Lecture 4. Semantic Types and Type-shifting.

1. Linguistic background:

1.1. Categorial grammar and syntax-semantics correspondence: the centrality of function-argument application

Synt. cat.	**Abbrev.**	**Sem. type (extensionalized)**	**Expressions**
e	e	e	*names (*John*)
t	t	t	sentences
t/e	IV	<e,t>	verb phrases (*runs*)
t//e	CN	<e,t>	common noun phrases (*cat*)
t/IV	T (or NP)	<<e,t>,t>	term phrases as generalized quantifiers (*John*, *every man*)
IV/e	TV1	<e,<e,t>>	*simple transitive verbs (*kicks*)
IV/T	TV2	<type(T),type (IV)>	transitive verbs (*kicks*, *seeks*)

*: not in PTQ

1.2. Tension between simplicity and generality, between uniformity and flexiblity

Example: Natural language NP's (noun phrases)

John, *every man* both NP's. Same type?

Montague: Yes: all NP's type <<e,t>,t>.

John: $\lambda P.P(j)$

every man: $\lambda P.\forall x[man(x) \rightarrow P(x)]$

$$\left\{\begin{array}{l} \textit{John} \\ \textit{every man} \\ \textit{no man} \end{array}\right\} + \textit{walks}$$

t/IV IV $\Rightarrow$ t

Montague's category-to-type correspondence: uniform and general, not "simple" (generalized to highest types ever needed), not flexible.

2. Conjunction and Type Ambiguity (from Partee & Rooth, 1983)

Structure of empirical argument: from cross-linguistic uniformity of generalized conjunction and elegance of its recursive definition, take its semantics as established. From that we get evidence for non-uniform typing of English transitive verb phrases and for type-shifting rules to shift simpler types to higher types by coercion as opposed to Montague's uniform typing at higher types.

2.0. To be explained: the cross-categorial distribution and meaning of 'and', 'or'.

With limited exceptions, it is apparently a linguistic universal that every major category can be conjoined with *and* and *or*. Partee and Rooth (1983) addressed the question of whether we could give a single meaning for *and* and a single meaning for *or* that covers their uses across the full range of categories. The core of that explanation has proven robust, and the semantics of cross-categorial conjunction now serves as one test in evaluating semantic proposals of various sorts.

We treat here only the central or "Boolean" *and*, whose core meaning is the meaning of ordinary logical conjunctio; examples are given in (1).

(1) (a) John and Mary are in Chicago.
(b) Bacon and eggs are (both) high in cholesterol.
(c) She was wearing a new and expensive dress.
(d) Cats purr, meow, and growl. Dogs bark and growl but they don't purr.
(e) Susan will retire and buy a farm.

Other uses which we do not treat are given in (2); these include the "group-forming" *and* of (2a-b), the "partly this and partly that" *and* of (2c-d), and the idiosyncratic *try and* construction of (2e). With the exception of the last of these, interesting proposals for further unification have emerged in more recent work that we will not discuss here: Krifka (19xx) gave an elegant unification of (2c-d) with (2a-b) based on a part-whole mereology, and Winter (1996, 1998) has shown a way to unify those with the Boolean *and* of (1).

(2) (a) John and Mary are a happy couple.
(b) Bacon and eggs is my favorite breakfast.
(c) She was wearing a blue and white dress.
(d) Can you rub your stomach and pat your head? [at the same time]
(e) Susan will try and sell her house.

- Early attempts to use syntactic transformations: "Conjunction-reduction"
- Derive (1a) from *John is in Chicago and Mary is in Chicago.*
- Implicit assumption: transformations are meaning-preserving; same meaning is to be captured by assigning same 'deep structure'.
- Downfall: *Every number is even or odd*; *Few rules are both explicit and easy to read.*
- Direction for cross-categorial unification of *and, or* suggested by Montague's (1973) treatment of conjunction of sentences, verb phrases, and noun phrases using the lambda-calculus. Main ideas for fully general recursive definition given by Gazdar (1980) and Keenan and Faltz (1978). Implications for type-shifting given by Partee and Rooth (1983).

2.1. Generalized conjunction

1. Conjoinable categories: S, NP, IV, TV, CNP, ADJP,...
2. Boolean *and* and *or* of basic type t, vs. "group-forming" *and* of basic type e.
3. Boolean *and, or* on type t: can be viewed in terms of truth tables, 2-element algebra, sets of possible worlds, or sets of assignment functions; all give familiar Boolean structure.
4. Recursive definition of **conjoinable types**:
   (i) t is a conjoinable type
   (ii) if b is a conjoinable type, then for all a, <a,b> is a conjoinable type.
5. Types for *and, or*: <X,<X,X>> for all conjoinable types X. (This is "curried" form, one-argument-at-a-time; in examples I will draw trees for uncurried form.)

6. Semantics for generalized *and* ($\sqcap$): pointwise lifting from codomain to function space.
   (i) for conjoinable type t, $\sqcap = \wedge$ (basic Boolean operation)
   (ii) for f_1, f_2 of conjoinable type <a,b>, $f_1 \sqcap f_2$ is defined by the condition
   $[f_1 \sqcap f_2](x) = f_1(x) \sqcap f_2(x)$.

7. Examples
   a. <e,t>: $walk' \sqcap talk' = \lambda x[walk'(x) \wedge talk'(x)]$
   b. <<e,t>,t>: $(every\ man)' \sqcap (some\ woman)' = \lambda P[(every\ man)'(P) \wedge (some\ woman)'(P)]$
   c. <<e,t>,<e,t>>: $old' \sqcap useless' = \lambda P[old'(P) \sqcap useless'(P)]$
   $= \lambda P[\lambda x[old'(P)(x) \wedge useless'(P)(x)]]$

2.2. Repercussions on the type theory: against uniformity, for "simplicity" and type-shifting

1. If the type of all transitive verbs (TV, or IV/NP) is, as Montague had it, <type(T), type(IV)>, then generalized conjunction predicts:
   $[\text{TVP1 and TVP2}] = \lambda \boldsymbol{P} \lambda x[\text{TVP'1}(\boldsymbol{P})(x) \wedge \text{TVP'2}(\boldsymbol{P})(x)]$

   -- Wrong result for:
   (1) John caught and ate a fish.
   (2) John hugged and kissed three women.
   -- Right result for:
   (3) John wants and needs two secretaries.
   (4) John needed and bought a new coat.

2. If the type of TV were <e, type(IV)>, then generalized conjunction would predict:
   $[\text{TVP1 and TVP2}] = \lambda y \lambda x[\text{TVP'1}(y)(x) \wedge \text{TVP'2}(y)(x)]$

-- Right for (1), (2), wrong for (3), (4)

-- Matches the first-order relations catch*, eat* predicted by Montague's meaning postulate for first-order-reducible transitive verbs.

2.3. Proposal:

(partly from Cooper, Dowty):

(i) Each verb entered lexically in its <u>minimal</u> type (to be defined) (give up Montague's strategy of putting <u>all</u> items of a given syntactic category in the "highest" type needed for <u>any</u> of them)

(ii) Each "low-type" verb has predictable homonyms of higher type. E.g. from *buy*$_1$ of type <e,<e,t>> predict *buy*$_2$ of type <type(T),<e,t>>:

$$buy_2' = \lambda \boldsymbol{P} \lambda x[\boldsymbol{P}(\lambda y[buy_1'(y)(x)])]$$

(iii) Conjoined expressions are interpreted at the lowest type they both have. Abbreviating as TV1 (*eat*, *buy*) and TV2 (*seek*, *need*, etc.), we have:

```
     TV1                   TV2                TV2
      9                     9                  9
 TV1  and  TV1        TV2  and  TV2      TV2  and  TV2
  |         |          |         |        |          |
 catch     eat        want      need     need       TV1
                                                     |
                                                    buy
```

(iv) This predicts all of (1)-(4) above correctly; (iii) may be taken as a "performance" strategy -- a natural "least effort" strategy.

(v) General form of above type-shifting operation. e-argument-functions to <e,t>,t>-argument-functions:

2.4. Parallel issues with intransitive verbs

a. IV as <e,t>: PTQ, Bennett, Partee (1975).

b. IV as <type(T),t>: UG, Keenan and Faltz, Gazdar, Bach and Partee (1980), Bach (1979)

c. Parallel differences in generalized conjunction;

-- lower type gives right result for

(5) A fish walked and talked.

(6) Every participant sent in an abstract or apologized.

-- higher type gives right result for

(7) An easy model theory textbook is badly needed and will surely be written within this decade (both high type)

(8) A tropical storm was expected to form off the coast of Florida and did form there within a few days of the forecast. (high type and low type)

d. Infinite ambiguity + 'least effort' principle.

2.5. General processing strategy:

"Use the simplest types consistent with coherent typing of entire sentence." Higher types invoked by "coercion": e.g. to conjoin *John and every woman*, *needed and bought*. There is in principle nothing wrong with infinite ambiguity if the system is designed to access higher types only when there is some reason to do so.

Query: What does it take to insure that such a system of flexible typing and type-shifting will always yield a unique "simplest" result? Under what conditions or by what measures does such a strategy offer greater overall simplicity than Montague's strategy of uniformly generalizing to the "hardest case"?

3. NP Type Multiplicity

3.1. Montague tradition:

Uniform treatment of NP's as generalized quantifiers, type (e→ t)→t.

John	$\lambda \boldsymbol{P}[\boldsymbol{P}(\mathbf{j})]$
a man	$\lambda \boldsymbol{P} \exists \boldsymbol{x}[\mathbf{man}(\boldsymbol{x}) \ \& \ \boldsymbol{P}(\boldsymbol{x})]$
every man	$\lambda \boldsymbol{P} \forall \boldsymbol{x}[\mathbf{man}(\boldsymbol{x}) \rightarrow \boldsymbol{P}(\boldsymbol{x})]$

Intuitive type multiplicity of NP's:

John	"referential use":	**j** (or **John)**	type e
a fool	"predicative use":	**fool**	type e→t
every man	"quantifier use":	as above	type (e→t)→ t

Resolution: All NP's have meaning of type (e→t)→t; some also have meanings of types e and/or e→ t. Find general principles for predicting these. Predicates may semantically take arguments of type e, e→t, or (e→t)→t, among others.

Type choice determined by a combination of factors including coercion by demands of predicates, "try simplest types first" strategy, and default preferences of particular determiners.

3.2. Evidence for multiple types for NP's.

Evidence for type e (Kamp-Heim): While any singular NP can bind a singular pronoun in its (c-command or f-command) domain, only an e-type NP can normally license a singular discourse pronoun.

(9) John /the man/ a man walked in. He looked tired.
(10) Every man /no man/ more than one man walked in. *He looked tired.

Evidence for type <e,t>: subcategorization for predicative arguments and conjoinablility of predicative NPs and APs in such positions.

(11) Mary considers John competent in semantics and an authority on unicorns.
(12) Mary considers that an island /two islands / many islands / the prettiest island / the harbor / *every island / *most islands / *this island / *?Hawaii / Utopia.

In general, the possibility of an NP having a predicative interpretation is predictable from the model-theoretic properties of its interpretation as a generalized quantifier; apparent counterexample (13) from Williams (1983) can be explained (see Partee (1987))

(13) This house has been *every color*.

3.3. Some type-shifting functors for NPs.

DIAGRAM 1

lift: $\mathbf{j} \rightarrow \boldsymbol{P}[\boldsymbol{P}(\mathbf{j})]$	total; injective
lower: maps a principal ultrafilter onto its generator	partial; surjective
lower (**lift** (**j**)) = **j**	
ident: $\mathbf{j} \rightarrow \boldsymbol{x}[\boldsymbol{x} = \mathbf{j}]$	total; injective
iota: $\boldsymbol{P} \rightarrow \iota \boldsymbol{x}[\boldsymbol{P}(\boldsymbol{x})]$	partial; surjective
iota(**ident**(**j**)) = **j**	
nom: $\boldsymbol{P} \rightarrow {}^{\cap}\boldsymbol{P}$ (Chierchia)	almost total; injective
pred: $\boldsymbol{x} \rightarrow {}^{\cup}\boldsymbol{x}$ (Chierchia)	partial;surjective
pred (**nom**(*P*)) = *P*	

3.4. "Naturalness" arguments: THE, A, and BE.

(14) **THE**: $\boldsymbol{Q} \Rightarrow \lambda \boldsymbol{P}[\exists \boldsymbol{x}[\forall \boldsymbol{y}[\boldsymbol{Q}(\boldsymbol{y}) \leftrightarrow \boldsymbol{y} = \boldsymbol{x}]] \,\&\, \boldsymbol{P}(\boldsymbol{x})]$

A: $\boldsymbol{Q} \Rightarrow \lambda \boldsymbol{P}[\exists \boldsymbol{x}[\boldsymbol{Q}(\boldsymbol{x}) \,\&\, \boldsymbol{P}(\boldsymbol{x})]$

BE: $\mathbf{P} \Rightarrow \lambda \boldsymbol{x}[\mathbf{P}(\lambda \boldsymbol{y}[\boldsymbol{y} = \boldsymbol{x}])]$ or $\lambda \boldsymbol{x}[\{\boldsymbol{x}\} \in \mathbf{P}]$

3.4.1 THE

The argument offered in Partee (1987) for the naturalness of **THE** comes largely from considering the interpretations of definite singular NPs like "the king" in all three types. I will not go through the argument here in detail, but will just summarize the main points with the aid of Diagram 2.

Diagram 2

[Solid lines indicate total functions, dotted lines partial ones.]

Iota and **THE** are related to each other by the fact that whenever **iota** is defined, i.e. whenever there is one and only one king, **lift** (**iota** (**king**)) = **THE** (**king**) and **lower** (**THE** (**king**)) = **iota** (**king**), and furthermore whenever **iota** is not defined, **THE** (**king**) is vacuous in that it denotes the empty set of properties.

(15) **<u>Proposal about BE</u>**: **BE** is not the meaning of English *be* but rather a type-shifting functor that is applied to the generalized quantifier meaning of an NP whenever we find the NP is an <e,t> position.

(16) **<u>Proposal about *be*</u>**: (following Williams (1983)) The English *be* subcategorizes semantically for an e argument and an <e,t> argument, and has as its meaning "apply predicate", i.e. $\lambda \boldsymbol{P} \lambda \boldsymbol{x}[\boldsymbol{P}(\boldsymbol{x})]$.

Then the predicative reading of *the king* is as given in (17).

(17) **Predicative reading of *the king*: BE**(**THE**(**king**))

In terms of logical formulas, **BE**(**THE**(**king**)) works out to be $\lambda \boldsymbol{x}[\mathbf{king}(\boldsymbol{x}) \,\&\, \forall \boldsymbol{y}[\mathbf{king}(\boldsymbol{y}) \leftrightarrow \boldsymbol{y} = \boldsymbol{x}]]$, or equivalently, $\lambda \boldsymbol{x} \exists \boldsymbol{y}[\mathbf{king}(\boldsymbol{x}) \rightarrow \boldsymbol{x} = \boldsymbol{y}]]$. This gives the singleton set of the unique king if there is one, the empty set otherwise. It is always defined, so the predicative reading also requires no presuppositions.

Note that if there is <u>at most</u> one king, then **king** = **BE**(**THE**(**king**))

(18) (a) John is {the president / president}
(b) John is {the teacher / *teacher}

The double-headed arrow on the **ident** mapping in Diagram 2 reflects the fact that for **iota** to be defined there must be one and only one king, hence **king** = **BE**(**THE**(**king**)) = **ident**(**iota**(**king**)). In fact, when iota is defined, the diagram is fully commutative: **king** = **BE**(**THE**(**king**)) = **ident**(**iota**(**king**)) = **ident**(**lower**(**THE**(**king**))) = **BE**(**lift**(**iota**(**king**))), etc. This property of the mappings lends some formal support to the idea that there is a unity among the three meanings of *the king* in spite of the difference in type.

3.4.2 A and BE

Let **A** be the categorematic version of Montague's treatment of *a/an*: in IL terms, $\lambda \boldsymbol{Q}[\ \lambda \boldsymbol{P}[\ \exists \boldsymbol{x}[\boldsymbol{Q}(\boldsymbol{x})\ \&\ \boldsymbol{P}(\boldsymbol{x})]]$. If we focus first on the naturalness of **BE**, we can then argue that **A** is natural in part by virtue of being an inverse of **BE**. The operation **BE** has some very nice formal properties that are summarized in (19) and (20) below.

(19) Fact 1: **BE** is a homomorphism from <<e,t>,t> to <e,t> viewed as Boolean structures, i.e:

$$\mathbf{BE}(\ \boldsymbol{P}_1 \cap \boldsymbol{P}_2) = \mathbf{BE}(\boldsymbol{P}_1) \cap \mathbf{BE}(\boldsymbol{P}_2)$$
$$\mathbf{BE}(\ \boldsymbol{P}_1 \cup \boldsymbol{P}_2) = \mathbf{BE}(\boldsymbol{P}_1) \cup \mathbf{BE}(\boldsymbol{P}_2)$$
$$\mathbf{BE}(\ \neg \boldsymbol{P}_1) = \neg \mathbf{BE}(\boldsymbol{P}_1)$$

(20) Fact 2: **BE** is the unique homomorphism *h* that makes Diagram 3 commute.

DIAGRAM 3

Now what exactly does **BE** do? We can write an expression equivalent to Montague's IL interpretation of English *be* but in set- theoretical terms as follows: $\lambda \mathbf{P} \lambda \boldsymbol{x}[\{\boldsymbol{x}\} \in \mathbf{P}]$. That is, it applies to a generalized quantifier, finds all the singletons therein, and collects their elements into a set. The commutativity of Diagram III is then straightforward. So **BE** is indeed a particularly nice, structure-preserving mapping from <<e,t>,t> to <e,t>.

(21) (MG) **be**(**TR**(*a man*)) $= \mathbf{be}\ (\lambda \boldsymbol{P} \exists \boldsymbol{x}[\mathbf{man}(\boldsymbol{x})\ \&\ \boldsymbol{P}(\boldsymbol{x})]]\ )$
$= \lambda \boldsymbol{x}[\mathbf{man}(\boldsymbol{x})]$
$= \mathbf{man}$
(MG) **be**(**TR**(*John*)) $= \mathbf{be}\ (\lambda \boldsymbol{P}\ \boldsymbol{P}(\mathbf{j})\ )$

$= \lambda x[x=\mathbf{j}]$
(MG) **be**(**TR**(*no man*)) $= \lambda x[\neg \mathbf{man}(x)]$
(MG) **be**(**TR**(*every man*)) $= \lambda x[\forall y[\mathbf{man}(y) \rightarrow y=x]]$

Now, having given some grounds for claiming that **BE** is a "natural" type-shifting functor, we can use that to support the naturalness of **A**, since it turns out that **A** is an inverse of **BE** in that $\mathbf{BE}(\mathbf{A}(P) = P$ for all P.

I would conjecture, in fact, that among all possible DET-type functors, **A** (which combines English *a* and *some*) and **THE** are the most "natural" and hence the most likely to operate syncategorematically in natural languages, or not to be expresses at all, and that **BE** is the most "natural" functor from <<e,t>,t> meanings to <e,t> meanings.

3.5. Further Discussion Topics.

If time remains, we may want to discuss some further topics related to type shifting and coercion, including the shiftability of weak NPs to predicate readings, the various possibilities for *two, three,* etc., and more about CN's and TCNs, all topics which are relevant to the upcoming discussion of genitives.

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HOMEWORK #4

From the questions below plus any questions from the previous homeworks that you haven't done yet, choose two or three that suit your level and your interests. Feel free to modify the problems if you wish, and/or to work on or discuss near-equivalents or related constructions in a language other than English.

1. a. Look back at Fragment 1 in Lecture 2. At the end of Section **1.3** is a translation of *is* a_{pred} *man*. Work out how that translation results from compositional semantic rules plus simplification via lambda-conversion. [You may have already done this in Homework 2.]

b. Show how the same translation could be achieved by first interpreting *a man* as a generalized quantifier, and then applying the **BE** type-shifting operator to shift the interpretation to type <e,t> (and simplifying the result by lambda-conversion); the remainder of the derivation (combining with the *be* of Lecture 2) would be the same, so you needn't repeat it.

2. (advanced) Explain, in the context of the Partee and Rooth article, why *most men and women* and *every man and woman* present a problem. What reading is predicted by the proposed scheme for generalized conjunction, given the normal CN (N-bar) type?

Optional: Partee and Rooth mention a proposal by Cooper for deriving the right reading by type-shifting the CNs to function types which would take the DET as argument. Is that proposal consistent with type-driven translation?

[The following is a note to this problem but not really part of it. Can you think of any other approach to this problem? (Additional references, totally optional: Merrie Bergmann in *Linguistics & Philosophy* in the mid-80's sometime, and especially Manfred Krifka on Non-Boolean conjunction. Krifka's approach is based on part-whole relations, extending the idea about e-conjoinable types discussed in Appendix B of Partee and Rooth to cases involving part-whole relations among plural or mass entities, and events, and accounting very nicely for the ambiguity of *green and yellow flags*.)

3. (advanced) Make a first attempt to think about the type(s) to be associated with partitive *of*-phrases, numerals, and determiners. (For some background on plurals, see Link (1983), also summarized briefly in Bach (1986), *The algebra of events*, L&P 9, 5-16.) Think about the syntactic categories and semantic types that might be proposed to achieve a compositional interpretation of the following expressions, and what sorts of type-assignment and type-shifting principles might be able to predict such results. (This does not have a single clear-cut answer by any means.)

six horses
six of the horses
the six horses
six of the ten horses

Assume that the whole NPs (or DPs) should end up either as type *e* or as type *ett* (that's my shorthand for <<*e,t*>,*t*>), and suppose that the lexical CN (or N-bar) is always of type *et*. There are some relevant suggestions in Partee (1987). See especially the mappings *link* and *delink* in 3.4.2 of Partee (1987) and the interpretations of *three* as <e,t>, as <<e,t>,<e,t>>, and as a DET (type(s)?).

Partial answers to Homework #4.

Question 1b: **TR**(a_{GQ} *man*) = $\lambda \boldsymbol{P}[\exists x[\mathbf{man}(x)\ \&\ \boldsymbol{P}(x)]]$ (From Lecture 2)
BE: $\mathbf{P} \Rightarrow \lambda x[\ \mathbf{P}(\lambda y[y = x])]$ (From this lecture; **P** is of type <<e,t>,t>)

So the first step of working out the result of applying the type-shifter **BE** to **TR**(a_{GQ} *man*) is to take **TR**(a_{GQ} *man*) as the input **P**. To avoid clash of variables, let's replace x in the input by z, yielding $\lambda \boldsymbol{P}[\exists z[\mathbf{man}(z)\ \&\ \boldsymbol{P}(z)]]$. Then as output, before lambda-conversion, we get:

$\lambda x[\lambda \boldsymbol{P}[\exists z[\mathbf{man}(z)\ \&\ \boldsymbol{P}(z)]](\lambda y[y = x])]$

Now we can apply λ-conversion with the operator $\lambda \boldsymbol{P}$ and the argument ($\lambda y[y = x]$). (To be sure that they are in the right configuration for λ-conversion, draw a syntactic tree for the whole expression and check types at each node.) The result will be:

$\lambda x[\exists z[\mathbf{man}(z)\ \&\ \lambda y[y = x](z)]]$. Then another λ-conversion gets us to:
$\lambda x[\exists z[\mathbf{man}(z)\ \&\ z = x]]$. From there some ordinary logical reasoning (x has the property of being identical to some man iff x is a man) tells us that this is equivalent to:
$\lambda x[\mathbf{man}(x)]$
And this is the same as **TR**(a_{pred} *man*), which is what we wanted to show.

Question 3: Note: The following notes were originally Partee's comments to a class about their answers to question 3 above; that explains remarks like "most popular choice", and other comments that don't fit the present context. But we are leaving it as is.
Possible types for:
NP (or DP) : e or ett
CN: et
six: et or <et,et> or <et, ett> or maybe in a Heim-like analysis <et,e>
Most popular choice: <et,et>
of the horses (PP or PartitiveP): <et> : this was by far your most popular choice, though it doesn't match what I did at the blackboard. I treated it as if it were another <et,et>, needing a "phantom CN" to combine with. But if as most of you suggested, and as Barwise and Cooper had also suggested, we make *of the horses* the same type as a CN, then of course there is no need for a "phantom CN" in a construction like *six of the horses*.
of: <e,et> or <ett,et>
the: <et, e> or <et, ett>

Points of agreement: (i) the partitive phrase *of the horses* or *of the six horses* should get the same type as a CN; that way *six* can combine with it the same way it can combine with a CN. (Issue not raised: what causes the distribution of partitive phrases to be different from the distribution of CNs? Something about their syntax, or some further semantic distinctions, or both?)

(ii) so *of* maps a term phrase meaning onto a set-type meaning, and its semantics should say that the resulting set is a subset of the set *corresponding to* the term phrase. We talked about how the Linkian view of definite plurals could help with that; see also the discussion of type-shifting operations *link* and *delink* in Partee (1987).

I didn't ask you to try to write down actual meanings of the given types, but some of you did, and I did some of that at the board yesterday. A meaning for *of the horses* consistent with the notes above could be:

ëX[X **sup**(*horsesN*)] , where **sup** *(supremum*) is Link's treatment of the definite article, mapping a set of plural entities onto its supremum element. X above is a variable over plural entities; it is a variable of type e. So both arguments of the *part-of* relation are of type e.

Points of variation: The greatest variation, as I expected, concerned the treatment of *six*: sometimes it seems like an adjective, sometimes like a determiner, and there is no single type that would let it be both. The four types listed above represent *adj-like* vs. *Det-like,* plus our two discussed possibilities for adjectives, and two possibilities for Det's reflecting the two possible resultant types for the DP. Possibilities (some of you raised more than one):

(a) *six* is always of type <et,et>. Whether it combines with a CNP or a PartitiveP (both type et), the result is of type et. Then a Det can combine with it to give an e or ett DP; if there is no overt Det, and yet the result can be used as a complete DP, then either we assume some covert Det, or we invoke some type-shifting operation to turn the et-type expression into an e or ett-type expression (the effect is the same as assuming a covert Det). (This is a proposal that could be considered for all sorts of determinerless NPs.)

(b) Same story but let *six* be itself of type et. I don't think anyone suggested it, but since *six* seems to be intersective, it could start out as et. (As a predicate, its meaning is *has cardinality 6*). Then it could combine with CNP or PartitiveP by predicate conjunction, same result as above.

(c) Basic type of *six* is <et,et>, but it can shift to Det type. On this account, it might be in different syntactic positions with different associated semantic types in *six horses* vs. *the six horses*.

(d) Basic type of *six* is Det type: <et,e> or <et,ett>, but it can shift to type <et,et>. I suspect this alternative would be harder to work out. The determiner meaning for *six* is presumably something like function composition of the meaning of the indefinite determiner with the meaning of adjectival *six*. If we started from that meaning, I'm not sure what kind of shifting rule could *subtract out* the determiner meaning; but there may well be a way to do it. Oh, yes, on second thought, I'm sure it could be done, using an idea that Ania had, making use of the type-shifting operator BE to map an ett meaning to an et meaning. On Ania's proposal, you leave the type of *six* alone, but if it gives you a resulting ett expression in a place where you want an et expression, you use BE to turn the ett expression into an et expression. (The category NP on that proposal is one which can be either et or ett; it's distinct from DP, which can only be e or ett.) That use of BE could be packed into a more complex type shifting rule to shift *six* itself.

To my mind, this alternative is an interesting example of what *could be done*; intuitively I don't think it's as nice as the others, but the issue is whether we can find arguments against it.