An interesting erroneous answer to the $A^0$ question.

**Note from BHP:** When I read this initially, I found it convincing. Only when later homeworks started coming in did I realize that the conclusion being argued for is not the one we wanted! We didn’t want an argument that $A^0 = \emptyset$, but that $A^0 = \{\emptyset\}$.

So here’s a new exercise: can you find any error in this proof? Or should we have decided that $A^0 = \emptyset$ (though that answer would mess up the nice cardinality generalization that $|A^n| = |A|^n$)?

1. $A^0 = \emptyset$. Why? Can you find any way of thinking about this to make an non-arbitrary decision?

For any $n \in \mathbb{N}$, $A^n$ is the set of all ordered $n$-tuples that can be formed by combining elements of $A$ (for instance, $A^2 = A \times A$, that is, the set of ordered pairs $<x,y>$ such that $x \in A$ and $y \in A$)

Therefore, $A^0$ is the set of all ordered 0-tuples that can be formed by combining elements of $A$. But what is a 0-tuple? We know that a pair consists of two elements, a triple consists of three elements, and so on for any $n \in \mathbb{N}$. Hence, a 0-tuple will consist of zero elements.

For any set $A$, no possible combination of the elements of $A$ will give us a 0-tuple. For instance, let $A= \{a,b\}$. Any possible combination of a with an element of $A$ will, at the very least, contain $a$, and hence it will not be a 0-tuple. And the same goes for $b$.

Therefore, for any set $A$, $A^0$ will not have any elements. And the set that does not have any elements is the empty set. Hence, $A^0 = \emptyset$