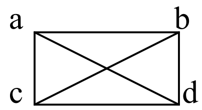
5) a. P = {a, b, c, d}
L = {p, q, r, s, t, u}

p = {a,b} q = {a,c} r = {a,d} s = {b,c} t = {b,d} u = {c,d)



b. By (2), we have at least 2 distinct points. Assume we have only these two, then the only possible lines are the singleton sets containing these points (the line containing both points is ruled out by (4)). But not having a line that both points are part of violates (3). So we have to have at least 3 points in P. By (3), any subset of Power(P) with the cardinality of 2 will be a line. This means that each element will be in at least two lines.

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VB comment: b above is good but I would prefer a slightly more straightforward modification, something like this:

b. By (2), in any model we have at least two distinct points, say **a** and **b**. Then by (3) we have a line, say **p**, such that {**a**, **b**} $\subseteq$ **p** . But then by (4) we should have a point not in **p**, let's name it **c**. Then, again by (3), for any point in **p** we have a line which contains this point and **c** and these lines are different for different points in **p** (also by (3)). The same holds for any other point outside of **p** in the case we have one (really, we should have at any rate one distinct from **c** but it is a different story). This means that each point will be in at least two lines. And we immediately have d.

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c. The crucial axiom that rules out the empty set as a member of L is Axiom 5. Any point in P will not be in l = {}. But if all the axioms are satisfied, then every point will be in more than one line, as shown in (b). Therefore, there will be more than one disjoint line containing a point that is not in {}, which violates axiom 5.

d. see b. If there are only two members, we cannot at the same time satisfy axioms (4) and (3). Either the line containing the two members violates (4) (i.e. there will be no point outside of that line), or we don't have a line containing the two members, and then (3) is violated, since there is not one line for every pair of elements in P.

e. No. Since (3) requires every pair of elements in P to be common members of one line, and since P has at least 3 members (as shown above), a model that only had singleton set lines would violate axiom 3.

6.

a.

ID1. Every node is immediately dominated by at most one node

n, m, o are node variables:

$\forall$n,m [IDOM(n,m) $\rightarrow$ $\forall$o [IDOM(o,m) $\rightarrow$ o = n]]

or

$\forall$n $\exists$m $\exists$o [[IDOM(m,n) & IDOM(o,n)] $\rightarrow$ o = n]

BHP (comment on second formula): NO. Not equivalent. Problems with the relative scope of $\exists$'s and $\rightarrow$. And if you make the scope of $\exists$ narrower, as you should, then the final o, m won't be bound. So you'd better stick with the first!
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ID2 There is exactly one node (the root) that is not dominated by any node.
$\forall n\ \forall m \sim\exists o \sim\exists p\ [[IDOM(o,n)\ \&\ IDOM(p,m)] \rightarrow n = m]$

BHP: No, I don't think this can work. Show why and try again.

ID 2 Second attempt
As in the second version for ID1 above, the problem is with the scope of $\exists$. As it stands, the formula above for ID2 says something like for any n and for any m, there is no o and there is no p such that if IDOM(o,n) and IDOM (p,m), then n = m.
What we want to say is that there is an n such that there is no m that dominates it, and for all o for which there is no p that dominates it, o is identical to n.

$\exists n\ [\sim\exists m\ IDOM(m,n)\ \&\ \forall o[\sim\exists p\ IDOM(p,o) \rightarrow o = n]]$

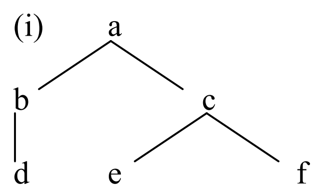
==

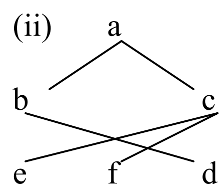
VB: What about the interaction between IDOM and $\Rightarrow$?
Interaction condition 1: For any pair of nodes in Node either they stand in the $\Rightarrow$ relation or they stand in the < relation, and not both.
$\forall x\ \forall y\ [\sim[\Rightarrow(x,y)\ \&\ <(x,y)]\ \&\ [\Rightarrow(x,y) \vee <(x,y)]]$

BHP : Almost right ! Just replace each $\Rightarrow(x,y)$ by $(\Rightarrow(x,y) \vee \Rightarrow(y,x))$ and each $<(x,y)$ by $(<(x,y) \vee <(y,x))$

The non-tangling condition : In any wf tree, for any nodes N and N', if N precedes N', then all nodes dominated by N precede all nodes dominated by N'.
$\forall x\ \forall y\ [<(x,y) \rightarrow [\forall q\ \forall r\ [[\Rightarrow(x,q)\ \&\ \Rightarrow(y,r)] \rightarrow <(q,r)]]]$

b. The definitions of linearally ordered trees in handout 3, p3, and in Blackburn et.al. (1993) are not equivalent, as far as I can see. In the Blackburn definition, the precedence relation is only defined for immediately dominated nodes. This allows for 'crossing branches'. Here's an example:

(i)


(ii)


The Blackburn definition will specifiy that b preceds c, and that e precedes f. It does not say anything about the ordering of the terminal nodes d, e, and f, and hence is compatible with either ordering above.

In the page 3 – definition, this is ruled out by the ‘non-tangling condition’, which states that if b precedes c, then all nodes dominated by b precede all nodes dominated by c.
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VB: That’s true that in Blackburn et al (1993) they give only the definition of immediate precedence relation on immediate constituents. But they don’t consider the order relation < on the nodes. So we cannot speak about ‘crossing branches’. Of course, we can consider the order < corresponding to their immediate precedence relation and in this case we need the conditions you considered above (interaction and ‘non-tangling’ conditions). But I don’t know whether they need this order for their goals.