Ling 726, September 26, 2001: Homework Notes

WHY A^0= {∅}?

Note from BHP: The answer below looks good to me. There was another good-looking answer which unfortunately reached the “wrong” conclusion, namely that A^0 = ∅. And there was one good partial answer that argued in terms of cardinality: For all n, $|A^n| = |A|^n$. And since we know that m^0=1 for all finite m, we can reason that $|A^0| = |A|^0 = 1$, and therefore that A^0 should be a singleton set. And from there it seems at least reasonable that it’s identified with{∅}, a singleton set with our most “basic” set as its member.

2. WHY A^0= {∅}?
[Note: the following is inspired by Prof. Hardegree’s notes on Set Theory].

We know n-tuples are ordered sequences. Our definition of <a,b> = {{a},{a,b}} allows us to prove that <a,b> = <b,a> iff a = b.
We can also define the natural numbers in set-theoretic terms and then define n-tuples in terms of functions from the natural numbers.

<u>Definition of N in terms of sets</u>
$0 =_{df}$ ∅
$1 =_{df}$ { ∅ }
$2 =_{df}$ {∅, {∅}}, etc..

<u>Definition of n-tuples in terms of functions from **N**</u>
For any n ∈ **N**, an n-tuple is any function whose domain is n.

<u>Examples:</u>
<Mickey Mouse, Donald Duck> $=_{def}$ { <∅ , Mickey Mouse>, < {∅}, Donald Duck>}
<Mickey Mouse> $=_{def}$ { <{∅}, Mickey…>}

<u>n-fold Cartesian product</u>

For any n ∈ **N**, $A^n =_{df}$ the set of all *n*-tuples of elements of A.

A^2 is the set of all 2-tuples of elements of A. A 2-tuple is defined as any function from 2 (i.e. {∅, {∅}}) to A.

A^1 is the set of all 1-tuples of elements of A. We have defined a 1-tuple as a function from 1 to A, so A^1 is the set of all functions from 1 to A.

A^0 is the set of all 0-tuples of elements of A. We have defined a 0-tuple as a function from 0 to A, so A^0 is the set of all functions from 0 to A. We have defined 0 in terms of ∅. So A^0 is the set of all functions from ∅ to A. f is a function from ∅ to A iff f = $\{\langle x,y\rangle \mid x \in \emptyset \;\&\; y \in A \;\&\; \forall x,y,z \in A\,[\langle x,y\rangle \;\&\; \langle x,z\rangle \rightarrow y = z]\}$ Since ∅ has no members, then f = ∅.

Hence, for any A, A^0 = {∅}