Lecture 26. Regular languages

Reading:
Section 17.2 of PtMW: Regular Languages. pp. 462-468; you can omit 17.2.1 on the Pumping Lemma for fall’s.

Homework:
Homework 26 on syllabus. Note: for all of exercises 1-3 for Chapter 17, the alphabet must be specified as \{0,1\}.

Erratum to PtMW p. 464: the single state \(q_0\) in Figure 17-5 a) is a final state as well as an initial state, and should therefore be drawn with a double circle, not a single circle.

17.1 Regular languages

We define two new operations on languages, considering languages as sets of strings.

Def 17.9 (p. 462) The concatenation \(AB\) (or set product) of two sets of strings \(A\) and \(B\).
(Sometimes written \(A \cdot B\) = \(\{x \cap y : x \in A, y \in B\}\)

The Kleene star or closure of a set of strings \(A\) is \(A^*\), the set of all strings formed by concatenating members of \(A\) any number of times (including zero) in any order and allowing repetitions. This is just like our existing notion \(A^*\), except that now \(A\) can be any set of strings, not just an alphabet (i.e. a set of strings of length one). The old notion is just a special case of the new more general notion; the old notion was used for the case where \(A\) is a set of strings of length one, i.e. an alphabet.

Using these two new operations on sets of strings, as well as standard set-theoretic notions, we can now define the regular languages recursively.

Definition 17.10 (p. 463)
Given an alphabet \(\Sigma\):
1. \(\emptyset\) is a regular language.
2. For any string \(x\) in \(\Sigma^*\), \(\{x\}\) is a regular language.
3. If \(A, B\) are regular languages, then so is \(A \cup B\).
4. If \(A, B\) are regular languages, then so is \(AB\).
5. If \(A\) is a regular language, then so is \(A^*\).
6. Nothing else is a regular language.

Theorem. (Kleene) A set of strings is a finite automaton language iff it is a regular language.

Examples: There are quite a few examples in the book, and we’ll do quite a few examples on the blackboard, and if there’s time, we’ll sketch at least part of the proof of Kleene’s theorem.