Question 6

(6c) I got to two definitions of Trees from Chris's handout…

Finite Transitive Trees

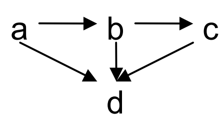
Dominates = $<$ = precedes

the definition of dominates is:

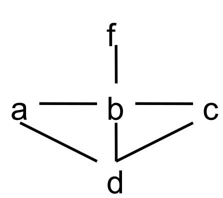
$\forall t \in T$, the set S { $s \in T \mid s < t$} is the set of predecessors of t and S is linearly ordered and finite.

To me this means that all the predecessors of each t is connected, and the set S is order before t, but does it follow that t is immediately dominated by only one node?. I.e. doesn't the tree below satisfy this part of the definition.

Where S = {a,b,c} and t = d (I use an arrow to identify domination)
Assume
$a < b < c$
$a < d$
$b < d$
$c < d$



In this case S is finite and linearly order, and S is the set of s that precede d. But this cannot be a well define syntactic tree. Part of the problem is that it has more than one root, but if we added a root node on top, that should fix that problem.



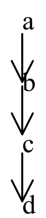
Still, I do not see how this definition rules out multiple domination. Am I missing something?

BHP Comment on ***'s discussion of Blackburn et al's definition of Finite Transitive Trees.

It took me some time to figure out whether the problem you found with Blackburn's definition was real. This is a good example of the power of pictures to either help or mislead.

The first thing to reflect on is whether the given relation < is to be thought of as "dominates" or "immediately dominates". We can figure out that it must be "dominates", because of the fact that the set of "predecessors" (i.e. "dominators") of any given node must be linearly ordered by <. Although we still need to try to figure out whether that linear order is strong (irreflexive, asymmetric, and transitive) or weak (reflexive, antisymmetric, and transitive), we know for sure that it's transitive. And we also know that immediate domination is intransitive. So this must be "dominates". [Aside: I think that if there are no typos, it must be a weak linear order, since the first condition says that the root dominates every node in the tree, and that would include itself, so dominates can't be irreflexive, and since a linear order is either irreflexive or reflexive (no half-way cases), it must be reflexive.]

OK: so this relation < is transitive. Now look at your first diagram again. It looks like it can't be a tree because of all that "multiple domination" – nodes a, b, and c all dominate node d. But that's "dominate", not "immediately dominate". What if I draw that same diagram like this:



There I've indicated only what I would take to be the "immediate dominance" lines, removing some of the lines that are in your diagram but which I take to be there only because of transitivity. Remember that we've mentioned in passing that when we draw diagrams of orderings, we usually leave out the lines that are just there because of transitivity, since we know they're always there and they just make the diagram harder to look at.

So my diagram is basically just a different visual layout of your diagram, with some transitivity lines suppressed. But mine shows that as far as immediate dominance goes, each node does indeed have only one mother.

Same goes for your further tree with the additional root node f; but notice that you were yourself slightly inconsistent when you added f, because you added only one line connecting f to b, not all the further lines that transitivity would require. By the way, I just realized that if you did all the arrows, and checked for the condition about all the nodes dominating d forming a linearly ordered set, you'd see that f has to immediately dominate a, not b: otherwise there's no way to get a,b,c,f to be linearly ordered. If you want f to be the (new) root, it has to dominate all the others, and since among a,b,c, it's a that dominates all the others, f should immediately dominate a. See if you can verify what I've just said by deriving a contradiction from the assumption that the arrows among a,b,c,d are as in your upper diagram, together with the addition of a new node f that immediately dominates b.

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An added note on that same definition by Blackburn et al of finite transitive tree: I am not convinced that the condition that the set of predecessors of each node is finite and linearly ordered by 'dominates' is enough to guarantee that the whole tree is finite. I believe that even in an infinite tree, the set of predecessors (dominators) of each node will be finite. It's the set of nodes dominated by some node that

would be infinite in the case of an infinite tree, not the set of nodes that dominate some node. If you want to consider only finite trees, surely the most straightforward way is to say that the set of nodes is finite, as is done in the Blackburn and Meyer-Viol definition and in the McCawley definition.
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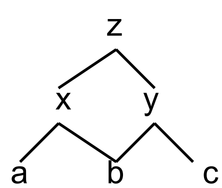
Finite binary branching trees

This definition takes immediate domination as basic. They use another symbol; I'll use <.
The relation is assymetric, since w < w'' iff w'' is a daughter. (a,b)
The relation is irreflexive since if w < w'' then w cannot equal w'' (e)
The relation is not transitive because if a < b and b < c, it cannot be that a < c since c is not a daughter of a.

I do not see that these definitions rule out multiple domination. Consider the tree below:



I'll go through each part of the definition:

i) there is a unique root (z)
a) for all w, w' $\in$ W, w $<_1$ w' iff w' is the first daughter of w; TRUE
b) and w $<_2$ w' iff w' is the second daughter of w. TRUE
c) … any node in an order binary tree has at most one first daughter and one second daughter. TRUE (both x and y each only have two daughters)
d) if w $<_2$ w', then there is a unique w'' such that w $<_1$ w'' TRUE—there is always a sister two each node (except for the root)
e) w'' cannot equal w TRUE the tree is irreflexive

So if the tree above really does satisfy (a)-(e) then this definition allows multiple domination (and perhaps other oddities)

BHP note on ***'s comments on Blackburn and Meyer-Viol 1997 "Finite binary branching trees" definition on the handout.

I agree. As I look over that "definition", it seems to presuppose a lot already; it really doesn't seem to constrain what counts as a tree in the ways we would expect. It seems to already presuppose a definition of root, a definition of terminal nodes, and maybe even a definition of 'ordered binary tree'. The only way I can make sense of it is that this is adding a definition of two new relations "first daughter of" and "second daughter of" to an existing definition of finite binary branching tree. No, not even that: the relations $>_1$ and $>_2$ are <u>defined</u> in terms of "first daughter of" and "second daughter of", so we must already have those notions from somewhere.