Homework 1 Supplementary Answers and Notes
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Questions Asked in Class and in the Homeworks

Q: Is a ∈ {{a}}?
A: No. {a} ∈ {{a}}.

The confusion here is with the notion 'member'. Informally, *x* is a member of a set if it appears between commas or the end braces. So, in {{a}, b, {{{c}}}}, the members are {a}, b, and {{{c}}}. Importantly, ∅ is not a member of this set.

Q: How do you tell whether to write a predicate or recursive rule?
A: Either way is fine. Our aim is to provide an unambiguous definition of a set. In principle, we can do that any way. By predicates or rules are just two possible ways. Neither is intrinsically better than the other, it is just that often one is easier to state than the other.

Q: What's the difference between a subset and a proper subset?
A: S is a proper subset of A (written S ⊂ A) if
(i) Every member of S is also in A
and (ii) A and S are not identical.
So {a} ⊂ {a,b}, but {a, b} ⊄ {a, b}. However, {a,b} ⊆ {a,b}.

Q: If K ∪ J = K, then what set is J?
A: K must be a subset of K. Of course, this includes K itself, and ∅.

Q: Difference: What happens when we take the difference of {∅, {∅}} and other sets with null elements?
A: (i) {∅, {∅}} - ∅ = {∅, {∅}}
The set produced by A-B contains all those members of A that are not in B. Since ∅ has no members, for all sets S, S - ∅ = S.
(ii) {∅, {∅}} - {∅} = {{∅}}
(iii) {∅, {∅}} - {{∅}} = {∅}

This question showed some confusion with the notion of 'empty set'. It is often more transparent to write it as {} rather than ∅. If we recast the question above in these terms, it may be more apparent why (i) comes out the way it does:
{{}, {{}}} – {} = {{}, {{}}}

Q: What is {a,b,c} – a, assuming that a is just the letter "a", not a set.?
A: There is no answer since this is not a well-formed statement. We can only talk about the difference between two *sets.* a is not a set.

Q: What are the subsets of S9 in Ex1.4d?
A: S9 = {∅, {∅}}

The members of S9 are $\emptyset$ and $\{\emptyset\}$.
The subsets of S9 are:
(i) $\emptyset$ (since $\emptyset$ is a subset of every set)
(ii) $\{\emptyset, \{\emptyset\}\}$ (since for every set S, $S \subseteq S$)
(iii) $\{\emptyset\}$
(iv) $\{\{\emptyset\}\}$

Try doing this with the set $\{\{a\}, \{\{b\}\}\}$. It might make things clearer.

Supplementary Answers
3b) Rule:
1. $7 \in A$
2. if $x \in A$ then $x+10 \in A$
3. There are no other members of A.

3c) Property:
$\{ x \mid x > 299 \text{ and } x < 401 \text{ and } x \in \mathbb{N}\}$

3e) Property:
{ x | x is an integer and a multiple of 2 }

3f) Rule:
1. $1 \in F$
2. if $x \in F$ then $1/x \in F$
3. There are no other members of F.

(4)
(a) Which are members of S1?
Answer: S2, S3, S7
Notes: $\emptyset$ is *not* a member of S1.
Contrast the statements:
(1) $\emptyset \in \{\emptyset\}$
(2) $\emptyset \notin \emptyset$
(3) $\emptyset \notin \{a\}$
(4) For every set S, $\emptyset \subseteq S$

(b) Which are subsets of S1?
Answer: S1, S3, S4, S5, S6, S8
Notes: (i) For every set S, $S \subseteq S$, so $S1 \subseteq S1$
(ii) For every set S, $\emptyset \subseteq S$, so $S6 \subseteq S1$
(iii) $S3 \subseteq S1$ because every member of S3 is also in S1 – i.e. A.
(iv) $S2 \not\subseteq S1$ because $A \in S1$, $A \not\subseteq S1$.
(v) $S5 \subseteq S1$ because every member of S5 is in S1 – i.e. {A} and A.

(vi) S7 ⊄ S1 because ∅ ∉ S1. Note that {∅} ∈ S1.

(vii) S9 ⊄ S1 because there is a member of S9 – namely ∅ -- that is not a member of S1.

(c) Which are members of S9?
Answer: S6, S7

(d) Which are subsets of S9?
Answer: S6, S7, S8, S9
Notes: (i) For every set S, S ⊆ S, so S9 ⊆ S9
(ii) For every set S, ∅ ⊆ S, so S6 ⊆ S1
(iii) {∅} ⊆ S9 because ∅ ∈ S9.
(iv) {{∅}} ⊆ S9 because {∅} ∈ S9

(e) Which are members of S4?
Answer: S3

(f) Which are subsets of S4?
Answer: S4, S6
Notes: Is S3 {A} a subset of S4?
If it was, A should be a member of S4. But it isn't: {A} ∈ S4; A ∉ S4.

5.
(a) ℘({a,b,c}) = { ∅, {a}, {b}, {c}, {a,b}, {a,c}, {b,c}, {a,b,c}}
(b) ℘({a}) = { ∅, {a}}

Notes on (5e): ℘(℘({a,b}))
First take the inner brackets: ℘({a,b}) = {∅, {a}, {b}, {a,b}}
Now, take ℘({∅, {a}, {b}, {a,b}})
Try listing all the subsets, starting with ∅, then listing singletons, doubletons, etc.

i.e.
1-member sets: ∅,
2-member sets: {∅}, {{a}}, {{b}}, {{a,b}},
{∅, {a}}, {∅, {b}}, {∅, {a, b}},
{{a}, {b}}, {{a}, {a, b}},
{{b}, {a,b}},
3-member sets: {∅, {a}, {b}}, {∅, {b}, {a,b}}, {∅, {a}, {a,b}}
{{a}, {b}, {a,b}}
4-member sets: {∅, {a}, {b}, {a,b}}

You should have 16 sets in all (4^2).

So:

$\wp(\{\emptyset, \{a\}, \{b\}, \{a,b\}\}) = \{\emptyset,$

$\{\emptyset\}, \{\{a\}\}, \{\{b\}\}, \{\{a,b\}\},$

$\{\emptyset, \{a\}\}, \{\emptyset, \{b\}\}, \{\emptyset, \{a, b\}\},$

$\{\{a\}, \{b\}\}, \{\{a\}, \{a, b\}\},$

$\{\{b\}, \{a,b\}\},$

$\{\emptyset, \{a\}, \{b\}\}, \{\emptyset, \{b\}, \{a,b\}\}, \{\emptyset, \{a\}, \{a,b\}\},$

$\{\{a\}, \{b\}, \{a,b\}\},$

$\{\emptyset, \{a\}, \{b\}, \{a,b\}\}$

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[BTW, there's an error in the answer for 5e. See if you can find it.]