-circle” Venn diagrams² are subalgebras of **Venn16**. Taking the algebra illustrated above as an example, an algebra whose carrier is diagrams where only the circle a and its outsides may be distinctly shaded is a subalgebra of **Venn16**, since (i) its carrier is a subset of the carrier **Venn16** (where four areas can be distinctly shaded), (ii) for every operator in the signature (given in III.0), this subalgebra is closed with respect to the operation as defined in **Venn16**, and (iii) for every operator in the signature, the operation (as defined in **Venn16**) and restricted to the carrier of this subalgebra is equivalent to the operation as defined in the subalgebra.
3. (Renaming each of the distinct shadable areas as follows:)



As long as the areas a, c, d are of the same hue (i.e. if the area a, c, d were treated as one larger area a), there will be a homomorphism between **Venn16** and its subalgebra **Venn4**.

4. It is false, even if we consider the word algebra $\mathbf{W}_{\Omega\text{BA}}(X)$ (where X corresponds to a set of variables) instead. In a word algebra, the expression $a \vee a \vee a$ is distinct from the expression a . In a **Venn** algebra, the diagram for $A \cup A \cup A$ and the one for A are not distinct.

² A more precise description is: Venn diagrams where only one circle is a distinct shadable area. Looking ahead, these are essentially **Venn4** algebras.

5. See answers to 2 and 3 above. The kernel ker of that homomorphism crucially treats the aggregate of the three distinct shadable areas in **Venn16** as a single area in **Venn4**. More concretely, the mapping looks as follows:

$$\begin{array}{llll}
 \emptyset & \rightarrow & 0 & \{b\} \rightarrow \{b\} \\
 \{a\},\{c\},\{d\} & & & \{a,b\},\{b,c\},\{b,d\} \\
 \{a,c\},\{c,d\},\{a,d\} & \rightarrow & \{a\} & \{a,b,c\},\{b,c,d\},\{a,b,d\} \rightarrow 1 \\
 \{a,c,d\} & & & \{a,b,c,d\}
 \end{array}$$

6. The quotient algebra **Venn16**/ ker : its carrier consists of the equivalence classes given in 5.

The operations of this algebra are defined as in **Venn16**, except for the fact that they ignore the boundaries between a , c , and d .

An isomorphic algebra in the **Venn** family of algebras is **Venn4**. Another would be the quotient algebra **Venn32**/ Q , where Q is an equivalence relation on the carrier **Venn32**, which, similarly to ker treats the aggregate of three distinct shadable areas (the ones in the overlapping two sets) as a single area in **Venn8**.

The isomorphic power set Boolean algebra is one whose carrier is the power set of a two-member set.