Homework 8.1
Linguistics 726

**one**

Signature $\Omega_{\text{Numb}} = \{\text{zero, one, } +, \times\}$. Homomorphism $h$ from $\mathbb{N}$ to Parity.

$$h(x) = \begin{cases} \text{even} & \text{if } x \text{ MOD } 2 = 0, \\ \text{odd} & \text{otherwise} \end{cases}$$

a) $\ker h = \{<x, y> \mid h(x) = h(y)\}$

that is the set $\{<0, \text{even}>, <1, \text{odd}>, <2, \text{even}>, \ldots\}$

b) $Q(x) = \ker h(x)$

Algebra $\mathbb{N}/Q$

- zero = even
- one = odd

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c) $h'(\llbracket x \rrbracket) = h(x)$

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\begin{tikzcd}
\mathbb{N} \arrow{r}{\text{nat}(Q)} & \mathbb{N}/Q \arrow{r}{h} \arrow{r}{h'} & \text{Parity}
\end{tikzcd}
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two

What kind of relation is “is older than”?

“is older than” is a strict partial order. Each two persons $a$ and $b$ are either comparable ($(a > b) \lor (b > a)) \land \neg((a > b) \land (b > a))$ or incomparable $(\neg((a > b) \lor (b > a)))$ with respect to that order.