I. a. Construct the quotient algebra **Nat/≡**$_{\text{Mod}n}$ corresponding to this congruence.

The carrier of this quotient algebra is the set of the equivalence classes $\{x_0 | x_0 = ns\}$, $\{x_1 | x_1 = ns+1\}$, …. , $\{x_{n-1} | x_{n-1} = ns+(n-1)\}$ ($s \in Nat$). The operations on these classes are defined in a natural way:

zero = 0

one = 1

(in the tables below, take the result of the operations to be the part excluding the *nX*; and 0 if there is none.)

+	*ns*	*ns*+1	*ns*+2	…	*ns*+*n*-1
nr	*n*(*r*+*s*)	*n*(*r*+*s*)+**1**	*n*(*r*+*s*)+**2**	…	*n*(*r*+*s*)+(***n*-1**)
nr+1	*n*(*r*+*s*)+**1**	*n*(*r*+*s*)+**2**	*n*(*r*+*s*)+**3**	…	*n*(*r*+*s*+1)
nr+2	*n*(*r*+*s*)+**2**	*n*(*r*+*s*)+**3**	*n*(*r*+*s*)+**4**	…	*n*(*r*+*s*+1)+**1**
…	…	…	…	…	
nr+(*n*-1)	*n*(*r*+*s*)+(***n*-1**)	*n*(*r*+*s*+1)	*n*(*r*+*s*+1)+**1**	…	*n*(*r*+*s*+1)+(***n*-2**)

×	*ns*	*ns*+1	*ns*+2	…	*ns*+(*n*-1)
nr	*n*(*nrs*)	*n*(*nrs*+*r*)	*n*(*nrs*+2*r*)	…	*n*(*nrs*+*n*-1*r*)
nr+1	*n*(*nrs*+*s*)	*n*(*nrs*+*r*+*s*)+**1**	*n*(*nrs*+2*r*+*s*)+**2**	…	*n*(*nrs*+*n*-1*r*+*s*)+***n*-1**
nr+2	*n*(*nrs*+2*s*)	*n*(*nrs*+*r*+2*s*)+**2**	*n*(*nrs*+2*r*+2*s*+1)	…	*n*(*nrs*+*n*-1*r*+2*s*+1)+**2**
…	…	…	…	…	
nr+(*n*-1)	*n*(*nrs*+(*n*-1)*s*)	*n*(*nrs*+*r*+(*n*-1)*s*) +(***n*-1**)	*n*(*nrs*+2*r*+(*n*-1)*s*+1) +**2**	…	*n*(*nrs*+(*n*-1)(*r*+*s*) +*n*-2)+**1**

To be more concrete: if, for instance, $n = 4$, then the carrier of **Nat/≡**$_{\text{Mod4}}$ is the set of the equivalence classes $\{x_1 | x_1 = 4s\}$, $\{x_2 | x_2 = 4s+1\}$, $\{x_3 | x_3 = 4s+2\}$, $\{x_4 | x_4 = 4s+3\}$ ($s \in Nat$).

b. Show that the quotient algebra **Nat/≡**$_{\text{Mod4}}$ is isomorphic to the algebra **Mod4**.

(i) The relation between the members of the carrier Nat/≡$_{\text{Mod4}}$ and Mod4 is a one-to-one onto mapping. (Each equivalence class mapping to a member of Mod4).

(ii) The operations of **Nat/≡**$_{\text{Mod4}}$ are defined as illustrated in (a). The operation tables given above and those for Mod4 are identical, once we leave out the *nX* parts out of consideration.