Homework 4: pp.51-2, 1,2,3

Questions

Q1: What's the difference between antisymmetric and asymmetric?

A1: R is asymmetric if for all $\langle x,y\rangle \in R$, $\langle y,x\rangle \notin R$.

R is antisymmetric if for all $\langle x,y\rangle \in R$, $\langle y,x\rangle \notin R$ *unless x=y*.

e.g. {<1,1>, <1,2>, <2,3>} is not asymmetric because of <1,1>, but it is antisymmetric.

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(1)

(a)

(i) "is a child of" is irreflexive, asymmetric, intransitive.

- No person is a child of him/herself
- For all people, if x is a child of y, then y is not a child of x.
- if x is a child of y and y a child of z, then for no person is x a child of z.

(ii) "is a brother of" is irreflexive, non-symmetric, nontransitive.

- No person is a brother of him/herself
- If x is a brother of y, then y may be x's brother.
  However, if x is a brother of y and y is a female, y is not x's brother.
- if John is a brother of Joe and Joe of Bill, then John is a brother of Bill.
  However, if John is a brother of Joe and Joe of John, John is not a brother of himself.

(iii) "is a descendent of" is irreflexive, asymmetric, transitive.

- No person is a descendent of themselves.
- For no pair of people x,y is x a descendent of y and y of x.
- If x is a descendent of y and y of z, then x is a descendent of z.

(iv) "is an uncle of" is non-reflexive, nonsymmetric, nontransitive.

- John can be an uncle an uncle of himself if he marries his aunt.
- If John is Joe's uncle, Joe is not John's uncle.
  But if John is John's uncle, then John is John's uncle, therefore not asymmetric.
- Ditto for nontransitive.

(b) (ii) would change if restricted to the set of all humans.

It would be symmetric. If x is y's brother, then y is x's brother as long as x,y are males.

(2) (a) "x,y are a minimal pair of utterances"

- irreflexive (no utterance forms a minimal pair with itself)
- symmetric
- nontransitive (e.g. {<cat,bat>, <bat, bag>}, but not <cat,bag>)
- nonconnected.

(b) "x,y are phones in complementary distribution"

- irreflexive (no phone is in complementary distrib. with itself)
- symmetric
- nontransitive (Real example: English [t]-[ɾ]-[t^h] are in complementary distribution, so the pairs <t,ɾ>, <ɾ,t^h>, <t,t^h> exist in the set. However, it is nontransitive because: [t] is in compl. distrib. with [ɾ] and [d] is in compl. distrib with [ɾ], but [t] is not in compl. distrib. with [d].

(c) "x,y are phones in free variation"
- reflexive or irreflexive, depending on whether a phone is in free variation with itself.
- symmetric
- nontransitive.

We could imagine a situation in which [b] and [β] are in free variation, and [v] and [β] are in free varation, but [b] and [v] are not.

i.e. [vata]~[βata], [bata]~[βata], *[vata]~[bata]

-nonconnected.

(d) "x,y are allophones of the same phoneme"
- reflexive
  [t] and [t] are allophones of /t/.
- symmetric
- non-transitive
  Similar to reasoning above.
  <t,ɾ>∈ A because [t] and [ɾ] are allophones of /t/.
  <ɾ,d>∈ A because [ɾ] and [d] are allophones of /d/.
  But <t,d>∉ A.
- nonconnected.

(e)
- reflexive
- symmetric
- transitive
- aanonconnected

Each equivalence class contains all the sets that have the same number of members.

(3)(a) R_1 and R_1^{-1}: reflexive, antisymmetric, nontransitive, nonconnected.
  (i) There are no pairs <x,y>, <y,x> for x≠y. Therefore, antisymmetric.

R'_1: irreflexive, nonsymmetric, nontransitive, connected.
  (i) book says non-connected.

R_2 and R_2^{-1}: irreflexive, asymmetric, transitive, connected.

R_2': reflexive, antisymmetric, transitive, connected.

R_3, R_3^{-1}: nonreflexive, symmetric, nontransitive, nonconnected.

R_3': nonreflexive, symmetric, nontransitive, nonconnected.

R_4 and R_4^{-1}: reflexive, symmetric, transitive, nonconnected.

R_4’: irreflexive, symmetric, intransitive, nonconnected.

R_4 is an equivalence relation.

The partition induced in A is {{1,3}, {2,4}}

(b) {<1,1>,<2,2>, <3,3>, <4,4>, <2,3>, <3,2>}

(c) 15

They are:

{{1,2,3,4}}	{{1,2,3},{4}}	{{1},{2,3},{4}}
{{1},{2,3,4}}	{{1,2,4},{3}}	{{1,2},{3},{4}}
{{1,2},{3,4}}	{{1,3,4},{2}}	{{1,3},{2},{4}}
{{1,3},{2,4}}	{{1},{2},{3,4}}	{{1,4},{2},{3}}
{{1,4},{2,3}}	{{1},{3},{2,4}}	{{1},{2},{3},{4}}