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I. The homework questions.

Homework 6.

Choose one or more problems that are challenging for you but not impossible – this is an area where some of you have much more background than others, so we’ve tried to provide a range.

1) Consider models M1 and M2 above. Find one or two closed formulas which are false in M1 and true in M2.

2) Write down axioms defining relations grandfather, grandmother (you will need the relation mother). Explain what motivates your axioms – what would go wrong if they were different or if you left one of them out. Mention any points at which you are having to make decisions about the constraints on those relations which could be empirically controversial. Mention any points at which you are having to make decisions about “How much of what we know about grandparents should be captured by the axioms?”

Write the axioms in predicate logic. Also state them in English.

3) Exercises from PtMW: [Note: if you are new to predicate logic, make sure you do these; if you find them difficult, check with us right away so we can help you find help. On the other hand, if you’re already comfortable with predicate logic and would be willing to volunteer to help someone who might need help, let us know.]

Chapter 7, pp. 173, 174. #1 (a,f,m), #3 a-e, #4 a-c, #5 a-d

4) The predicate logic formula \( x(\text{love}(\text{Mary}, x) \land \text{happy}(x)) \) is equivalent to the English sentence “Everyone who Mary loves is happy.” Draw a syntactic tree (analogous to the trees 13-11, p.326 and 13-12, p. 328), which shows how that formula (not the English sentence) is built up from its parts according to the syntactic rules of the predicate calculus (in Section 1.1 above).

(a) Give each node a label that identifies both the syntactic category of the expression it dominates and the number of the syntactic rule by which its immediate constituents were combined (or “Basic”, if that node dominates a basic expression.) [The trees on pp 326, 328 have only rule numbers.]

(b) Below is a derivation of the truth-conditions of the formula according to the semantic rules of the predicate calculus. Annotate each line by identifying the semantic rule that was applied anywhere within that line (show where), and the node of the tree to which it corresponds. (According to the principle of compositionality, there should be a perfect match between syntactic rule and semantic rule applied at each node.)

(c) In addition, further annotate the syntactic tree by adding to the label of each non-terminal node the number of the semantic rule which was used to combine the meanings of the daughter-node expressions to get the meaning of the whole expression dominated by that node. For nodes dominating basic expressions, indicate whether the semantic rule to use is Rule A or Rule B. (If you’ve done it
right, there should be a perfect correspondence between syntactic rules and semantic rules applied at a given node.)

Semantic derivation of truth conditions:
1. \_x(love(Mary, x) 6 happy(x)) \text{M1,}g =1 iff for each d in D,
   \_love(Mary, x) 6 happy(x) \text{M1,}g[d/x] =1.

2. That will hold iff for each d in D,
   \_love(Mary, x) \text{M1,}g[d/x] = 0 or \_happy(x) \text{M1,}g[d/x] =1.

3. That will hold iff for each d in D,
   if \_Mary \text{M1,}g[d/x], \_x \text{M1,}g[d/x] > 0 \_love \text{M1,}g[d/x], then \_x \text{M1,}g[d/x] 0 \_happy \text{M1,}g[d/x].

4. And that will hold iff for each d in D,
   if \_Mary \text{M1,}g[d/x], d> 0 \_love \text{M1,}g[d/x], then d 0 \_happy \text{M1,}g[d/x].

5. I.e., if \_l(Mary), d> 0 l(love), then d 0 l(happy).

5) Here is a nice ‘axioms and models’ problem from Chapter 8: PtMW p. 235, Ex. 13.
   (BHP learned that one from Hartley Rogers, Jr., in a logic course in graduate school.) Try it after you have read 8.5.1 – 8.5.4. This is relatively challenging. If the previous ones are easy for you, be sure to try this one.

6) Axiomatizing the notion of a syntactic tree structure. More challenging. And we will probably return to it again later. This is strictly optional for this time. But it may be fun to begin thinking about a linguistic example.

Consider syntactic structures (trees of immediate constituency) as sets of nodes together with some relations (e.g., is a constituent, is an immediate constituent of; the order from left to right (is to the left of, maybe something else). Try to write axioms which define all “well-formed” trees of this kind. Here is a suggestion for how to begin. Let a tree consist of a set Node of nodes, with two basic relations defined on Node: Dom (immediate dominance) and [] (immediate left-right precedence). Write axioms characterizing Dom and[]. Then you can define further relations and properties for trees, such as Dom* for (not-necessarily-immediate) dominance, < for (not-necessarily-immediate) precedence, Root as a unary property of a node that is the “top” node of a tree, Leaf as a unary property of a node that is a “bottom” or “terminal” node of a tree (note that linguistic trees are always “upside down”, with their root on the top and their leaves on the bottom!). Then try writing axioms for when a set of nodes Node forms a well-formed tree. (If you want to keep going, you can try characterizing the relation C-command axiomatically, if you are familiar with it, and any other relations and properties of nodes...
that you can think of. And you could think about adding labels to the nodes – this can be done in several different ways, either by introducing a new set called Node-labels and a binary relation Label relating elements of Node to elements of Node-labels, or just by introducing a family of unary relations S, NP, VP, etc.)

If you want to read about one way of axiomatically characterizing trees, look at Chapter 16, pp 431-448, of Partee, ter Meulen and Wall.

If you want to play with it yourself for a while before looking at PtMW, one good way is to work with a partner and be devil’s advocates for each other. One of you proposes an axiomatization of some of the various tree-related notions and/or of what it takes to be a well-formed tree, and the other one tries to construct an unintended model – something that intuitively doesn’t fit those notions but does conform to the proposed axioms. Or maybe the axioms are too strong and you can think of something they wrongly exclude. Take turns being the proposer and the counter-example person. Keep a written record of the process and let us see it – it’s all interesting, not just the endpoint you may reach.

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Student answers:

Solution 1: Exercise 1. Several answers to question 1, all good, collected from several participants.

Solution 2: Excercises 1-6.

Solution 3: Exercise 2.

Solution 4: Excercises 2-4.

Solution 5: Exercise 4.

Note on Exercise 2. The *grandparent, grandmother* problem. Several thoughtful answers. (There is no uncontroversial “correct” answer, but some of these are annotated.)