

Index to Homework 14. Infinities.

Questions of Homework 14.

I. PtMW, pp 71-73, Exercises 2, 3, 4, 5, 6. Optional 7, 8. Note: when it just asks “show that set S is infinite”, as in question 3, use the definition of *infinite*, rather than showing what specific infinite cardinality the set has.

II. PtMW, p. 84, Exercise 4. For (4b), when it says “without using the results of Chapter 4”, it means to do a “diagonal proof”, rather than proving by putting the set into 1-1 correspondence with some set known to be denumerably infinite.

III. Question from Quiz 1 in Ling 409:

For all of this question, let V be the alphabet $\{a,b\}$. We will consider finite strings on V (the empty string e and strings like a , abb , $bbababb$, etc.)

Define a *language on V* to be any set (empty, finite, or infinite) of finite strings on V .

So: each string must be of finite length, but the number of strings that constitute a “language” can be either finite or infinite.

Your task: Classify the 3 following sets as finite, denumerably infinite, or non-denumerably infinite. Give reasons, but I’m not asking for a complete proof. [Note: for 726, since you aren’t under time pressure, go ahead and give a proof if you can.]

If the answer is “finite”, explain why, informally.

If the answer is “denumerably infinite”, show how you would go about putting the set into 1-1 correspondence with the set of natural numbers or with some other set we already have shown to be denumerably infinite.

If the answer is “non-denumerably infinite”, you can give me either of two kinds of answers. Either (i) give a sketch of how a proof might go, or (ii) show how you could put the set into a 1-1 correspondence with some other set we already know to be non-denumerably infinite.

a. The set of all strings on V whose length is less than or equal to 10 symbols.

b. The set of all finite strings on V .

c. The set of all languages on V .

Note to 726 about answers: For the ones not in the back of the book, and the quiz question, the answers can be found on the web page of 409. www.umass.edu/linguist/409/

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Going up on web:

HW14, Set 1. Includes Part I, questions 2,3,4,6,7; 7 is the optional question about cardinal arithmetic. All correct modulo one small ‘instructors’ note’ inserted in question 7. Also Part III, the 409 quiz question, all correct.

HW14, Set 2. Includes Part I, questions 2,3,4,6 (numbered 1,2,3,4), Part II question 4 (numbered 5), and Part III, the quiz question (numbered 6). All correct modulo a few inserted “instructors’ notes”.

HW14, Set 3. Includes Part I, questions 2,3,4,6,8; 7 is the optional question about \aleph_0 being the smallest infinite cardinal number. Part II question 4, and Part III, the quiz question. All but 8 correct; one inserted “instructors’ note”.

Note to Homework 14. A fallacious diagonal proof.