I. a. Construct the quotient algebra \( \mathbb{N}/\equiv\equiv\equiv\equiv \mathbb{M}_{\text{Mod}} \) corresponding to this congruence.

The carrier of this quotient algebra is the set of the equivalence classes \( \{x_0 \mid x_0 = ns\} \), \( \{x_1 \mid x_1 = ns+1\} \), \ldots, \( \{x_{n-1} \mid x_{n-1} = ns+(n-1)\} \) \((s \in \mathbb{N})\). The operations on these classes are defined in a natural way:

- **zero** = 0
- **one** = 1

(in the tables below, take the result of the operations to be the part excluding the \( nX \); and 0 if there is none.)

\[
\begin{array}{|c|c|c|c|c|}
\hline
+ & ns & ns+1 & ns+2 & \ldots & ns+(n-1) \\
\hline
nr & n(r+s) & n(r+s)+1 & n(r+s)+2 & \ldots & n(r+s)+(n-1) \\
\hline
nr+1 & n(r+s)+1 & n(r+s)+2 & n(r+s)+3 & \ldots & n(r+s+1) \\
\hline
nr+2 & n(r+s)+2 & n(r+s)+3 & n(r+s)+4 & \ldots & n(r+s+1)+1 \\
\hline
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
\hline
nr+(n-1) & n(r+s)+(n-1) & n(r+s+1) & n(r+s+1)+1 & \ldots & n(r+s+1)+(n-2) \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|c|c|}
\hline
\times & ns & ns+1 & ns+2 & \ldots & ns+(n-1) \\
\hline
nr & n(nrs) & n(nrs+r) & n(nrs+2r) & \ldots & n(nrs+n-1r) \\
\hline
nr+1 & n(nrs+s) & n(nrs+r+s)+1 & n(nrs+2r+s)+2 & \ldots & n(nrs+n-1r+s)+n-1 \\
\hline
nr+2 & n(nrs+2s) & n(nrs+r+2s)+2 & n(nrs+2r+2s+1) & \ldots & n(nrs+n-1r+2s+1)+2 \\
\hline
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
\hline
nr+(n-1) & n(nrs+(n-1)s) & n(nrs+r+(n-1)s)+n-1 \\
\hline
\end{array}
\]

To be more concrete: if, for instance, \( n = 4 \), then the carrier of \( \mathbb{N}/\equiv\equiv\equiv\equiv \mathbb{M}_{\text{Mod}} \) is the set of the equivalence classes \( \{x_1 \mid x_1 = 4s\} \), \( \{x_2 \mid x_2 = 4s+1\} \), \( \{x_3 \mid x_3 = 4s+2\} \), \( \{x_4 \mid x_4 = 4s+3\} \) \((s \in \mathbb{N})\).

b. Show that the quotient algebra \( \mathbb{N}/\equiv\equiv\equiv\equiv \mathbb{M}_{\text{Mod}} \) is isomorphic to the algebra \( \mathbb{M}_{\text{Mod}} \).

(i) The relation between the members of the carrier \( \mathbb{N}/\equiv\equiv\equiv\equiv \mathbb{M}_{\text{Mod}} \) and \( \mathbb{M}_{\text{Mod}} \) is a one-to-one onto mapping. (Each equivalence class mapping to a member of \( \mathbb{M}_{\text{Mod}} \)).

(ii) The operations of \( \mathbb{N}/\equiv\equiv\equiv\equiv \mathbb{M}_{\text{Mod}} \) are defined as illustrated in (a). The operation tables given above and those for \( \mathbb{M}_{\text{Mod}} \) are identical, once we leave out the \( nX \) parts out of consideration.