Lecture 9: Statement Logic: Syntax and Semantics

Reading: Chapter 6, Sections 6.1 and 6.2, pp. 97-104.

6.1. Syntax

The syntax of statement logic is very simple.

Basic vocabulary: An infinite set of *atomic statements*, represented by the symbols *p, q, r, s, …*, with the addition of primes or subscripts as needed (p', p'', …, p_1, p_2, …).

Definition 6.1, p. 97.

1. Any atomic statement is itself a sentence (or well-formed formula: wff).
2. Any wff preceded by the symbol '~' (negation) is also a wff.
3. Any two (not necessarily distinct!) wff's can be made into another wff by writing the symbol '&' (conjunction), '∨' (disjunction), '→' (conditional), or '↔' (biconditional) between them and enclosing the result in parentheses.

Some examples of wff's of statement logic according to these rules:

p

q'

$(p \vee q)$

$\sim(p' \leftrightarrow p')$

$\sim\sim r$

$\sim(p \rightarrow (q \ \& \ r))$

Some examples that are not wffs:

pq

$\vee q'$

$\sim\sim$

$p \vee q$ **Why?**

$\sim(p' \leftrightarrow p'$

$\sim(r)$ **Why?**

Convention about outer parentheses: According to the rules, there should always be an outermost pair of parentheses in a formula like $(p \rightarrow (q \ \& \ r))$. But there is an abbreviatory convention: in practice, the <u>outermost</u> parentheses may be dropped. Then we write $p \rightarrow (q \ \& \ r)$. We regard the latter as an abbreviation for the former, and count

it as well-formed. Similarly we may write $p \vee q$, although in the list above we had declared it to be ill-formed. With this abbreviatory convention in place, we now regard it as perfectly OK; it's an abbreviation for a formula that contains outermost parentheses.

Nearest natural language counterparts. (But caution! See the semantics in the next section. Don't read too much into these.)

~ : not

&: and

∨ : or ("inclusive" *or*)

→ : if- then

↔ : iff

On the use of the term *'statement'*. We will say that a sentence such as *John saw a black cat* expresses a *statement*. We ignore pragmatic concerns such as speaker and context. Sometimes the term *proposition* is used instead, and this is called *propositional logic*. An ambiguous English sentences such as "Mary saw the man with the telescope" can express more than one statement or proposition.

6.2. Semantics. Truth values and truth tables.

Each atomic statement is assumed to have one of the two truth values 1 (or *true*) or 0 (or *false*). Each complex wff also receives a truth value, which depends on the truth values of its parts and on its syntactic structure. We specify the semantic rules that correspond to pieces of syntactic structure via *truth tables* for each of the five connectives.

6.2.1. – 6.2. 5: the Truth Tables. [blackboard]

Notes on the various connectives: Disjunction (*or*) is **inclusive.** Conjunction (*and*) is symmetrical – no "and then" interpretation, just pure logical 'and'. The conditional may not be intuitive: → means no more and no less than what its truth table specifies. It's not always clear whether a given natural language sentence is best translated with "if" or with "iff".

Computing the truth conditions of a complex wff: We'll illustrate at the blackboard. Let's, for instance, examine whether the following two statements are equivalent.

$(p\&q) \vee r$ vs. $p \ \& \ (q \vee r)$

In principle, we can compute the truth conditions for any complex statement at all in this way.