

HW5, Ex.1, ANS4

Example 1

Show that the Poset in Figure 11-1 is a lattice.

$$\begin{array}{llll} \sup\{0,0\} = 0 & \sup\{0,a\} = a & \sup\{0,b\} = b & \sup\{0,1\} = 1 \\ \inf\{0,0\} = 0 & \inf\{0,a\} = 0 & \inf\{0,b\} = 0 & \inf\{0,1\} = 0 \end{array}$$

$$\begin{array}{lll} \sup\{a,a\} = a & \sup\{a,b\} = 1 & \sup\{a,1\} = 1 \\ \inf\{a,a\} = a & \inf\{a,b\} = 0 & \inf\{a,1\} = a \end{array}$$

$$\begin{array}{ll} \sup\{b,b\} = b & \sup\{b,1\} = 1 \\ \inf\{b,b\} = b & \inf\{b,1\} = b \end{array}$$

$$\begin{array}{l} \sup\{1,1\} = 1 \\ \inf\{1,1\} = 1 \end{array}$$

Therefore, for all $x, y \in A$ $\sup\{x,y\}$ and $\inf\{x,y\}$ exists. Therefore A is a lattice.