Homework 7

Throughout this homework, “[“ = “(“ and “[]” = “)”. I use square brackets alongside parentheses for ease of interpretation. I also assume that if v and w are terms, “v = w” is a formula with the value 1 if they have the same value in the interpretation, on the assignment of values to variables, and 0 otherwise.

(2) Assuming that the relevant domain is of human beings, I propose the following system of axioms to define grandparent and grandmother:

(A1): ∀x∃y∃z [~(y = z) & parent(y,z) & parent(x,z) & ∀w [~(w = y) & ~(w = z)] -- > ~parent(w,x)]

“Everyone has exactly two parents, who are distinct from each other”.

I intend “parent of x” to mean “one of the two people who biologically created x” - i.e. adopting does not make one a parent, nor does marrying a parent of x’s make one x’s parent. Issuing sperm, then undergoing a sex change operation and impregnating oneself with one’s own sperm would intuitively make one a parent, but my definition would not allow it, assuming the pre-operation and post-operation person are the same entity. To resolve this contradiction, I assume that in such a situation, the pre-operation individual and the post-operation individual are not the same person. Another difficulty arises from the question of origin. It must be empirically false that every individual has two parents who are themselves individuals in the domain, assuming that the past is finite, because there must have been a first pair of ancestors who did not have parents but were parents themselves. This could be solved if we define a couple of individuals as parentless parents and stipulate in (A1) that they are exceptions to the rule. For instance, let’s assume that these two individuals are Adam and Steve (“a” and “s” in our model). We can then make all of (A1) except “∀x” the consequent of a new axiom which explicitly excepts it from applying to Adam and Steve, as follows:

(A1’) ∀x [~(x = a) & ~(x = s)] -- > ∃y∃z [~(y = z) & parent(y,z) & parent(x,z) & ∀w [~(w = y) & ~(w = z)] -- > ~parent(w,x)]]

“Everyone who is not Adam or Steve has exactly two parents, who are distinct from each other”.

Another assumption concerns death. I assume that the parent relationship continues to hold regardless of whether the individuals involved are alive or dead.

(A1) and (A1’) allow for a pair of parents to have multiple children, and also allows parents to have different children with different individuals (but not the same children with different individuals). Both of these accord with my intuitions.

(A2) ∀x [~(x = a) & ~(x = s)] -- > ∃y [mother(y,x) & ~∃z [mother(z,x) & ~(y = z)]]

“Everyone who is not Adam or Steve has exactly one mother”

(A3) ∀x∀y [mother(x,y) -- > parent(x,y)]
“One’s mother is always one’s parent”

Together, (A2) and (A3) ensure that everyone except Adam and Steve has exactly one mother, who is always a parent. It follows that everyone except Adam or Steve has one parent who is not a mother. These results are consistent with our intuitions that of one’s two parents, one is a mother and the other is a father.

(A4) \( \forall x \forall y [\text{parent}(x,y) \rightarrow \text{BET}(x,y)] \)
“Always, if x is y’s parent, then x is born earlier than y”

I am introducing the relation \( \text{BET} \) to exclude models where one can counterintuitively be one’s own parent, or parent’s parent, or parent’s parent’s parent, etc. In order to make such models invalid, \( \text{parent} \) needs to be related to a transitive relation. \( \text{Parent} \) is not itself transitive - intuitively, a x’s parent’s parent is generally not x’s parent\(^1\). The transitivity of \( \text{BET} \) will ensure that the domain is ordered in such a way that self-ancestry is disallowed.

The transitivity axiom is stated as follows:

(A5) \( \forall x \forall y \forall z [\text{BET}(x,y) & \text{BET}(y,z) \rightarrow \text{BET}(x,z)] \)
“Always, if x is born earlier than y and y is born earlier than z, then x is born earlier than z”.

\( \text{BET} \) is also asymmetric, which entails that it is irreflexive. The asymmetry is stated as follows:

(A6) \( \forall x \forall y [\text{BET}(x,y) \rightarrow \neg \text{BET}(y,x)] \)
“Always, if x is born before y, it is not the case that y is born before x”.

The irreflexivity that follows from (A6) will together with (A4) entail that one cannot be one’s own parent.

Now that \( \text{parent} \) and \( \text{mother} \) are defined and restricted to domains that correspond to our intuitive notion of order, we can define \( \text{grandparent} \) and \( \text{grandmother} \) in terms of \( \text{parent} \) and \( \text{mother} \).

(A7) \( \forall x \forall y \forall z [(\text{parent}(x,y) & \text{parent}(y,z)) \rightarrow \text{grandparent}(x,z)] \)
“One’s parent’s parent is one’s grandparent”

(A8) \( \forall x \forall y \forall z [(\text{mother}(x,y) & \text{parent}(y,z)) \rightarrow \text{grandmother}(x,z)] \)
“One’s parent’s mother is one’s grandmother”

\(^1\) It’s not intransitive either. Consider the case of Oedipus, who married his mother. If they had a child, then we would consider Oedipus to be the child’s parent, Oedipus’ wife/mother to be Oedipus’ parent, and Oedipus’ wife/mother to be the child’s parent. This is a counterexample to the claim that \( \text{parent} \) is intransitive. \( \text{Mother} \), however, is intuitively intransitive.
Since these are defined in terms of *parent*, the ordering restrictions imposed by (A4 - A6) will carry over to the relations *grandparent* and *grandmother*.

In summary, (A1') - (A8) entail the following theorems, all of which correspond to our intuitive ideas about kinship relations:

- Everyone except the original couple has two distinct parents, one of whom is a mother, one of whom is not.
- One cannot be one’s own ancestor, i.e. parent, parent’s parent, etc.
- It is possible (i.e. there are consistent models that allow) for one’s parent’s parent to be one’s parent; however, it is impossible (no model allows) for one’s mother’s mother to be one’s mother, or one’s father’s father (i.e. one’s non-mother parent’s non-mother parent) to be one’s mother or father (see note 1).
- One’s parent’s parent is one’s grandparent; one’s parent’s mother is one’s grandmother.
- A pair of people can be parents to more than one person.