Lecture 8: Basic Concepts of Logic and Formal Systems

Reading: Chapter 5.

5.1. Formal systems and models

Formalization and axiomatization is an outgrowth of scientific systematization. The goal is to find a small set of laws or principles from which the full array of "facts" in a given domain follow. Replace an unsystematic set of "truths" with a systematized "system" of basic premises and all that follows from them.

Euclid – geometry; Newton – physics.

- Issue: the status of the basic premises, or initial assumptions.
  - First idea: Those are basic truths, self-evident (Euclid) or discovered (Newton).
  - A big change, with far-reaching consequences, after the discovery of **non-Euclidean geometry.**
    - Sketch the history. The anomalous status of the Parallel Postulate. Attempts to derive it from the other postulates. Attempts to show that there's a contradiction if one denies it.
    - Lobachevsky, Bolyai : more than one parallel
    - Riemann: less than one parallel
    - No contradiction: they found "models". Show model of Lobachevsky-Bolyai non-Euclidean geometry.

- Initially: model first, then systematize "what we know about it".
- After non-Euclidean geometry, the beginnings of the modern era: propose systems of axioms, and "play with them", and see what if any models they might have. The axioms are no longer regarded as basic truths (or not); now one asks "what are they true *of*?", i.e. what sorts of *models* do they have?

Formal systems have:

(i) A non-empty set of *primitives* (notions, concepts, terms)
(ii) A set of statements about the primitives, the *axioms*
(iii) A means of deriving further statements (*theorems*) from the axioms; either
(a) An explicit set of recursive rules of derivation, or
(b) Appeal to a background logic for the language in which the axioms are stated, usually predicate logic, or
(c) No explicit means of derivation. Just derive "whatever logically follows" from the axioms.

In linguistics, sometimes we are reasoning *about* language. In that case, we need to distinguish *object language* (the language we are talking about) from *metalanguage* (the "scientific" language we use for describing the object language).

Axiomatizing a logic itself:

(i) A syntax defining the expressions of the logical language
(ii) A set of axioms, chosen from among the formulas of the language

(iii) A set of explicit rules of inference for deriving further formulas (the *theorems*).

5.2. Natural languages and formal languages.

Formal languages are *designed* for particular purposes.

- Languages of logic
- Computer languages, programming languages
- The language of arithmetic
- (musical notation, …)

Formal languages can be of interest for linguists in two different ways:

(i) We can study formal languages as simplified and schematized versions of natural languages; they are simpler than natural languages, but may share important relevant properties with natural languages. Here we see formal languages as objects of study, hoping that what we can learn about them may be applied to natural languages as well.
(ii) Formal languages may be used as tools for studying natural languages.

5.3. Syntax and semantics.

What do *syntax* and *semantics* mean in the study of formal systems?

Syntax	Semantics
Expressions	Their meanings
Form	Content
Language	Models
Well-formedness rules, derivations, proofs	Truth, reference, entailment
Proof theory	Model theory

Hilbert and the formalist program. Autonomous syntax.

The oddness of the name "formal semantics"
Broader uses of the term "semantics".

5.4. About statement logic and predicate logic

Other names: Statement logic: propositional logic, sentential logic
Predicate logic: First-order predicate logic, first-order quantificational logic
Other notations: See appendices.
Purpose of logic: studying valid/invalid forms of reasoning; "regimentation": simplified languages stripped down to certain logical essentials.