Addendum to
Supplement to Logic Unit: Logical Structure in Natural Language

Appendix 2: Returning to the quantifier scope ambiguity question.
The problem described in Section 1 of the main handout:
Consider the following sentence containing a universal quantifier-word every and an indefinite article a.
The sentence is semantically ambiguous: we can think of the indefinite article as introducing an existential quantifier and every as introducing a universal quantifier, and the two quantifiers can be interpreted in either 'scope order'.

(1) a. Every student read a book. (Quantifier scope ambiguity)
   Just one (surface) syntactic structure:

   S: every student read a book
   NP: every student
   VP: read a book

   Compositional representations of the two readings:
(2) (i) \( \forall x \) (Student (x) \( \rightarrow \) \( \exists y \) (Book (y) & Read (x,y)))
(ii) \( \exists y \) (Book (y) \& \( \forall x \) (Student (x) \rightarrow Read (x,y)))

Compositional interpretation of the English sentence: How do we derive the meaning of the whole from the meaning of the parts?

Here is (an informal statement of) Montague’s Quantifying In rule; it is similar to the Quantifier-Lowering rule of Generative Semantics and Quantifier Raising (QR) of May (1977); various alternative treatments of quantifier scope ambiguity exist, including Cooper-storage (Cooper 1975) and Herman Hendriks’s flexible typing approach (Hendriks 1988, 1993).

Quantifying In Rule, Syntax: (informally stated): An NP combines with a sentence with respect to a choice of variable (“he” in MG). Substitute the NP for the first occurrence of the variable; change any further occurrences of the variable into pronouns of the appropriate number and gender.

Semantic rule: NP(\( \lambda x [S'] \)) (“The set of properties denoted by the NP includes the property denoted by the \( \lambda \)-expression derived from the sentence.” You can pretend that \( \lambda x \phi \) is simply the characteristic function of the set of entities that satisfy \( \phi \).)

We illustrate with two derivations for the ambiguous sentence Every student read a book.

**Syntactic derivation (i)** (rough sketch; read from bottom to top. **Bold** is used here to show which variables are substituted for at each step.)

S: every student read a book
NP: every student
S: he read his book
NP: a book
S: he read it

Compositional Translation: \( (\lambda x [\lambda y [\lambda z [\text{book} (z) \& \text{read} (x,z)]]]) \)
Rough paraphrase: Every student has the property that there is a book that he read.

If you write out the interpretations of the NPs and apply Lambda-Conversion as many times as possible, the result will be (some alphabetic variant of) the first-order PC formula \( \forall x (\text{student}(x) \rightarrow \exists y (\text{book}(y) \& \text{read}(x,y))) \).

**Syntactic derivation (ii)**

S: every student read a book
NP: a book
S: every student read him
NP: every student
S: he read him

Compositional Translation: \( (\lambda x [\lambda y [\lambda z [\text{book} (z) \& \lambda x (\text{student}(x) \rightarrow \text{read} (x,z))]]) \)

Homework problem 6.

Paraphrase: Some book has the property that every student read it.
After applying Lambda-Conversion as many times as possible, the result will be (some alphabetic variant of) the first-order PC formula \( \exists y (\text{book} (y) \& \forall x (\text{student}(x) \rightarrow \text{read}(x,y))) \).

Observation: Compositional semantics requires that every ambiguous sentence be explainable on the basis of ambiguous lexical items and/or multiple syntactic derivations. Semantic structure mirrors syntactic part-whole structure, which in Montague Grammar is represented by syntactic derivational structure, not surface structure. There are different theories of the semantically relevant syntactic structure: "Derivation trees" or "analysis trees" (MG), LF (Chomskian GB or Minimalist theory), Tectogrammatic Dependency Trees (Prague), Deep Syntactic Structure (Melčuk) Underlying Structure (Generative Semantics), … GPSG, HPSG, and various contemporary versions of Categorial Grammar are attempts to represent all the necessary syntactic information directly in a single “level” of syntax.