

Homework 2: Supplemental Answers

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(9a) I have expanded this each detail at a time for the sake of clarity. There are several ways to prove this problem; this is only one.

$$((A \cup C) \cap (B \cup C')) \subseteq (A \cup B)$$

$$((A \cup C) \cap (B \cup C')) \cup (A \cup B) = (A \cup B) \quad \text{Consistency Principle (8a)}$$

$$(((A \cup C) \cap B) \cup ((A \cup C) \cap C')) \cup (A \cup B) = (A \cup B) \quad \text{Distributive}$$

$$(((A \cup C) \cap B) \cup ((C' \cap A) \cup (C' \cap C))) \cup (A \cup B) = (A \cup B) \quad \text{Distributive}$$

$$(((A \cup C) \cap B) \cup ((C' \cap A) \cup \emptyset)) \cup (A \cup B) = (A \cup B) \quad \text{Complement (c)}$$

$$(((A \cup C) \cap B) \cup (C' \cap A)) \cup (A \cup B) = (A \cup B) \quad \text{Identity (a)}$$

$$(((C' \cap A) \cup (A \cup C)) \cap ((C' \cap A) \cup B)) \cup (A \cup B) = (A \cup B) \quad \text{Distributive}$$

$$(((A \cup C) \cup C') \cap ((A \cup C) \cup A)) \cap ((C' \cap A) \cup B) \cup (A \cup B) = (A \cup B) \quad \text{Distrib}$$

$$(((A \cup (C \cup C')) \cap ((A \cup C) \cup A)) \cap ((C' \cap A) \cup B)) \cup (A \cup B) = (A \cup B) \quad \text{Assoc}$$

$$((A \cup U) \cap ((A \cup C) \cup A)) \cap ((C' \cap A) \cup B) \cup (A \cup B) = (A \cup B) \quad \text{Compl}$$

$$(U \cap ((A \cup C) \cup A)) \cap ((C' \cap A) \cup B) \cup (A \cup B) = (A \cup B) \quad \text{Identity}$$

$$(U \cap (A \cup C)) \cap ((C' \cap A) \cup B) \cup (A \cup B) = (A \cup B) \quad \text{Distrib}$$

$$(A \cup C) \cap ((C' \cap A) \cup B) \cup (A \cup B) = (A \cup B) \quad \text{Identity}$$

$$(A \cup B) \cup (A \cup C) \cap ((A \cup B) \cup ((C' \cap A) \cup B)) = (A \cup B) \quad \text{Distrib}$$

$$(A \cup B \cup C) \cap ((A \cup B) \cup ((C' \cap A) \cup B)) = (A \cup B) \quad \text{Assoc + Identity}$$

$$(A \cup B \cup C) \cap ((A \cup B) \cup ((B \cup C') \cap (B \cup A))) = (A \cup B) \quad \text{Distrib}$$

$$(A \cup B \cup C) \cap ((A \cup B) \cup (A \cup B)) \cap ((A \cup B) \cup (B \cup C')) = (A \cup B) \quad \text{Distrib}$$

$$(A \cup B \cup C) \cap ((A \cup B) \cap ((A \cup B) \cup (B \cup C'))) = (A \cup B) \quad \text{Idempotent}$$

$$(A \cup B \cup C) \cap ((A \cup B) \cap (A \cup B \cup B \cup C')) = (A \cup B) \quad \text{Distrib + Assoc}$$

$$(A \cup B \cup C) \cap ((A \cup B) \cap (A \cup B \cup C')) = (A \cup B) \quad \text{Idempotent}$$

$$(A \cup B \cup C) \cap (A \cup B) = (A \cup B) \quad \text{Distrib, etc.}$$

$$(A \cup B) = (A \cup B) \quad \text{Distrib, etc.}$$

QED

(11d)

(i) $A + A$

$$= (A \cup A) - (A \cap A) \quad \text{by definition}$$

$$= A - A \quad \text{Idempotent}$$

$$= A \cap A' \quad \text{Complement (d)}$$

$$= \emptyset \quad \text{Complement (c)}$$

(ii) $A + U$

$$= (A \cup U) - (A \cap U) \quad \text{by definition}$$

$$= U - (A \cap U) \quad \text{Identity}$$

$$= U - A \quad \text{Identity}$$

$$= U \cap A' \quad \text{Complement}$$

$$= A' \quad \text{Identity}$$

$$\begin{aligned} & \text{(iii) } A + \emptyset \\ &= (A \cup \emptyset) - (A \cap \emptyset) && \text{by definition} \\ &= A - \emptyset && \text{identity} \\ &= A \cap \emptyset' && \text{complement} \\ &= A \cap U && \text{rewrite} \\ &= A && \text{identity} \end{aligned}$$

$$\begin{aligned} & \text{(iv) } A + B \text{ where } A \subseteq B \\ &= (A \cup B) - (A \cap B) && \text{by definition} \\ &= (A \cup B) - A && \text{consistency} \\ &= B - A && \text{consistency} \end{aligned}$$

$$\begin{aligned} & \text{(v) } A+B \text{ where } A \cap B = \emptyset \\ &= (A \cup B) - (A \cap B) && \text{by definition} \\ &= (A \cup B) - \emptyset && \text{given} \\ &= A \cup B \end{aligned}$$
