Logistics:

My email address: florian@linguist.umass.edu

Feel free to contact me with questions!

Office Hours: Monday, 9/12, 3:45 – 4:45
South College, 317 (Prof. Partee’s office)

Let me know if that time doesn’t work for you, and we can make an appointment!

Homework: If we forget to list what exercises are no the homework, you can always check on the course website: http://www.people.umass.edu/partee/409/

Review:

- Set: abstract collection of objects (concrete or abstract)
- Identity: same members
- Cardinality: number of members in the set
- Membership vs. Subset relation
- Power sets (where does the name come from?)

See also the ‘Cheat Sheet’ on the reverse side
# Set Theory Cheat Sheet

Some basic symbols and what they mean:

- $x \in A$: $x$ is a member of the set $A$
- $x \notin A$: $x$ is not a member of the set $A$
- $\emptyset$: the empty set
- $\mathcal{P}(A)$: The Power Set of $A$ – the set containing all subsets of $A$
- ‘/’: Used in various symbols expressing relations to say that the relation does not hold [e.g. $\notin$, $\not\subset$, etc.]

## Venn Diagram Schema

![Venn Diagram](image)

## Operation / Relation & Symbol

<table>
<thead>
<tr>
<th>Operation / Relation &amp; Symbol</th>
<th>Formal description</th>
<th>Informal description</th>
<th>Areas to be shaded in Venn-diagram</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Union: $A \cup B$</td>
<td>${x \mid x \in A \text{ or } x \in B}$</td>
<td>The set of all objects that are in $A$ or in $B$ (or in both)</td>
<td>1, 2, 3</td>
<td>${a, b, c} \cup {b, c, d} = {a, b, c, d}$</td>
</tr>
<tr>
<td>Intersection: $A \cap B$</td>
<td>${x \mid x \in A \text{ and } x \in B}$</td>
<td>The set of all objects that are in $A$ and in $B$</td>
<td>1</td>
<td>${a, b, c} \cap {b, c, d} = {b, c}$</td>
</tr>
<tr>
<td>Difference: $A – B$</td>
<td>${x \mid x \in A \text{ and } x \notin B}$</td>
<td>The set of all objects that are in $A$ but not in $B$</td>
<td>2</td>
<td>${a, b, c} – {b, c, d} = {a}$</td>
</tr>
<tr>
<td>Complement: $A'$</td>
<td>${x \mid x \notin A}$</td>
<td>The set of all objects that are not in $A$</td>
<td>3, 4</td>
<td>${a, b, c}' = {d, e}$ [assuming that $U = {a, b, c, d, e}$]</td>
</tr>
<tr>
<td>Subset Relation: $A \subseteq B$</td>
<td>$\text{All members of } A \text{ are members of } B$</td>
<td>True if 2 is empty</td>
<td>${a, b, c} \subseteq {a, b, c, d}$</td>
<td></td>
</tr>
<tr>
<td>Proper Subset: $A \subset B$</td>
<td>$\text{All members of } A \text{ are members of } B \text{ and there are members in } B \text{ that are not in } A$</td>
<td>True if a) 2 is empty b) 3 is not empty</td>
<td>${a, b, c} \subset {a, b, c, d}$ But: ${a, b, c} \not\subset {a, b, c}$</td>
<td></td>
</tr>
</tbody>
</table>
1.8. Set-theoretic equalities

There are a number of general laws about sets which follow from the definitions of set-theoretic operations, subsets, etc. A useful selection of these is shown below. They are grouped under their traditional names. These equations hold for any sets $X, Y, Z$:

1. **Idempotent Laws**
   - (a) $X \cup X = X$
   - (b) $X \cap X = X$

2. **Commutative Laws**
   - (a) $X \cup Y = Y \cup X$
   - (b) $X \cap Y = Y \cap X$

3. **Associative Laws**
   - (a) $(X \cup Y) \cup Z = X \cup (Y \cup Z)$
   - (b) $(X \cap Y) \cap Z = X \cap (Y \cap Z)$

4. **Distributive Laws**
   - (a) $X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z)$
   - (b) $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$

5. **Identity Laws**
   - (a) $X \cup \emptyset = X$
   - (b) $X \cup U = U$
   - (c) $X \cap \emptyset = \emptyset$
   - (d) $X \cap U = X$

6. **Complement Laws**
   - (a) $X \cup X' = U$
   - (b) $(X')' = X$
   - (c) $X \cap X' = \emptyset$
   - (d) $X - Y = X \cap Y'$

7. **DeMorgan’s Laws**
   - (a) $(X \cup Y)' = X' \cap Y'$
   - (b) $(X \cap Y)' = X' \cup Y'$

8. **Consistency Principle**
   - (a) $X \subseteq Y$ iff $X \cup Y = Y$
   - (b) $X \subseteq Y$ iff $X \cap Y = X$

We will see later that operations on subsets of a set form a Boolean algebra.

Another Boolean algebra familiar to you is arithmetics. Some of the laws above also hold there, in particular 2, 3, and 4 (in one direction).

How about the others?
Idempotency does not hold: $x + x \neq x$ (unless $x = 0$)

Identity does hold to some extent: $x + 0 = x$, $x * 0 = 0$, and $x * 1 = 1$
   But not: $x + 1 = 1$…

The laws above can be helpful in simplifying expressions denoting sets. Let’s simplify the following expression together: $(A \cup B) \cup (B \cap C)'$. 