Notes on Homework 2 (Ch 2, PtMW)

1. Most homework was excellent, so we have just some notes.

2. Sometimes some of you give only answers without any comments. We would prefer you to give **short** proofs or at any rate some comments.

3. In 1 (b) (iii) you were asked:

   Given A = {b,c}, <c,c> $\subseteq$ A $\times$ A ?

   The answer “no” is correct, but some said “no, <c,c> can’t be a subset because <c,c> is not a set.” But we defined ordered pairs in terms of sets, so if one goes back to that definition, <c,c> *is* a set, namely {{c},{c,c}} = {{c}}.

   So it is a set. But it’s not a subset of A $\times$ A. It would take more work to really prove that (which wasn’t demanded for the problem). You could try constructing an argument about the number of nested braces all subsets of A $\times$ A have in their standard representations (Does {{c}} have too few nested braces to be a subset of A $\times$ A?). Or you could just list all subsets of A $\times$ A and show that none of them are equal to {{c}}. Or you could try other general arguments.

   Note that the fact that <c,c> is a *member* of A $\times$ A is not an argument that it’s not also a *subset* of A $\times$ A. (Exercise 4 of Chapter 1 illustrates that point.)

4. Exercise 1(c). It is important to understand that if we have a relation P from M to N, the relation P^{-1} is by definition from N to M. So in this exercise the relation R^{-1} is from ($A \cup B$) to A and its complement $(R^{-1})'$ is also from ($A \cup B$) to A.

   The book gives **wrong** answer to 1(c)(iii). The right answer is: $(R')^{-1} = (R^{-1})'$.

5. Exercise 2 (How many relations, how many onto, etc): See Paula's and Andries's homeworks. They both give very explicit reasoning, sometimes using different strategies, but both correct.

6. Exercise 4. F and G are functions (from K to L and from L to M respectively) but their inverses are not, they are relations. But we consider not only compositions of functions but also compositions of relations.

   F^{-1}, G^{-1}, $(G \circ F)^{-1}$, $F^{-1} \circ G^{-1}$ are relations. F^{-1} is a relation from L to K, G^{-1} is a relation from M to L, $(G \circ F)^{-1}$, $F^{-1} \circ G^{-1}$ are relations from M to K. And we have $(G \circ F)^{-1} = F^{-1} \circ G^{-1}$. We can verify it by building these concrete relations following our definitions. Most of you did. It is a good way to understand the structure of these constructions through concrete example.

   But it is also possible to give a general proof of such an equality for any relations of the kind F from K to L and G from L to M. And some of you did it. It is a more sophisticated task and, of course, also very useful.

   The proof:

   A pair <*y*,*x*> belongs to $(G \circ F)^{-1}$ iff the inverse pair <*x*,*y*> belongs to $G \circ F$.

   <*x*,*y*> belongs to $G \circ F$ iff there exists some $z \in L$ such that <*x*,*z*> belongs to F and <*z*,*y*> belongs to G.

   If there exists some $z \in L$ such that <*x*,*z*> belongs to F and <*z*,*y*> belongs to G, then there exists some $z \in L$ such that <*z*,*x*> belongs to F^{-1} and <*y*,*z*> belongs to G^{-1} and *vice versa*.

Pair $<y,x>$ belongs to $F^{-1} \circ G^{-1}$ iff there exists some $z \in L$ such that $<z,x>$ belongs to F^{-1} and $<y,z>$ belongs to G^{-1}.

So $<y,x>$ belongs to $(G \circ F)^{-1}$ iff it belongs to $F^{-1} \circ G^{-1}$. QED.