

Homework 3  
pp.36-7: 1,2,3a

(1)

- (a) (i)  $\{\langle b,2\rangle, \langle b,3\rangle, \langle c,2\rangle, \langle c,3\rangle\}$   
 (ii)  $\{\langle 2,b\rangle, \langle 2,c\rangle, \langle 3,b\rangle, \langle 3,c\rangle\}$   
 (iii)  $\{\langle b,b\rangle, \langle b,c\rangle, \langle c,b\rangle, \langle c,c\rangle\}$   
 (iv)  $\{\langle b,2\rangle, \langle b,3\rangle, \langle c,2\rangle, \langle c,3\rangle, \langle 2,2\rangle, \langle 2,3\rangle, \langle 3,2\rangle, \langle 3,3\rangle\}$   
 (v)  $\emptyset$ , since  $A \cap B = \emptyset$ .  
 (vi) Same as  $A \times B$

- (b) (i) True  
 (ii) False  
 (iii) False,  $\langle c,c\rangle \in (A \times A)$   
 (iv) True  
 (v) True  
 (vi) True  
 (vii) True

- (c) (i)  $\text{dom}(R) = A$ ,  $\text{range}(R) = \{b,2,3\}$   
 (ii)  $R' = \{\langle b,c\rangle, \langle b,3\rangle, \langle c,b\rangle, \langle c,c\rangle\}$   
 $R^{-1} = \{\langle b,b\rangle, \langle 2,b\rangle, \langle 2,c\rangle, \langle 3,c\rangle\}$   
 (iii) Yes (answer in the book is wrong)

(2) (a) Relations from A to B.

Each of a,b,c can be paired with 1, 2, 1 and 2, or neither – i.e. in four possible ways.

Therefore, there are  $4 \times 4 \times 4 = 64$  distinct relations.

In functions from A to B, a,b,c can either be paired with 1 or 2, but not both or neither. So, there are  $2 \times 2 \times 2 = 8$  distinct relations.

- \* $\{\langle a,1\rangle, \langle b,1\rangle, \langle c,1\rangle\}$   
 $\{\langle a,1\rangle, \langle b,1\rangle, \langle c,2\rangle\}$   
 $\{\langle a,1\rangle, \langle b,2\rangle, \langle c,1\rangle\}$   
 $\{\langle a,1\rangle, \langle b,2\rangle, \langle c,2\rangle\}$   
 $\{\langle a,2\rangle, \langle b,1\rangle, \langle c,1\rangle\}$   
 $\{\langle a,2\rangle, \langle b,1\rangle, \langle c,2\rangle\}$   
 $\{\langle a,2\rangle, \langle b,2\rangle, \langle c,1\rangle\}$   
 \* $\{\langle a,2\rangle, \langle b,2\rangle, \langle c,2\rangle\}$

Six of these are onto. The starred ones are not onto.

None of these are 1 to 1 and onto, so non have inverses that are functions.

(b) Relations from B to A.

1 can be paired with (i) a, (ii) b, (iii) c, (iv), a & b, (v) a & c, (vi) b & c, (vii) a,b,c, and (viii) none. Same for 2, therefore  $8 \times 8 = 64$  possible relations.

9 are functions.

None are onto.

Six are 1 to 1.

None have inverses that are functions.

(3a)

$$R_2 \circ R_1 = \{ \langle 1,2 \rangle, \langle 1,4 \rangle, \langle 1,3 \rangle, \langle 2,2 \rangle, \langle 2,4 \rangle, \langle 2,3 \rangle, \langle 3,4 \rangle, \langle 4,2 \rangle, \langle 4,3 \rangle, \langle 4,4 \rangle \}$$

$$R_1 \circ R_2 = \{ \langle 3,4 \rangle, \langle 3,1 \rangle, \langle 1,1 \rangle, \langle 1,2 \rangle, \langle 1,4 \rangle, \langle 2,4 \rangle, \langle 2,3 \rangle, \langle 2,1 \rangle, \langle 1,3 \rangle \}$$