

## Index to Homework 10

### Questions of Homework 10:

I. All of the first set of problems are based on the elementary formal system  $L$  defined by Axioms A1-A6 in Section 8.5.4. of PtMW, though some of the questions, like 2 and 3, are fully general. [Most of these questions are from Partee (1978).]

1. Find three models for  $L$  other than the one given in the text.
2. If the deletion of a certain axiom from a formally complete system changes the system into one which is not formally complete, then that axiom is independent. Why?
3. If the deletion of a certain axiom changes a formal system from categorical to non-categorical, must that axiom be independent? Why?
4. Find two models for axioms A2-A6 which are not isomorphic to the models of  $L$  nor two each other. What does this tell you about axiom A1?
5. If A2 is replaced by A2':  $\exists x Rxx$ , is the resulting system consistent? If so, find a model for it. If not, deduce a contradiction from the new set of axioms.
6. If A2 is replaced by A2' as above and A3 and A4 are deleted, is the resulting system consistent? Justify as above. Is the resulting system categorical? If not, find two non-isomorphic models for it.
7. What happens if we replace A5 by A5':  $(\forall y)(\exists x) Rxy$ ? Is the resulting axiom system consistent or inconsistent? Justify as above.
- \*8. [\* means maybe harder] Let axioms A2 – A6 be replaced by the single axiom A2'':  $\exists x \exists y \exists z ((Rxy \ \& \ Rxz) \ \& \ y \neq z)$ . Is this system consistent? Is it categorical? Justify your answers.
- \*9. Show that axiom A3 is not independent in the system  $L$ .

II. Return to the question about the axiom system  $W$  with “points” and “lines” from PtMW Ch8, p. 235, question 13. [This was Homework 7, Question 5.] If you already did it, try the questions below. If you didn't do it before, you might try it now together with a subset of the questions below.

10. Is the axiom system  $W$  consistent? \*Is it categorical? If you already did that homework problem, you have the answer to the first question and most of the arguments that you need for the second question.
  11. What do your answers to the questions 13a-e tell you about the independence of various of the axioms of  $W$ ?
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For this homework, it seems most straightforward to put complete homeworks on the web, rather than break them up by question. Annotations are added as needed.

**HW10 Solution1:** All of questions 1-9, all excellent. Question 8 was a trap (and the instructors both fell into it, even though one of them had invented this question many

years ago), but this seminar participant discovered it (but was uncertain of his/her answer, see student's query and our response.)

**HW10 Solution2:** All of questions 1-9, excellent, only minor errors aside from falling into the trap of Question 8. Comments embedded.

**HW10 Solution 3.** All of questions 1-11, all correct except for the trap in Question 8. Question 10 is answered only "suggestively", not definitively, but we think it's correct.