

# The Tail in the Volatility Index

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## **Abstract**

Both volatility and the tail of the stock return distribution are impacted by discontinuities (“large jumps”) in the stock price process. In this paper, we construct model-free volatility and tail indexes in a manner that clearly distinguishes one from the other. Our tail index measures time-variation in jump intensity, and is constructed non-parametrically from the identical set of 30-day index options used to construct a volatility index. We use the indexes to examine the relative economic importance of volatility and tail in predicting market returns over 1996-2009. Over this period, our predictability regressions indicate that the first order impact of jumps is through stock return volatility rather than the tail of the distribution.

*Keywords:* index option, VIX, quadratic variation, tail risk, return predictability

*JEL classification:* G1, G12, G13

# 1 Introduction

Understanding time variation in volatility is important in asset pricing as it impacts pricing of both equities and options through the equity and variance risk premia, respectively.

More recently, there has been interest in whether tail risk - the possibility of an extreme return - should be considered an additional channel of risk and risk premia. Gabaix (2008) argues that a time-varying probability of disaster may explain a range of asset pricing puzzles. But discontinuities - large jumps - in the stock price process impact both the risk of extreme returns and stock return variability. Before one can understand the potentially distinct roles of stock return variability and tail risk, it is essential to differentiate one from the other.

In this paper, we demonstrate how a volatility and a jump/tail index, respectively, can be constructed in a model-free manner from the same set of option prices. Using the identical set of options to measure time-variation in volatility and tail risk is useful as it allows researchers to clearly distinguish between the two channels. We use the indexes to examine the relative economic importance of volatility and tail risk in predicting equity returns using the setup of Bollerslev, Tauchen and Zhou (2009).

There already exist measures of tail risk and volatility in the literature. By far the most commonly used option-based measure of stock return variability is the Chicago Board Options Exchange's VIX index. However, as we explain below, the VIX is *not* model-free. It is biased in the presence of discontinuities which makes it difficult to distinguish correctly between volatility and tail risk. Bollerslev and Todorov (2010) and Kelly (2010) propose measures of tail risk. These measures rely on the notion that tail risk can be inferred from either observation of extreme returns (Kelly, 2010) or the pricing of extreme events by short maturity deep out-of-the-money options (Bollerslev and Todorov, 2010).

Our approach to measuring tail risk is different and new. Instead of focusing on extreme returns, we consider the impact of discontinuities on stock return variability. We show that a jump and tail index measuring time-variation in jump intensity can be constructed from the differential impact of discontinuities on two measures of stock return variability - the Bakshi, Kapadia and Madan (2003) measure of the variance of the holding period return and the VIX ("integrated variance") index. The advantage of our approach is that the jump and tail index is directly related to higher ( $n \geq 3$ ) order moments of the jump distribution. Therefore, even though tail and volatility are each impacted by discontinuities, our volatility and tail indexes measure distinct statistics. Importantly, time-variation in volatility and tail can be distinguished even though jumps impact the former and determine the latter.

The VIX index plays a central role in our analysis because its accuracy in measuring quadratic variation - a measure of stock return variability - deteriorates rapidly as the contribution of jumps to stock return variability increases. Technically, the VIX index is constructed to measure the integrated variance, i.e., the quadratic variation of a strictly continuous stock return process.<sup>1</sup> When there are discontinuities in the stock price process, the VIX is a biased estimator of quadratic variation. Building on prior analysis by Carr and Wu (2009), we show that the bias is proportional to the jump intensity. Because the time variation in jump intensity determines the time variation in tail risk, a jump - and, therefore, tail - index can be created based on how *inaccurately* the VIX measures quadratic variation.

But to construct a jump and tail index based on the inaccuracy of the VIX, it is necessary to have a more accurate measure of stock return variability. How should a risk-neutral model-free measure of quadratic variation be constructed from option prices that is more accurate than the VIX in the presence of jumps? This question is also of independent interest.

Consider two commonly used measures of stock return variability - the quadratic variation and the variance of the holding period log-return, respectively. In general, these two measures of variability are related but different as the former measures the sum of squared log-returns, while the latter relates to the square of summed log-returns. Given that the VIX is constructed to measure quadratic variation of a continuous process, it would appear reasonable to conclude that the VIX is also the most accurate non-parametric measure of quadratic variation when there are discontinuities.<sup>2</sup> We show that such a conclusion would be incorrect.

Indeed, we demonstrate that the variance of the holding period log-return is a more accurate measure of quadratic variation for a wide range of models when jumps contribute 20% or more to total volatility. Moreover, it is almost as accurate when jumps contribute less to the quadratic variation. The variance of the holding period return can be estimated model-free from option prices as shown by Bakshi, Kapadia and Madan (2003) (BKM, henceforth). Not surprisingly, the BKM variance measure is different from the integrated variance as it correctly accounts for the higher order impact of jumps. We are the first to observe that the BKM variance can measure the quadratic variation more accurately than the VIX.

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<sup>1</sup>The VIX was constructed to measure the integrated variance using the log-contract based on the analysis of Carr and Madan (1998), Demeterfi, Derman, Kamal and Zou (1999a, 1999b), Britten-Jones and Neuberger (2000) and earlier work by Neuberger (1994) and Dupire (1996).

<sup>2</sup>Jiang and Tian (2005) argue in their Proposition 1 that the VIX is an accurate measure of quadratic variation. Carr and Wu (2009) show that it is biased, but provide numerical simulations that suggest that the bias is negligible. Consequently, a number of recent papers have used the VIX as a “model-free” measure of stock return variability; see Bollerslev, Gibson and Zhou (2011), Drechsler and Yaron (2011), Wu (2011), and Bakshi, Panayotov, and Skoulakis (2011) amongst others.

We proceed to empirically estimate the volatility indexes and the jump and tail index from S&P 500 index option data over 1996-2009. We also construct a jump and tail index from individual options.

We document significant time-variation in jump risk in the index. In recessions, the jump index for the market is five-fold that in non-recessions. At the peak of the financial crisis, fears of jumps in the market are an extraordinary 50-fold those of the median month. For individual stocks, the jump and tail index demonstrates the central role of financial firms in the recent crisis. Tail risk, which shows little difference between financial and non-financial firms for much of the decade, starts increasing for financial firms from August of 2007. Tail risk in non-financial firms increases only after the bankruptcy of Lehman Brothers.

Jump risk has an economically significant impact on stock return variability. Over the entire sample period, on average, the integrated volatility is biased downwards relative to the BKM volatility measure by an annualized 0.49%. The negative bias is more severe in recessions and times of crisis. In the recent financial crisis, we estimate that the integrated variance measure underestimates stock return variability by 12%.

Using the volatility indexes and the tail index, we next examine the two potential channels of risk and risk premia through predictability regressions following Bollerslev, Tauchen and Zhou (2009). They demonstrate that the “variance risk premium” (VRP), defined as the difference between the VIX and the realized variance, predicts short-horizon index returns over horizons of less than one year. If jumps are of economic importance for stock return variability, then the variance risk premium based on the BKM variance should be of greater statistical and economic significance. This is precisely what we find. Accounting for jumps increases the predicted return over a one-year horizon by the VRP by over 40%.

Finally, we consider if the jump-induced tail risk has an economic role to play in *addition* to the contribution of jumps to stock return variability. Once the impact of jumps on volatility is accounted for by including a model-free measure of stock return variability, we find little evidence that there remains much additional variability in returns that can be further explained by jump risk. The first-order impact of jump risk is through stock return volatility rather than the extreme tail of the stock return distribution.

Our analysis relates to a number of recent papers. The motivation for studying tail risk for risk premia is first provided by Rietz (1988), and further elaborated by Barro (2006). Gabaix (2010) argues that, in addition, it is important to allow for time-variation in disaster risk. Our tail index clearly demonstrates that jump intensity is time-varying and that tail risk increases

dramatically during times of crisis.

Empirical evidence that jump risk in asset prices is important for modeling option prices is in Bakshi, Cao and Chen (1997), Bates (2000), Pan (2002), and others. Carr and Wu (2003) and Bollerslev and Todorov (2010) both infer jump risk from option data as we do. While both these papers focus on short-dated deep out-of-the-money options, we infer jump and tail risk from the entire continuum of option strikes.

That index returns are predictable using information from option data is observed in Bollerslev, Tauchen and Zhou (2009), Drechsler and Yaron (2011), and Bakshi, Panayotov and Skoulakis (2011). Bali and Hovakimian (2009) use equity option data to study cross-sectional variation in individual stock returns. Our predictability regressions refine and strengthen the finding of Bollerslev, Tauchen and Zhou (2009) and Drechsler and Yaron (2011) that the VRP constructed using the VIX predicts excess returns. The VRP constructed using the BKM variance has even greater statistical and economic significance than demonstrated by them, underscoring the role of the variance risk premium in predicting short horizon returns.<sup>3</sup>

Kelly (2010) demonstrates that a tail risk measure constructed from cross-sectional dispersion of individual stock returns predicts index returns. Bali, Cakici and Whitelaw (2011) show that past extreme stock returns predict cross-sectional variation in individual stock returns.

Empirical evidence that jump risk contributes a significant proportion to the variability of observed stock log-returns under the physical measure is in Ait-Sahalia and Jacod (2008, 2009a, 2009b, 2010), Lee and Mykland (2008), and Lee and Hannig (2010). Broadie and Jain (2008), Carr, Lee and Wu (2011) and Cont and Kokholm (2010) observe that jumps can significantly bias the integrated variance measure of stock return variability.

The rest of the paper is as follows. Section 2 provides an illustration of our approach using the Merton jump diffusion model. Section 3 contains our primary theoretical results. Section 4 delineates the empirical questions which we address in following sections. Section 5 describes the data. Sections 6 and 7 contain our empirical results. The last section concludes.

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<sup>3</sup>The economic importance of the variance risk premium in option markets is noted in Bakshi and Kapadia (2003) and Carr and Wu (2009).

## 2 An Illustration: Quadratic Variation, Variance of the Holding Period Return, the VIX, and Tail Risk for the Merton Jump-Diffusion Model

For the Merton jump-diffusion model, we demonstrate (i) how quadratic variation can be estimated using the variance of the holding period return, and (ii) how a tail measure can be constructed from the difference between the more precise measure of quadratic variation and the “integrated variance” measure of stock return variability used to construct the VIX.

Let the stock price  $S_T$  at time  $T$  be specified by a jump-diffusion model under the risk-neutral measure  $\mathbb{Q}$ ,

$$S_T = S_0 + \int_0^T (r - \lambda\mu_J)S_{t-} dt + \int_0^T \sigma S_{t-} dW_t + \int_0^T \int_{\mathbb{R}^0} S_{t-} (e^x - 1) \mu(dx, dt), \quad (1)$$

where  $r$  is the constant risk-free rate,  $\sigma$  is the volatility,  $W_t$  is standard Brownian motion,  $\mathbb{R}^0$  is the real line excluding zero, and  $\mu(dx, dt)$  is the Poisson random measure for the compound Poisson process with compensator equal to  $\lambda \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}(x-\alpha)^2}$  with  $\lambda$  as the jump intensity.  $\mu_J = e^{\alpha + \frac{1}{2}\sigma^2} - 1$  ensures that the discounted stock price remains a martingale. From Ito’s lemma, the log of the stock price is,

$$\ln S_T = \ln S_0 + \int_0^T \frac{1}{S_{t-}} dS_t - \int_0^T \frac{1}{2} \sigma^2 dt + \int_0^T \int_{\mathbb{R}^0} (1 + x - e^x) \mu(dx, dt), \quad (2)$$

$$= \ln S_0 + \int_0^T (r - \frac{1}{2}\sigma^2 - \lambda\mu_J) dt + \int_0^T \sigma dW_t + \int_0^T \int_{\mathbb{R}^0} x \mu(dx, dt). \quad (3)$$

Let the quadratic variation of the log stock process over the period  $[0, T]$  be denoted as  $[\ln S, \ln S]_T$ . The quadratic variation will be the sum of the quadratic variation of the continuous portion (“integrated variance”) and the discontinuous portion of the stochastic process (e.g., Cont and Tankov, 2003),

$$[\ln S, \ln S]_T = \int_0^T \sigma^2 dt + \int_0^T \int_{\mathbb{R}^0} x^2 \mu(dx, dt). \quad (4)$$

The objective is to compute  $E^{\mathbb{Q}}[\ln S, \ln S]_T$  from option prices.

Consider the stochastic process for the square of the log return,  $\ln^2 S_t/S_0$ . Applying the

(generalized) Ito's lemma to equation (3),

$$\ln^2(S_T/S_0) = \int_0^T 2 \ln(S_{t-}/S_0) d \ln S_t + \int_0^T \sigma^2 dt + \int_0^T \int_{\mathbb{R}^0} x^2 \mu(dx, dt). \quad (5)$$

Taking expectations and rearranging using the definition of quadratic variation in equation (4), we get

$$\mathbb{E}^{\mathbb{Q}} [\ln S, \ln S]_T = \mathbb{E}^{\mathbb{Q}} \ln^2(S_T/S_0) - \mathbb{E}^{\mathbb{Q}} \int_0^T 2 \ln(S_{t-}/S_0) d \ln S_t. \quad (6)$$

To evaluate the integral, first observe from equation (3),

$$\mathbb{E}^{\mathbb{Q}} \ln(S_{t-}/S_0) = (r - \frac{1}{2}\sigma^2 - \lambda\mu_J)t + \lambda\alpha t, \quad (7)$$

Therefore,

$$\begin{aligned} \mathbb{E}^{\mathbb{Q}} \int_0^T 2 \ln(S_{t-}/S_0) d \ln S_t &= 2 \mathbb{E}^{\mathbb{Q}} \left[ \int_0^T \ln(S_{t-}/S_0) (r - \frac{1}{2}\sigma^2 - \lambda\mu_J) dt + \int_0^T \int_{\mathbb{R}^0} \ln(S_{t-}/S_0) x \mu(dx, dt) \right], \\ &= 2 \left[ \int_0^T \mathbb{E}^{\mathbb{Q}} (\ln(S_{t-}/S_0)) (r - \frac{1}{2}\sigma^2 - \lambda\mu_J) dt \right. \\ &\quad \left. + \left[ \int_0^T \int_{\mathbb{R}^0} \mathbb{E}^{\mathbb{Q}} (\ln(S_{t-}/S_0)) x \frac{\lambda}{\sqrt{2\pi\sigma_J^2}} e^{-\frac{(x-\alpha)^2}{2}} dx dt \right] \right], \\ &= \left( (r - \frac{1}{2}\sigma^2 - \lambda\mu_J) + \lambda\alpha \right)^2 T^2, \end{aligned} \quad (8)$$

where the the last equality is the end result of evaluating the integrals after substituting for  $\mathbb{E}^{\mathbb{Q}} \ln(S_{t-}/S_0)$  using equation (7). From equations (6), (7), and (8), it follows

$$\mathbb{E}^{\mathbb{Q}} [\ln S, \ln S]_T = \mathbb{E}^{\mathbb{Q}} \ln^2(S_T/S_0) - \left( \mathbb{E}^{\mathbb{Q}} (\ln S_T/S_0) \right)^2. \quad (9)$$

$$= \text{var}^{\mathbb{Q}} (\ln S_T/S_0) \quad (10)$$

That is, the quadratic variation over a horizon  $T$  is equal to the variance of the log-return over the same horizon. Proposition 1 of Bakshi, Kapadia and Madan (2003) (BKM) demonstrates how the variance can be estimated model-free from option prices.

Next, with some abuse of notation, denote  $\mathbb{E}^{\mathbb{Q}} [\ln S, \ln S]_T^c$  as the *estimate* of the expectation of integrated variance. Carr and Madan (1998), Demeterfi, Derman, Kamal and Zou (1999a, 1999b) and Britten-Jones and Neuberger (2000) note that for a purely continuous process,

$$\mathbb{E}^{\mathbb{Q}} [\ln S, \ln S]_T^c = \mathbb{E}^{\mathbb{Q}} \int_0^T \sigma^2 dt = 2 \mathbb{E}^{\mathbb{Q}} \left[ \int_0^T \frac{1}{S_t} dS_t - \ln S_T/S_0 \right], \quad (11)$$

can be estimated from option data. But because of discontinuities in the jump-diffusion model, equation (2) implies,

$$2\mathbb{E}^{\mathbb{Q}} \left[ \int_0^T \frac{1}{S_{t-}} dS_t - \ln S_T/S_0 \right] = \mathbb{E}^{\mathbb{Q}} \int_0^T \sigma^2 dt - 2\mathbb{E}^{\mathbb{Q}} \int_0^T \int_{R^0} (1+x-e^x) \mu(dx, dt). \quad (12)$$

Adding and subtracting  $\int_0^T \int_{R^0} x^2 \mu(dx, dt)$  to (12) and combining with equation (10), we get,

$$\text{var}^{\mathbb{Q}}(\ln S_T/S_0) - 2\mathbb{E}^{\mathbb{Q}} \left[ \int_0^T \frac{1}{S_{t-}} dS_t - \ln S_T/S_0 \right] = 2\mathbb{E}^{\mathbb{Q}} \int_0^T \int_{R^0} \left( 1+x+\frac{x^2}{2}-e^x \right) \mu(dx, dt). \quad (13)$$

The difference between the variance of the holding period return (BKM) and the integrated variance measure (VIX) is determined solely by the discontinuities in the stock price process.<sup>4</sup>

The time variation in the LHS of equation (13) measures the time variation in jump and tail risk, and we propose it as a tail index. In the next section, we discuss the extent and limitation to which we can generalize and formalize this statement.

### 3 Measuring Stock Return Variability and Tail Risk

#### 3.1 Quadratic variation and Variance of the Holding Period Return

Let the log-stock price  $\ln S_t$  at time  $t$ ,  $t \geq 0$ , be a semimartingale defined over a filtered probability space  $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathbb{Q})$  with  $S_0 = 1$ . Denote the quadratic variation over a horizon  $T > 0$  as  $[\ln S, \ln S]_T$  and the variance of the holding period return as  $\text{var}^{\mathbb{Q}}(\ln S_T/S_0)$ .<sup>5</sup> We consider the relation between the two measures of stock return variability.

**Proposition 1** *Let  $E^{\mathbb{Q}}[\ln S, \ln S]_T$  and  $\text{var}^{\mathbb{Q}}(\ln S_T/S_0)$  be the expected quadratic variation and variance of the holding-period return, respectively, over a horizon  $T < \infty$ . Denote the difference*

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<sup>4</sup>Our analysis differs from Carr and Madan (1998), Demeterfi, Derman, Kamal and Zou (1999a, 1999b) and Britten-Jones and Neuberger (2000) because we consider the contract that pays the *square* of the log-return as opposed to the contract that pays the log-return. Earlier literature focused on the log-return (in short, “log”) contract because it provides a replication strategy for the variance swap when the underlying stochastic process is continuous. However, in the presence of jumps, the variance swap cannot be replicated nor hedged. Defining the variance swap in terms of the square of the holding-period return would allow the swap to be priced and replicated by options. It is fortuitous that the log-contract is useful for measuring tail risk.

<sup>5</sup>The two measures are different measures of stock return variability. To see this simply, suppose  $\ln S_t$  was defined in discrete time over dates  $t_i$ ,  $i = 0, 1, \dots, n$  with  $E^{\mathbb{Q}} \ln S_{t_i}/S_{t_{i-1}} = 0$ . Then, the quadratic variation would be defined as  $\sum_{i=1}^n (\ln S_{t_i} - \ln S_{t_{i-1}})^2$  and the variance of the holding period return as  $E^{\mathbb{Q}} (\sum_{i=1}^n (\ln S_{t_i} - \ln S_{t_{i-1}}))^2$ . That is, the quadratic variation depends on the *sum* of the squared log-returns while the variance of the holding period return depends on the *square* of the summed log-returns.

between the two measures of variability as  $\mathbb{D}(T) = \text{var}^{\mathbb{Q}}(\ln S_T/S_0) - E^{\mathbb{Q}}[\ln S, \ln S]_T$ . Then,

$$\mathbb{D}(T) = E^{\mathbb{Q}} \left[ \int_0^T 2 \ln S_{t-}/S_0 d \ln S_t \right] - \left( E^{\mathbb{Q}}(\ln S_T/S_0) \right)^2.$$

In addition, suppose the log return process decomposes as  $\ln S_t/S_0 = A_t + M_t$ , where  $A_t$  is a continuous finite variation process with  $A_0 = 0$  and  $M_t$  is a square-integrable martingale with  $M_0 = 0$ . Then if  $A_t$  is deterministic,  $\mathbb{D}(T) = 0$ .

**Proof:** Given that  $\ln S_t$  is a semimartingale, the quadratic variation exists and is defined by stochastic integration by parts,

$$\ln^2 S_T = \ln^2 S_0 + 2 \int_0^T \ln S_{t-}/S_0 d \ln S_t + [\ln S, \ln S]_T. \quad (14)$$

From the definition of the variance,

$$\text{var}^{\mathbb{Q}}(\ln S_T/S_0) \equiv E^{\mathbb{Q}}(\ln S_T/S_0)^2 - (E^{\mathbb{Q}} \ln S_T/S_0)^2. \quad (15)$$

From equations (14) and (15), it follows that  $\mathbb{D}(T) = (E^{\mathbb{Q}}(\ln S_T/S_0))^2 - E^{\mathbb{Q}} \left[ \int_0^T 2 \ln S_{t-}/S_0 d \ln S_t \right]$  and that  $\mathbb{D} = 0$  if and only if  $2E^{\mathbb{Q}} \int_0^T \ln S_{t-}/S_0 d \ln S_t = (E^{\mathbb{Q}} \ln S_T/S_0)^2$ .

Next, when  $\ln S_t/S_0 = A_t + M_t$  with  $A_t$  deterministic and  $M_t$  a martingale,  $E^{\mathbb{Q}} \int_0^T 2 \ln S_{t-}/S_0 d \ln S_t = E^{\mathbb{Q}} \int_0^T \ln S_{t-}/S_0 E_t d \ln S_t = \int_0^T 2A_t dA_t = (A_T)^2 = (E^{\mathbb{Q}} \ln S_T/S_0)^2$ . Therefore,  $\mathbb{D} = 0$ . In the above equations, the first equality follows from the law of iterative expectations and the second because  $M_t$  is a martingale. The third equality follows because the drift is of finite variation with continuous paths (Theorem I.53 of Protter (2004)).  $\square$

The intuition behind why the quadratic variation and the holding period variance are equivalent when the process can be decomposed into a martingale and a deterministic drift is that the deterministic drift does not add to the quadratic variation (because it is a continuous finite variation process) nor to the variance (because it is deterministic).<sup>6</sup> However, if the drift of the log-stock process is stochastic, then the variance will need to account for this stochasticity and will differ from the quadratic variation by  $\mathbb{D}(T)$ .

To illustrate, suppose the stock price is lognormally distributed,  $S_T = S_0 \exp \left( (r - \frac{1}{2}\sigma^2)T + \sigma\sqrt{T}\tilde{z} \right)$ , where with usual symbols,  $r$  is the risk-free rate,  $\sigma$  is the diffusion parameter, and  $\tilde{z}$  is the standard normal variate. Now evaluating the stochastic integral,  $E^{\mathbb{Q}} \int_0^T \ln S_{t-}/S_0 d \ln S_t = ((r - \frac{1}{2}\sigma^2)T)^2 = (E^{\mathbb{Q}}(\ln S_T/S_0))^2$ . Therefore,  $\mathbb{D}(T) = 0$ , and the expected quadratic variation

<sup>6</sup>We thank G. Lowther for this intuition.

and variance of the holding period return are equivalent for the log-normal process. Similarly, the equivalence between the two measures holds for the Merton jump-diffusion model as noted earlier by the explicit evaluation of the stochastic integral in equation (8). We arrive at the same conclusion without evaluating the stochastic integral by observing that for the Merton model,  $\ln S_t/S_0$  can be decomposed as,

$$\begin{aligned}\ln S_t/S_0 &= \left( \left( r - \frac{1}{2}\sigma^2 \right) + \lambda\alpha \right) t + M_t, \\ &= A_t + M_t,\end{aligned}$$

where  $M_t$  is the sum of a continuous and pure jump martingale. The conclusion follows because  $A_t$  is deterministic.

The variance of the holding period return can be estimated model-free from option prices as shown by Bakshi, Kapadia and Madan (2003) in their Proposition 1. For stochastic processes for which the variance and quadratic variation are equivalent, we can, therefore, also estimate the quadratic variation in a model-free manner. For stochastic processes for which the two measures of variability are not equivalent, the difference between them is equal to  $\mathbb{D}(T)$ .

### 3.2 Lévy vs. Stochastic Volatility

For which general class of models does the equivalence between holding-period variance and quadratic variation (not) hold?

A Lévy process with characteristic function given by the Lévy-Khintchine theorem and characterized by  $(\gamma, \sigma^2, \nu)$  can be decomposed into a deterministic drift and a martingale. Therefore, quadratic variation and the variance of the holding period return are equivalent for the entire class of Lévy processes.<sup>7</sup> This class includes many commonly used models including the geometric Brownian motion (Black and Scholes, 1973), jump diffusion models (e.g., Merton, 1976; Kou, 2002), and infinite activity Lévy processes (e.g., the variance gamma process of

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<sup>7</sup>We can easily verify. From the Lévy-Khintchine theorem, the characteristic function of the log return when it follows a Lévy process characterized by  $(\gamma, \sigma^2, \nu)$  is,

$$\mathbb{E}e^{iu \ln S_t/S_0} = \phi_t(u) = e^{t\psi(u)} \tag{16}$$

where

$$\psi(u) = i\gamma u - \frac{1}{2}\sigma^2 u^2 + \int_{\mathbb{R}^0} \left( e^{iux} - 1 - iux\mathbb{1}_{\{|x|<1\}} \right) \nu(dx).$$

The triplet  $(\gamma, \sigma^2, \nu)$  uniquely characterizes the Lévy process. Here,  $\gamma$  is the drift of the diffusion component of the Lévy process,  $\int_0^t \sigma^2 dt$  is the quadratic variation of the continuous component, and  $\nu$  is the Lévy measure. We can explicitly compute the moments from the characteristic function. First, consider  $\mathbb{E}^{\mathbb{Q}} \ln S_T/S_0$ , or the

Madan and Seneta (1990), normal inverse Gaussian process of Barndorff-Nielsen (1998), and the finite-moment stable process of Carr, Geman, Madan and Yor (2002)).

We *cannot* decompose the log-stock process with stochastic volatility into a deterministic drift and martingale. In stochastic volatility models, the log-return process has a stochastic drift. Therefore, the two measures of variability will not be identical for any model with stochastic volatility (e.g. Heston (1993) and Bates (2000)).

To demonstrate, let  $\ln S_T/S_0$  be a continuous semimartingale,

$$\ln S_T = \ln S_0 + \int_0^T \left(r - \frac{1}{2}\sigma_t^2\right)dt + \int_0^T \sigma_t dW_{1,t}, \quad (17)$$

where  $\sigma_t^2$  is another continuous semimartingale, orthogonal to  $W_{1,t}$ . Let the quadratic variation for the continuous log-stock process (“integrated variance”) be denoted  $[\ln S, \ln S]_T^c = \int_0^T \sigma_t^2 dt$ . By the law of total variance,

$$\text{var}^{\mathbb{Q}}(\ln S_T/S_0) = \text{var}^{\mathbb{Q}}\left(\mathbb{E}^{\mathbb{Q}} \ln S_T/S_0 \mid \{\sigma_t^2\}_{0 \leq t \leq T}\right) + \mathbb{E}^{\mathbb{Q}}\left(\text{var}^{\mathbb{Q}}(\ln S_T/S_0) \mid \{\sigma_t^2\}_{0 \leq t \leq T}\right). \quad (18)$$

Now,

$$\begin{aligned} \text{var}^{\mathbb{Q}}\left(\mathbb{E}^{\mathbb{Q}} \ln S_T/S_0 \mid \{\sigma_t^2\}_{0 \leq t \leq T}\right) &= \frac{1}{4} \text{var}^{\mathbb{Q}} \int_0^T \sigma_t^2 dt, \\ &= \frac{1}{4} \text{var}^{\mathbb{Q}}([\ln S_0, \ln S_0]_T^c), \end{aligned} \quad (19)$$

and

$$\mathbb{E}^{\mathbb{Q}}\left(\text{var}^{\mathbb{Q}}(\ln S_T/S_0) \mid \{\sigma_t^2\}\right) = \mathbb{E}^{\mathbb{Q}}[\ln S_0, \ln S_0]_T^c, \quad (20)$$

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first moment,

$$\begin{aligned} \mathbb{E}^{\mathbb{Q}} \ln S_T/S_0 &= \left. \frac{1}{i} \frac{\partial \phi_T(u)}{\partial u} \right|_{u=0} \\ &= T \left( \gamma + \int_{\mathbb{R}^0} (x - ix \mathbb{1}_{[|x|<1]}) \nu(dx) \right). \end{aligned}$$

Next, compute the second moment,  $\mathbb{E}^{\mathbb{Q}}(\ln S_T/S_0)^2$ ,

$$\begin{aligned} \mathbb{E}^{\mathbb{Q}} \ln^2 S_T/S_0 &= \left. \frac{1}{i^2} \frac{\partial^2 \phi_T(u)}{\partial u^2} \right|_{u=0} \\ &= T^2 \left( \gamma + \int_{\mathbb{R}^0} (x - ix \mathbb{1}_{[|x|<1]}) \nu(dx) \right)^2 + T \left( \sigma^2 + \int_{\mathbb{R}^0} x^2 \nu(dx) \right), \\ &= \left( \mathbb{E}^{\mathbb{Q}} \ln S_T/S_0 \right)^2 + \mathbb{E}^{\mathbb{Q}}[\ln S, \ln S]_T. \end{aligned}$$

Rearranging the last equation, it follows that the quadratic variation is equivalent to the variance of the holding-period return.

from Ito isometry. Putting it together into (18) we get,

$$\mathbb{D}(T) = \text{var}^{\mathbb{Q}}(\ln S_T/S_0) - \mathbb{E}^{\mathbb{Q}}[\ln S, \ln S]_T^c = \frac{1}{4} \text{var}^{\mathbb{Q}}([\ln S, \ln S]_T^c) \quad (21)$$

The variance is not equivalent to the quadratic variation and  $\mathbb{D}(T) \neq 0$ . The conclusion also holds when stochastic volatility is correlated with the log-return process. Because we can write the diffusion process for the volatility as  $W_\sigma = \rho W_1 + \sqrt{1 - \rho^2} W_2$ , where  $W_2$  is independent of  $W_1$ ,  $\mathbb{D}(T)$  will have an additional component corresponding to  $W_1$  whose magnitude depends on  $\rho$ .

In summary, for the entire class of Lévy processes, the variance of the holding period return is equivalent to the quadratic variation and can be estimated model-free by the Bakshi, Kapadia and Madan (2003) measure of stock return variability. For a purely continuous process with no tail risk from discontinuities, the quadratic variation can be estimated by the VIX measure. However, if there are both discontinuities and stochastic volatility, then both the BKM and the VIX are biased measures of quadratic variation.

### 3.3 Quadratic Variation, Integrated Variance, and Tail Risk

We put more structure on the stock return process. Let the log of the stock price be a general diffusion with jumps,

$$\ln S_T = \ln S_0 + \int_0^T (a_t - \frac{1}{2} \sigma_t^2) dt + \int_0^T \sigma_t dW_t + \int_0^T \int_{\mathbb{R}^0} x \mu(dx, dt). \quad (22)$$

Here, the time-variation in  $\sigma_t$  is left unspecified while  $a_t$  is restricted to ensure that the discounted stock price is a martingale. Let the Poisson random measure have the intensity measure  $\mu(dx, dt) \equiv \nu(dx)dt$ . Given our focus on tail risk and because we have already included a diffusion component, we assume that  $\int_{\mathbb{R}^0} \nu(dx) < \infty$  and that all moments  $\int_{\mathbb{R}^0} x^n \nu(dx)$ ,  $n = 1, 2, \dots$ , exist.

As in Carr and Madan (1998), Demeterfi, Derman, Kamal and Zou (1999a, 1999b), and Britten-Jones and Neuberger (2000), define the VIX measure of integrated variance as,

$$\mathbb{E}^{\mathbb{Q}}[\ln S, \ln S]_T^c \equiv \mathbb{E}^{\mathbb{Q}} \left[ 2 \left( \int_0^T \frac{dS_t}{S_t} - \ln \frac{S_T}{S_0} \right) \right]. \quad (23)$$

If there are no discontinuities in the stock return process, then  $\mathbb{E}^{\mathbb{Q}} \left[ 2 \left( \int_0^T \frac{dS_t}{S_t} - \ln \frac{S_T}{S_0} \right) \right] = \mathbb{E}^{\mathbb{Q}} \int_0^T \sigma_t^2 dt$ . If there are discontinuities, Carr and Wu (2009) show in their Proposition 1 that

the difference between  $E^{\mathbb{Q}}[\ln S, \ln S]_T$  and  $E^{\mathbb{Q}}[\ln S, \ln S]_T^c$  is determined by the jump distribution. Specifically,

$$E^{\mathbb{Q}}[\ln S, \ln S]_T - E^{\mathbb{Q}}[\ln S, \ln S]_T^c = 2E^{\mathbb{Q}} \int_0^T \int_{R^0} \psi(x) \mu(dx, dt), \quad (24)$$

where  $\psi(x) = 1 + x + \frac{1}{2}x^2 - e^x$ . The difference is determined by the higher order moments ( $\geq 3$ ) of the jump distribution. Economically, the difference is equal to the expected loss because of jumps to a variance swap priced under the (erroneous) assumption that the underlying log-stock process is continuous.

From iterated expectation, the RHS of equation (24), is determined completely by the compensator of the jump process. But tail risk is also determined by the expectation of jump intensity because when there are jumps, the tail of the stock return distribution is completely determined by (large) jumps. Therefore, the time variation in the difference between the quadratic variation and the VIX measure of integrated variance measures the time variation in the tail of the stock return distribution.

**Proposition 2** *Let the log price process be specified as in equation (22). Let the intensity measure  $\nu(dx)$  be of the form,*

$$\nu(dx) = \lambda_t f(x) dx,$$

where  $\lambda_t$  is the jump arrival intensity of jump of any size with jump size distribution  $f(x)$ . Then,

$$E^{\mathbb{Q}}[\ln S, \ln S]_T - E^{\mathbb{Q}}[\ln S, \ln S]_T^c = 2 \Psi(f(x)) \Lambda_{0,T} \quad (25)$$

where

$$\Lambda_{0,T} = E^{\mathbb{Q}} \int_0^T \lambda_t dt,$$

with  $\Psi(f(x)) = \int_{R^0} \psi(x) f(x) dx$ , and  $\psi(x) = 1 + x + \frac{1}{2}x^2 - e^x$ .

**Proof:** Given that jumps of all sizes have the same arrival intensity,  $E^{\mathbb{Q}} 2 \int_0^T \int_{R^0} \psi(x) \mu(dx, dt) = 2 \int_0^T E^{\mathbb{Q}} [\int_{R^0} \psi(x) f(x) dx] \lambda_t dt = 2 \Psi(f(x)) E^{\mathbb{Q}} \int_0^T \lambda_t dt$ , where  $\Psi(\cdot)$  is determined by the jump size distribution.  $\square$

Intuitively, Proposition 2 states that the difference between the quadratic variation and the measure of integrated variance over an interval  $T$  is proportional to the expectation of the number of jumps over that interval. Thus, the time variation in the difference is an index of tail risk as it measures the expectation of extreme returns.

In addition, if we divide equation (25) by  $E^{\mathbb{Q}}[\ln S, \ln S]_T$ , then we can also measure the time-variation in the contribution of discontinuities to the total quadratic variation which is also of interest in itself (Bollerslev and Todorov, 2010).

### 3.4 Holding Period Variance and the Integrated Variance in SVJ Model

If the stock return process is a Lévy process with discontinuities, Proposition 1 notes that the variance of the holding period return is equivalent to the quadratic variation, and thus unlike the integrated variance measure, the BKM variance can perfectly measure the quadratic variation. In this section, we compare the accuracy with which the BKM and VIX, respectively, measures quadratic variation when there are both stochastic volatility and jumps.

Let  $\mathbb{V}$  be the annualized variance of the log-return,  $\mathbb{V} = \frac{1}{T} \left( E^{\mathbb{Q}} (\ln S_T/S_0)^2 - \mu_{0,T}^2 \right)$ , where  $\mu_{t,T} = E^{\mathbb{Q}} \ln S_T/S_t$ . From Bakshi, Kapadia and Madan (2003), the price of the variance contract is estimated from out-of-the-money (OTM) calls and puts of maturity  $T$ . Denoting  $C(S_t; K, T)$  and  $P(S_t; K, T)$  as the call and put of strike  $K$  and remaining time to expiration  $T$ , BKM demonstrate,

$$E^{\mathbb{Q}} e^{-rT} (\ln S_T/S_0)^2 = \int_{K>S_0} \frac{2(1 - \ln(K/S_0))}{K^2} C(S_0; K, T) dK + \int_{K<S_0} \frac{2(1 + \ln(S_0/K))}{K^2} P(S_0; K, T) dK, \quad (26)$$

where  $r$  is the constant risk-free rate.

Similarly, let the annualized integrated variance be denoted as  $\mathbb{IV} = \frac{1}{T} E^{\mathbb{Q}} \int_0^T \sigma_t^2 dt$ .  $\mathbb{IV}$  for a continuous stochastic process can also be computed from OTM option prices,

$$e^{-rT} \mathbb{IV} = \frac{2}{T} \left[ \int_{K>S_0} \frac{1}{K^2} C(S_0; K, T) dK + \int_{K<S_0} \frac{1}{K^2} P(S_0; K, T) dK - e^{-rT} (e^{rT} - 1 - rT) \right]. \quad (27)$$

This particular formulation is in Demeterfi, Derman, Kamal and Zou (1999b) (their equation (26)), and equivalent but slightly different formulations are in Carr and Madan (1998) and Britten-Jones and Neuberger (2000).

In Table 1, we compare the theoretical quadratic variation,  $E^{\mathbb{Q}}[\ln S, \ln S]_T$ , with those estimated from option prices using  $\mathbb{V}$  and  $\mathbb{IV}$ , respectively, for the Merton (1974) jump-diffusion model, and the stochastic volatility and jump (SVJ) model of Bates (2000). Except for the initial variance ( $\sigma_0^2$ ) and the mean of the jump size distribution ( $\alpha$ ), we calibrate the parameters to those empirically estimated by Pan (2002). The initial volatility and the mean of the jump size distribution are adjusted to vary the contribution of the variance from jumps to the total variance from 0 to 90%.

Panel A provides the comparison for the Merton model. For the Merton jump-diffusion model,  $\mathbb{V}$  accurately measures the quadratic variation, but  $\mathbb{IV}$  does so with error because of tail risk. The error increases as jumps contribute a larger fraction to the total variance.

In Panel B, we provide numerical results for the SVJ model. As in Panel A, the proportion of the variance from the jump to the total variance is varied from 0 to 90%. Neither  $\mathbb{V}$  nor  $\mathbb{IV}$  measures the true quadratic variation of the process without error. First, consider the BKM variance of holding period return.  $\mathbb{V}$  will be most inaccurate in relative terms when there are no jumps. As can be observed, even when the error is at its maximum, the bias in BKM is only 0.61% in relative terms. While the true  $\sqrt{[\ln S, \ln S]_T}$  is 20%, the volatility is estimated to be 20.05%.

Next, consider the accuracy of  $\mathbb{IV}$ . As in the Merton model, the inaccuracy increases as the contribution of jumps to the quadratic variation increases. When the contribution of jumps to volatility is below 20%, the relative error is less than 1% of the variance, and for all practical purposes, both  $\mathbb{V}$  and  $\mathbb{IV}$  are accurate measures of the quadratic variation. The error in  $\mathbb{IV}$  increases as jumps become more important, exceeding 9% of the variance when jumps contribute over 70% of the total variance. In this case,  $\sqrt{\mathbb{IV}}$  is 19% instead of the correct 20% - an economically significant bias. Comparing  $\mathbb{V}$  and  $\mathbb{IV}$ , the bias in  $\mathbb{IV}$  is twice that of  $\mathbb{V}$  when jumps contribute 20% to the total variance, increasing multi-fold as jump risk increases. In summary, the results indicate that in the presence of jumps, the BKM variance is a more accurate measure of quadratic variation than the integrated variance as it correctly accounts for jump risk while the stochasticity of the drift adds negligible error for the model.

Why is the error from the stochasticity of drift small? When the correlation is zero, we noted in equation 21 that the bias is proportional to  $\text{var}^{\mathbb{Q}}([\ln S, \ln S]_T^c)$ . Bollerslev and Zhou (2002) demonstrate for the Heston (1993) model,<sup>8</sup>

$$\text{var}^{\mathbb{Q}}([\ln S, \ln S]_T^c) = A_T \sigma_0^2 + B_T, \text{ where } A_T \sim O(T^2) \text{ and } B_T \sim O(T^3). \quad (28)$$

Thus for short time to maturities ( $\leq 30$ -day), the contribution of  $\frac{1}{4} \text{var}^{\mathbb{Q}}([\ln S, \ln S]_T^c)$  of  $O(T^2)$  is very small (and decreases even further for shorter maturities).<sup>9</sup>

<sup>8</sup>Bollerslev and Zhou (2002) (Appendix A) show that when  $d\sigma_t^2 = \kappa(\theta - \sigma_t^2)dt + \eta\sigma_t dW_{\sigma,t}$ , we have  $A_T = \frac{\eta^2}{\kappa^2 T} [\frac{1}{\kappa} - 2e^{-\kappa T} - \frac{1}{\kappa}e^{-2\kappa T}]$ ,  $B_T = \frac{\eta^2}{\kappa^2 T} [\theta T(1 + 2e^{-\kappa T}) + \frac{\theta}{2\kappa}(e^{-\kappa T} + 5)(e^{-\kappa T} - 1)]$ . A Taylor expansion on the quantity  $e^{\kappa T}$  in the above formulas at  $T = 0$  shows that  $A_T \sim O(T^2)$  and  $B_T \sim O(T^3)$ .

<sup>9</sup>For a given maturity, the exact magnitude of the error depends on the coefficients. We also investigate the impact of the stochastic volatility parameters  $\kappa$ ,  $\theta$  and  $\eta$  on the error of  $\mathbb{V}$  with other parameters taken from the first row of Panel B of Table 1.  $\kappa$  and  $\theta$  have negligible influence on the error. Specifically, the relative error of  $\mathbb{V}$  is between 0.42% and 0.65% when  $\kappa \in (0, 20)$  and between 0.44% and 0.61% when  $\theta \in (0, 2)$ . The volatility of volatility parameter  $\eta$  is relatively more significant; when  $\eta$  is 50%, the relative error in  $\mathbb{V}$  is 1.02% - but this

When  $\rho \neq 0$ , the difference between variance and the quadratic variation has another additive component linearly related to the correlation  $\rho$ . A more negative  $\rho$  makes  $\mathbb{V}$  less accurate. However, even for the extreme case where  $\rho = -0.9$ , the relative error of  $\mathbb{V}$  is less than 1% of the quadratic variation. For  $\rho = -0.9$ ,  $\sqrt{\mathbb{V}}$  gives an estimate of 20.09% instead of the correct 20%.

Overall, for the SVJ model when jumps contribute less than 20% to the variance, both  $\mathbb{V}$  and  $\mathbb{IV}$  are good approximations for the true expected quadratic variation.<sup>10</sup> However, when jumps are a significant component of volatility as might be expected in times of stress, the integrated variance significantly underestimates the quadratic variation. The difference between  $\mathbb{V}$  and  $\mathbb{IV}$  is predominantly a function of jump risk, increasing as the contribution of jumps to quadratic variation increases.

## 4 Defining a Jump and Tail Index

We define a jump and tail index based on the difference between  $\mathbb{V}$  and  $\mathbb{IV}$ . From Propositions 1 and 2, the difference between  $\mathbb{V}$  and  $\mathbb{IV}$  is the sum of two components, the first determined by the stochasticity of the drift and the second determined by jump risk. Our simulation evidence indicates that the impact of the stochasticity of the drift can be neglected for standard jump-diffusion models. If so, the time variation in  $\mathbb{V} - \mathbb{IV}$  is determined by the time variation in jump intensity. Denoting  $JTIX$  as the *jump and tail index*,

$$\begin{aligned} JTIX &= \mathbb{V} - \mathbb{IV}, \\ &= \frac{1}{T} \left( \mathbb{E}^{\mathbb{Q}} \int_0^T \ln S_T/S_0 d \ln S_t - \mathbb{E}^{\mathbb{Q}} (\ln S_T/S_0)^2 \right) + 2\Psi \frac{1}{T} \Lambda_{0,T} \end{aligned} \quad (29)$$

$$\approx 2\Psi \frac{1}{T} \Lambda_{0,T}. \quad (30)$$

Equation (29) combines the results of Propositions 1 and 2. From the definition of  $\Lambda_{0,T} = \mathbb{E}^{\mathbb{Q}} \int_0^T \lambda_t dt$ , it follows that  $JTIX$  measures jump and tail risk. Extending the popular analogy of the Chicago Board Options Exchange VIX index to a fear index, we can view  $JTIX$  as an index of extreme fear.

To validate and investigate the economic significance of the jump and tail risk index, we pose the following questions.

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is still lower than the error in  $\mathbb{IV}$  when jumps contribute 20% or more to the total variance.

<sup>10</sup>Here, our analysis concurs with Jiang and Tian (2005) and Carr and Wu (2009). The parameterizations chosen in their numerical experiments correspond to a (low) jump contribution of 14% and 11.7% to the quadratic variation, respectively. When jump risk is of low economic importance, we can accurately measure quadratic variation using either the VIX or the BKM measure.

### 1. Is $JTIX = 0$ ( $\mathbb{V} \approx \mathbb{IV}$ )?

The first question we consider is whether jump risk is significant. This question is equivalent to the question of whether the VIX is an accurate estimator of quadratic variation. When jump risk is negligible,  $\mathbb{V}$  and  $\mathbb{IV}$  are approximately equal and  $JTIX$  is close to zero.

The sign of  $JTIX$  is determined by  $\Psi = \int_{R^0} \psi(x)f(x)dx$  where  $\psi(x) = 1 + x + \frac{1}{2}x^2 - e^x$ . To a first order approximation, the sign is determined by the negative of the third moment,  $\int_{R^0} -x^3 f(x)dx$ . When the jump size distribution is negatively skewed,  $\mathbb{V}$  is greater than  $\mathbb{IV}$ .

### 2. Is there time variation in $JTIX$ , and is the time variation related to jump risk?

If the difference between  $\mathbb{V}$  and  $\mathbb{IV}$  is related to  $\lambda_t$ , then it should increase in periods when fears of jump risk are higher. Therefore, we expect the time variation in the jump and tail index to be countercyclical. We expect the  $JTIX$  to be especially high in times of severe market stress as in the most recent financial crisis. Finally, we expect it to be higher for firms or industries experiencing severe stress.

### 3. What is the channel through which $JTIX$ impacts market returns?

Our primary question is whether jump risk impacts future market returns. If it does, then we are interested in understanding the channel through which jump risk is significant. Does jump risk impact market returns through volatility or through the tail of the distribution? A number of recent papers have raised the question of whether the equity risk premium can be explained by tail risks (specifically, see the rare risk literature of Rietz, 1988; Barro, 2006; Gabaix, 2010).

In investigating these questions, we follow Bollerslev, Tauchen and Zhou (2009). Specifically, they demonstrate that the difference between the integrated variance and the historical variance, a measure of the “variance risk premium” ( $VRP$ ), predicts index returns. Over short to medium horizons of less than a year, the  $VRP$  is more significant than traditionally used predictors of index returns. The robustness of their finding is corroborated by Drechsler and Yaron (2011).

We proceed as follows. First, to consider the economic importance of jump risk, we investigate the relative significance of  $\mathbb{V}$  and  $\mathbb{IV}$ . If jump risk is significant, then the  $VRP$  defined using  $\mathbb{V}$  will be of greater economic significance than the one defined using  $\mathbb{IV}$  as the former accounts for the contribution of jumps to volatility. In terms of the sign of the coefficient on  $JTIX$ , we expect it to be positive, i.e., the  $VRP$  based on  $\mathbb{IV}$  will underestimate the predicted return. Second, we consider whether volatility and tail should be considered separate channels. If tail risk is an additional channel of risk and risk premia, then there there should be vari-

ability in returns that can be further explained by jump risk after accounting for the impact of jumps in volatility. Therefore, both *JTIX* and the *VRP* defined in terms of  $\mathbb{V}$  should be significant.

## 5 Data

Our data consists of option prices on S&P 500 index (SPX) and 189 individual stocks that have been components of the S&P 100 index during our sample period of January 1996 to October 2009. We use four different maturities of 30 days, 23 days, 16 days, and 9 days for the empirical work in Section 6. For the construction of month-end volatility and tail indexes used in predictive regressions of Section 7, we construct a 30-day implied volatility curve by linear interpolation across the two near-month implied volatility curves. The options data, underlying securities' price data and the zero coupon yield are from OptionMetrics. Dividend yield for the index is also from Optionmetrics, while dividends for individual stocks are from CRSP.

In line with previous literature, we clean the data using several filters. First, we exclude options that have (i) a missing implied volatility in OptionMetrics, (ii) zero open interest, and (iii) bid equal to zero, or negative bid-ask spreads. Second, we verify that options do not violate the no-arbitrage bounds. For an option of strike  $K$  maturing at  $T$  with current stock price  $S_t$  and dividend  $D$ , we require that  $\max(0, S_t - PV[D] - PV[K]) \leq c(S_t; T, K) \leq S_t - PV[D]$  for European call options, and  $\max(0, PV[K] - (S_t - PV(D))) \leq p \leq PV[K]$  for European put options, where  $PV[\cdot]$  is the present value function. Similarly, for American call and put options, we require  $\max(S_t - K, 0) \leq C(S_t; T, K) \leq S_t$  for calls and  $\max(K - S_t, 0) \leq P(S_t; T, K) \leq K$  for puts. Third, for individual equity options, we exclude options if the underlying stock pays a dividend over the remaining maturity of the option. We treat the remaining equity options as European. Fourth, if two calls or puts with different strikes have identical mid-quotes, i.e.  $c(S_t; T, K_1) = c(S_t; T, K_2)$  or  $p(S_t; T, K_1) = p(S_t; T, K_2)$ , we discard the further away from the money quote. Finally, we keep only out-of-the-money (OTM) options, and include observations for a given underlying stock and date only if there are at least two valid OTM call and put quotes. Table 2 provides summary statistics of our sample. Our final sample consists of 42,692 and 172,521 prices for index and individual equity options, respectively.

To construct the indexes, we require a continuum of strikes. Following Jiang and Tian (2005) and Carr and Wu (2009), we interpolate the implied volatility across the range of observed strikes using a cubic spline, and assume the smile to be flat beyond the observed

range of strikes. Using the interpolated curve, we compute 1001 option prices over a range of zero to three times the current stock price.

We use two measures of realized variance,  $\mathbb{RV}$ . The first is based on daily return data,

$$\mathbb{RV} = \frac{252}{T} \sum_{t=1}^T \ln^2 \frac{S_t}{S_{t-1}}. \quad (31)$$

In addition, we also use the realized variance computed from high frequency data from Hao Zhou’s web site.

## 6 Time Variation in Jump and Tail Risk

Table 3 reports summary statistics for  $\sqrt{\mathbb{V}}$ ,  $\sqrt{\mathbb{IV}}$ , the difference,  $\Delta = \sqrt{\mathbb{V}} - \sqrt{\mathbb{IV}}$ ,  $JTIX = \mathbb{V} - \mathbb{IV}$ , and the realized volatility,  $\sqrt{\mathbb{RV}}$ , for S&P 500 index options and individual options. For index options, the reported values are the time series average of the 166 monthly observations and for individual options, the statistics are computed by pooling all observations in each sub-sample.

Consider, first, the 30-day options for both index and individual firms. On average, for the entire sample,  $\sqrt{\mathbb{V}}$  is larger than  $\sqrt{\mathbb{IV}}$  by 0.49% for 30-day index options and by 0.72% for 30-day equity options. This difference is larger during recessions, increasing to 1% for the market and to 1.39% for individual equity options. Panel A of Figure 1 provides a plot of  $\Delta = \sqrt{\mathbb{V}} - \sqrt{\mathbb{IV}}$  for 30-day options over our sample period. In addition, for comparison, the plot also shows  $\sqrt{\mathbb{IV}}$  and the realized volatility,  $\sqrt{\mathbb{RV}}$ . The shaded area in the plots represents the NBER defined recession period in our sample: March 2001- November 2001 and December 2007- June 2009. The volatility measured by  $\sqrt{\mathbb{V}}$  is always higher than that estimated by  $\sqrt{\mathbb{IV}}$ , consistent with the expectation that the jump size distribution is negatively skewed.

Next, consider the jump and tail index  $JTIX$ . The time variation in  $JTIX$  is countercyclical. Comparing the magnitude in recessions versus non-recessions, there is a five-fold increase in jump risk in recessions. Panel B of Figure 1 shows that there are spikes in the index in every crisis period. The most prominent spike with a severe increase in expected jumps is in the second half of 2008 when jump risk is over 50-fold that in the median month. Additional spikes are during the previous recession of 2001 and the Russian bond and LTCM crisis in 1998. Unlike the integrated variance measure  $\mathbb{IV}$ , the jump and tail index clearly differentiates between the 2001 recession and LTCM crisis; the latter had about twice the tail risk compared

to the former.

To sharpen the distinction between the holding period volatility and the integrated volatility, we also present the plot of  $JTIX/\mathbb{V}$  (the difference between  $\mathbb{V}$  and  $\mathbb{IV}$  as a proportion of the holding period variance) in Panel B. It can be seen that  $\mathbb{IV}$  underestimates quadratic variation by almost 12% at the peak of the recent financial crisis. Using the numerical analysis of Table 1 as a guide, this magnitude of underestimation suggests that jump risk might comprise 80-90% of total volatility in the peak months of the crisis.

The tail index for individual firms also show a revealing pattern. Although there is little difference between  $JTIX$  for financial (0.56%) versus non-financial firms (0.62%) in non-recession periods, the corresponding comparison in recession periods presents a contrast with  $JTIX$  for financial firms being more than five-fold that of non-financial firms (8.18% vs. 1.42%, respectively, for 30-day options). Figure 2 demonstrates that the higher tail risk in financial firms comes solely from the recent recession. In Panel A, we present the average time variation in  $\mathbb{V}$  and  $\mathbb{IV}$  across all firms in our sample. In Panel B, we plot  $JTIX/\mathbb{V}$  and  $JTIX$  for sub-samples of financial firms and non-financial firms. The time series patterns for the two sub-groups are similar before the subprime mortgage crisis. However, the two series start to diverge with tail risk increasing dramatically for financial firms from August 2007. Non-financial firms exhibit increases in tail risk only after the bankruptcy of Lehman. The tail risk measure clearly places financial firms at the epicenter of the crisis.

Results for maturities ranging from 9 to 23 days present a consistent picture. For each maturity,  $\mathbb{V}$  is higher than  $\mathbb{IV}$  on average for both index and individual equity options, and the difference between the two measures of variability is higher in recessions.

To check statistical significance, we regress  $JTIX$  computed from the 30-day option series on an intercept and a recession dummy variable,

$$\begin{aligned}
 JTIX_t &= \alpha + \beta \cdot \mathbb{1}_{[Recession,t]} + \epsilon_t, \\
 \mathbb{1}_{[Recession,t]} &= \begin{cases} 1 & \text{if month } t \text{ is in recession period;} \\ 0 & \text{if otherwise.} \end{cases}
 \end{aligned}
 \tag{32}$$

The intercept  $\alpha$  is the average magnitude of  $JTIX$  in the non-recession period, and  $\beta$  is the additional bias in the recession period. The hypothesis is that  $\alpha$  and  $\beta$  are significantly positive. We also report the results using  $JTIX/\mathbb{V}$  as the dependent variable.

For individual firms, we run a panel regression including fixed firm effects,

$$\begin{aligned}
 JTIX_{it} &= \alpha_i + \beta \cdot \mathbb{1}_{[Recession,t]} + \epsilon_{it}, \\
 \mathbb{1}_{[Recession,t]} &= \begin{cases} 1 & \text{if month } t \text{ is in recession period;} \\ 0 & \text{if otherwise.} \end{cases}
 \end{aligned}
 \tag{33}$$

Similarly, we also run the regression with the  $JTIX/\mathbb{V}$  as the dependent variable. As with the regression for index options,  $\beta$  represents the impact of a recession.

Table 4 reports the regression results across the sub-samples. Panel A reports the regression results for the index options for each of the four maturities. As expected, the signs of both  $\alpha$  and  $\beta$  are positive. Both coefficients are also significant at 95% confidence levels. Results are consistent whether we use  $JTIX$  or  $JTIX/\mathbb{V}$  as the dependent variable. Panel B presents the results for individual firms. The overall results are consistent with those observed in Panel A in that  $\beta$  is significant and positive. Overall, both Panels A and B demonstrate that tail risk is higher during recessions as opposed to non-recessions.

In Panel C and D of Table 4, we focus on the difference between financial and non-financial firms in the recent financial crisis. We now define the dummy variable in equation (33) as 1 only if the period belongs to the 2008-09 recession related to the financial crisis (the exact recession period is from December 2007 to June 2009 based on NBER’s definition). All firms show significant increase in jump risk during the recent financial crisis. But the risk in financial firms is starkly different than that in non-financial firms. In the regression with  $JTIX_{it}$  as the dependent variable, the coefficient for 30-day options for non-financial firms is 0.0085, while the coefficient for the financial firms is 0.11 - about 13-fold higher.

In summary, the results demonstrate that (i)  $\mathbb{V}$  is significantly larger than  $\mathbb{IV}$ , (ii) jump and tail risk (both  $JTIX$  and  $JTIX/\mathbb{V}$ ) is higher in times of economic stress and crises, and (iii) jump and tail risk is larger by an order of magnitude for financial firms compared to non-financial firms in the recent financial crisis.

## 7 Jump Risk: Tail vs. Volatility

We next investigate the economic importance of jump risk and the channels of volatility and tail using predictability regressions.

## 7.1 Setup for Predictability Regressions

Our setup follows Bolleslev, Tauchen and Zhou (2009) (BTZ, henceforth). They demonstrate that the difference between the integrated variance and historical variance,  $\mathbb{IV}_t - \mathbb{RV}_{t-1}$ , predicts market excess returns after controlling for usual set of traditional predictors. The predictive regression is specified as,

$$R_{t+j} - r_{t+j}^f = \alpha + \beta_1 \cdot (VRP_t) + \Gamma' \cdot \bar{Z}_t + \epsilon_t, \quad (34)$$

$VRP_t$  is alternatively defined as either  $\mathbb{V}_t - \mathbb{RV}_{t-1}$  or  $\mathbb{IV}_t - \mathbb{RV}_{t-1}$ .<sup>11</sup>  $R_{t+j}$  denotes the log return of the S&P 500 from the end of month  $t$  to the end of month  $t + j$ .  $r_{t+j}^f$  denotes the risk-free return for the same horizon. We use five different values for  $j$ : 1 month, 3 months, 6 months, 1 year and 2 years.  $\bar{Z}_t$  denotes the commonly used predictors. We also investigate the impact of  $JTIX$  directly by including  $JTIX$  and  $JTIX/\mathbb{V}$  in the base specification.

Because of the pre-specified expiration calendar of index options, there does not exist a 30-day option at the end of each month. Therefore, we interpolate from the two near-month maturities to construct a 30-day implied volatility curve. This interpolated implied volatility curve is used to generate option prices, which we then use to construct the indexes.

To be consistent with BTZ, we download the monthly realized variance  $\mathbb{RV}$  calculated from high frequency intra-day return data from Hao Zhou’s website. For common predictors  $\bar{Z}_t$ , we use the identical set of variables used in BTZ. Quarterly P/E ratios and dividend yields ( $P/E$  and  $D/P$ ) for S&P 500 are from Standard & Poor’s Web site. Quarterly  $CAY$  as defined in Lettau and Ludvigson (2001) is downloaded from Lettau and Ludvigson’s website. When monthly data are not available, we use the most recent quarterly data. The term spread ( $TERM$ ) is defined as the difference between ten-year T-bond and three-month T-bill yields. The default spread ( $DEF$ ) is defined as the difference between Moody’s BAA and AAA corporate bond spreads. Data needed to calculate the term spread and the default spread are from the website of the Federal Reserve Bank of St. Louis. RREL, the detrended risk-free rate is the one-month T-bill rate minus the preceding twelve-month moving average. The risk-free rate is the one-month T-bill rate from Kenneth R. French’s online Data Library.

Table 5 presents description for all variables. Panel A of Table 5 reports the basic summary statistics.  $\mathbb{V}$  and  $\mathbb{IV}$  have very similar distributional properties, except that the mean and

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<sup>11</sup>BTZ define  $VRP_t = \mathbb{IV}_t - \mathbb{RV}_{t-1}$  as the “variance risk premium”, although it would be more conventional to define  $\mathbb{RV}_t - \mathbb{IV}_t$  as the (negative) variance risk premium (Bakshi and Kapadia, 2003; Carr and Wu, 2009). Here,  $\mathbb{RV}_t$  is the variance computed from returns over the remaining lifetime of the option. For predictability regressions, the BTZ definition is straightforward to interpret. We, therefore, follow their convention here.

median values of  $\mathbb{V}$  are higher than those of  $\mathbb{IV}$ . The mean value for  $JTIX/\mathbb{V}$  is 3.92% with a maximum about 14%, which is close to the maximum  $JTIX/\mathbb{V}$  constructed from raw 30-day SPX options (of 12%) reported earlier.

Panel B reports the correlation matrix of the variables. Both  $JTIX$  and  $JTIX/\mathbb{V}$  are strongly positively correlated with  $\mathbb{V}$  and  $\mathbb{IV}$ . In addition, they are also positively correlated with  $\log(P/E)$ ,  $TERM$ ,  $DEF$  and  $CAY$ , and negatively correlated with  $\log(P/D)$  and  $RREL$ . The tail index is negatively correlated with the risk-neutral skew ( $SKEW$ ) and risk-neutral kurtosis ( $KURT$ ) (computed as in Bakshi, Kapadia and Madan, 2003), indicating that a more negative skew and a fatter tail in the risk-neutral distribution is associated with higher jump risk. In summary, correlations are about as expected.

More to the point of our study,  $JTIX$  has low correlations with  $VRP$ . The correlation of  $JTIX$  with  $\mathbb{V} - \mathbb{RV}$  and  $\mathbb{IV} - \mathbb{RV}$  is 0.16 and  $-0.06$ , respectively. When we scale  $JTIX$  by  $\mathbb{V}$ , correlations increase as would be expected, but are nevertheless low.  $JTIX/\mathbb{V}$  has a correlation of 0.36 with  $\mathbb{V} - \mathbb{RV}$  and 0.18 with  $\mathbb{IV} - \mathbb{RV}$ . In contrast to these low correlations,  $JTIX$  and  $JTIX/\mathbb{V}$  are highly correlated with the default spread  $DEF$  with correlations of 0.62 and 0.58, respectively. Overall, the correlations are consistent with the economic interpretation of  $JTIX$  as a measure of jump and tail risk.

In the last row of Panel A, we report first order autocorrelations for the control variables. As noted in previous literature, autocorrelations are extremely high for many of the control predictors - as high as 0.97 for dividend yield and the price-earnings ratio - raising concerns about correct inference. In contrast, as can be noted from Panel C,  $VRP$ ,  $JTIX$  and  $JTIX/\mathbb{V}$  have much lower autocorrelations. In Panel D, we test for a unit root in  $JTIX$ ,  $JTIX/\mathbb{V}$ ,  $VRP$  (as well as  $SKEW$  and  $KURT$ ). The Phillips-Perron unit root test strongly rejects the null hypothesis of a unit root in all of the option-based variables.

## 7.2 Significance of Jump Risk in Volatility

In our first set of predictability regressions, we focus on the significance of jump risk in volatility.

### 7.2.1 Statistical Significance

Table 6 reports the regression results for each horizon. We report  $t$  statistics based on Hodrick (1992) 1B standard error under the null of no-predictability.<sup>12</sup> The one-month horizon does

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<sup>12</sup>Ang and Bekaert (2007) show that inference in predictability regressions is critically dependent on the choice of standard errors, and they recommend Hodrick (1992) 1b standard errors under the null of no predictability.

not have overlapping periods and, therefore, we also report Newey-West (1994)  $t$  values.

First, in univariate regressions,  $VRP$  (defined as either  $\mathbb{V} - \mathbb{R}\mathbb{V}$  or  $\mathbb{IV} - \mathbb{R}\mathbb{V}$ ) is consistently significant at horizons of less than a year.  $VRP$  has the highest R-square at the three- to six-month horizons with both significance and R-square declining for longer horizons. Control variables have lower significance. Overall, our results corroborate those of BTZ in that on a univariate basis over short horizons,  $VRP$  has higher predictive power than variables used in previous literature, and is often the only variable that is significant. Next, in multivariate regressions, when control variables are included with  $VRP$ , we again find that  $VRP$  is highly significant for horizons up to one year. Many of the control variables are also significant in the multivariate setting; dividend-yield and  $DEF$  are significant at the shorter horizons, while  $CAY$  is significant at longer horizons of one- and two-years.  $RREL$  is significant for horizons exceeding one month.

To examine the importance of jump risk, we focus on the multivariate specifications. First, compare the results when  $VRP$  is defined as  $\mathbb{V} - \mathbb{R}\mathbb{V}$  with those when  $VRP$  is defined as  $\mathbb{IV} - \mathbb{R}\mathbb{V}$ . Hodrick  $t$ -statistics and adjusted R-square in the multivariate regressions are higher for the former specification of  $VRP$  for all horizons. For example, for the 6-month horizon, adjusted R-square increases from 32.2% when  $VRP$  is defined using the integrated variance to 34.7% when the BKM holding-period variance is used. For the one-year horizon, the  $VRP$  defined using  $\mathbb{IV} - \mathbb{R}\mathbb{V}$  is barely significant at the 10% level with a Hodrick  $t$ -value of 1.73. Bollerslev, Tauchen and Zhou (2009) have a similar finding for the identical horizon in their dataset; they find a  $t$ -value of 1.68 for  $\mathbb{IV} - \mathbb{R}\mathbb{V}$ . However, when we replace  $\mathbb{IV} - \mathbb{R}\mathbb{V}$  by  $\mathbb{V} - \mathbb{R}\mathbb{V}$ , the coefficient on  $VRP$  is now significant at conventional levels with a  $t$ -value of 2.05, and we reach a very different conclusion.

In the last two columns of Table 6, we add  $JTIX$  and  $JTIX/\mathbb{V}$  into the regression specification with  $\mathbb{IV} - \mathbb{R}\mathbb{V}$ .  $JTIX$  is significant at the 95% level for the six-month horizon, and at the 90% level for all other horizons up to 1-year using the robust Hodrick  $t$ -value.  $JTIX/\mathbb{V}$  is significant at the 95% level for 3-month and 6-month horizon and at the 90% level for 1-year horizon. The signs of the coefficients are positive indicating a higher expected return with greater jump risk. Interestingly, the significance of  $\mathbb{IV}$  also increases when  $JTIX$  is added to the regression.

The significance of  $JTIX$  in these specifications does not indicate that the tail of the distribution is significant in addition to volatility. Because  $\mathbb{IV}$  does not correctly account for

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For 1 month non-overlapping predictions, Hodrick(1992) 1B standard error is the same as Hansen-Hodrick (1980) standard error and very close to White standard error.

the contribution of jumps to stock return variability, the significance of *JTIX* simply reiterates the importance of accounting for jump risk in the measurement of return volatility. The results indicate that the jump-induced bias in  $\mathbb{IV}$  is statistically significant, but not necessarily that the tail is an additional channel of jump risk.

### 7.2.2 Economic Significance

To understand the economic significance of the jump-induced bias in  $\mathbb{IV}$ , we compare the predicted return by each measure of stock return variability.

In Panel A of Table 7, we tabulate *VRP* in terms of both  $\mathbb{V} - \mathbb{RV}$  and  $\mathbb{IV} - \mathbb{RV}$ . Over the entire sample period, the variance risk premium in terms of  $\mathbb{V} - \mathbb{RV}$  is 0.33% (volatility points) higher. The difference in recessions months increases to 0.91%. In the most recent financial crisis, the difference is even higher at 1.18% not only because  $\mathbb{V} - \mathbb{RV}$  increases, but because  $\mathbb{IV} - \mathbb{RV}$  is *lower* than in the previous recession. The integrated measure particularly underestimates return variability in the financial crisis because of the extraordinarily high degree of jump risk in this period.

Next, we compare the magnitude of the predicted return. Panel B of Table 7 reports the magnitude of the return predicted by  $\mathbb{V} - \mathbb{RV}$  and  $\mathbb{IV} - \mathbb{RV}$ . The returns are predicted using the coefficients estimated in Panel D of Table 6 for each of the *VRPs* for the one-year horizon. Over the entire sample period, the one-year excess return predicted by  $\mathbb{V} - \mathbb{RV}$  is 4.51% in comparison with 3.17% by  $\mathbb{IV} - \mathbb{RV}$ . That is, on average, the integrated variance underestimates the predicted excess return by 30%. As noted earlier, jump risk reaches its highest level in the most current recession. The excess return predicted by  $\mathbb{V} - \mathbb{RV}$  in the most recent recession is 6.06%, which is 80% higher than the 3.35% predicted by  $\mathbb{IV} - \mathbb{RV}$ . The return predicted by  $\mathbb{IV} - \mathbb{RV}$  during the financial crisis is lower than the return it predicts for the prior recession. The results for the financial crisis period underscore the economic importance of jump risk. Very simply, if jump risk is not correctly accounted for in the measurement of stock return variability (i.e., if one assumes the integrated variance is an accurate measure of stock return variability), one would arrive at the extraordinary conclusion that the financial crisis period was of lower risk compared to the previous 2001-02 recession.

In summary, the bias in  $\mathbb{IV}$  is both statistically and economically significant. It is necessary to account for time-variation in jump risk in computing stock return variability.

### 7.3 Tail vs. Volatility

Having ascertained that volatility is an important channel for jump risk, we next investigate whether the jump-induced tail of the distribution should be considered a separate channel. We now include  $JTIX$  and  $JTIX/V$  to the regression specification that includes  $VRP$  computed using the BKM variance.

Table 8 reports the results for all horizons previously considered. As in previous tables,  $VRP$  defined as  $\mathbb{V} - \mathbb{R}\mathbb{V}$  is significant at the highest levels for all horizons up to one-year. Including  $JTIX$  does not impact the economic significance of  $\mathbb{V} - \mathbb{R}\mathbb{V}$ ; the sign and magnitude of the estimated coefficients remain about the same. For example, the coefficient for the three-month horizon in Table 6 when  $JTIX$  is excluded is 1.41. In Table 8, it is now estimated as 1.45. The results are about the same for  $JTIX/V$  with the exception of the coefficient for  $VRP$  for the one-year horizon which reduces in significance from 95% to 90% level.

Next, consider the economic importance of the tail index. The statistical significance of both  $JTIX$  and  $JTIX/V$  is much reduced compared to the results of table 6. In particular, neither  $JTIX$  nor  $JTIX/V$  is significant at the 95% or higher level for any horizon.  $JTIX$  is barely significant at the 90% level for the six-month horizon with a Hodrick t-value of 1.68, and insignificant for all other horizons.  $JTIX/V$  is significant at the 90% level at the six-month horizon and one-year horizon. While  $JTIX/V$  is significant at the 95% level at the 3-month horizon (Hodrick t-value of 2.04) and 90% level for the 1-year horizon (Hodrick t-value of 1.85) in the regression with  $\mathbb{I}\mathbb{V} - \mathbb{R}\mathbb{V}$ , it is now insignificant for these horizons in the regression with  $\mathbb{V} - \mathbb{R}\mathbb{V}$ .

Hodrick standard errors are conservative. For the 1-month horizon, there is no overlap of observations, and we verify the conclusion using Newey-West errors. In Table 6,  $JTIX$  and  $JTIX/V$  are significant at conventional levels for the 1-month horizon when we use Newey-West to compute standard errors. When we compute Newey-West standard errors for  $JTIX$  and  $JTIX/V$  in the regression with  $\mathbb{V} - \mathbb{R}\mathbb{V}$ , we get slightly higher but still insignificant t-values of 1.04 and 0.34, respectively.

Together, Tables 6 and 8 lead us to the following conclusions. First, it is evident that volatility is an important channel through which the risk of discontinuities manifests itself. Thus, it is necessary to account for jumps in measuring volatility. But once the impact of jumps on volatility is accounted for by a model-free measure of stock return variability, it is less evident that there remains much additional variability in returns to be explained by the jump and tail index. The weak significance at the six-month horizon, at best, suggests a

second order role for tail risk compared with volatility risk. Of the two channels of jump risk, volatility is more significant than the tail.

## 8 Conclusion

When the risk-neutral stock return process incorporates fears of discontinuities, not only is the tail of the return distribution determined by these fears of jumps but so is stock return volatility. In this paper, we investigate the relative economic importance of both channels by constructing volatility and tail indexes that distinguish between the two. Our jump and tail index demonstrates the extreme time-variation in jump risk with intensity of jumps increasing 50-fold in times of crises. Not surprisingly, jump risk has a significant impact on stock return volatility. Using a measure of the integrated variance (like the square of the VIX) in times of crisis can underestimate market variance by as much as 12%, the variance risk premium by 30%, and the predicted equity return by over 40%. Although the economic impact of jump risk on stock return volatility is important, it is less clear that jump-induced tail constitutes an additional channel of risk for index returns. When stock return variability correctly accounts for jumps by using a model-free measure of stock return variability, the statistical significance of the tail index is much reduced. At best, the tail of the distribution is of second order significance relative to volatility. In summary, our results indicate that jump risk is economically important, but the channel through which it manifests itself is primarily volatility.

Overall, our analysis underscores the importance of understanding the variance risk premium from option prices, and its relation to the equity risk premium.

Our analysis and comparison of the Bakshi, Kapadia and Madan (2003) measure of holding period variance with the integrated variance measure is of independent interest. In particular, our analysis indicates that the VIX index should be used with caution as jump-induced bias may be significant in times of crisis. This is particularly important as the Chicago Board Options Exchange is introducing derivatives on the VIX, and also because the formulation of the VIX is now being duplicated and implemented for local indices in other countries (e.g. India, Canada). It is fortuitous that information about tail risk is in the VIX.

Our analysis also has implications for volatility derivative markets such as those of variance swaps. Instead of basing volatility derivatives on the quadratic variation (sum of squared log-returns), we recommend their payoffs be based on the second moment of the holding period return (square of summed log-returns). Derivatives based on the holding period return variability can be priced truly in a model-free manner.

Our tail index, easily constructed from option data using now standard techniques, is novel. It would be useful to compare it with other recently proposed measures of tail risk. We view both Kelly's and Bollerslev and Todorov's measures of tail risk as complementary to our own as each uses a different methodology. It would be interesting to understand in practice how the three measures relate to each other.

We used our jump and tail index to predict index returns and distinguish between volatility and tail risk. Gabaix (2010) suggests additional applications. We intend to pursue some of these applications in future research.

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**Table 1. Numerical Comparison:  $E^Q[\ln S, \ln S]_T$ ,  $\mathbb{V}$  and  $\mathbb{IV}$** 

Numerical comparison of annualized expected quadratic variation,  $E^Q[\ln S, \ln S]_T$  with  $\mathbb{V}$  and  $\mathbb{IV}$  for the jump-diffusion model of Merton (1976) (Panel A), and the jump-diffusion and stochastic volatility (SVJ) model of Bates (2000) (Panel B). The specification for the Merton model is:  $d \ln S_t = (r - \mu_J \lambda - \frac{1}{2} \sigma^2) dt + \sigma dW_t + x dJ_t$ , where  $x \sim N(\alpha, \sigma_J^2)$ , and  $\mu_J = e^{\alpha + \frac{1}{2} \sigma_J^2} - 1$ . The specification for the SVJ model is:  $d \ln S_t = (r - \frac{1}{2} \sigma_t^2 - \lambda \mu_J) dt + \sigma_t dW_t^1 + x dJ_t$ ,  $d\sigma_t^2 = \kappa(\sigma_t^2 - \theta) dt + \eta \sigma_t dW_t^2$ , where  $\text{corr}(dW_t^1, dW_t^2) = \rho$ ,  $x \sim N(\alpha, \sigma_J^2)$  and  $\lambda = \lambda_0 + \lambda_1 \cdot \sigma_t^2$ . Parameters are from Pan (2002);  $\lambda_0 = 0$ ,  $\lambda_1 = 12.3$  (for Merton's model,  $\lambda = \lambda_1 \cdot 0.04$ ),  $\sigma_J = 0.0387$ ,  $\kappa = 3.3$ ,  $\theta = 0.0296$ ,  $\eta = 0.3$ , and  $\rho = -0.53$ . In addition, interest rate  $r = 0.03$ , and time to maturity  $\tau = 30/365$ . Remaining parameters are shown in the table. The first column denotes the contribution of jumps to the total variance, defined as holding period variance of the pure jump component of the stochastic process divided by the holding period variance of the combined jump-diffusion process.

**Panel A: Merton model**

$\frac{\text{var}_{Jump}}{\text{var}}$	$\sigma_0^2$	$\alpha$	$E^Q[\ln S, \ln S]_T$	$\mathbb{V}$	$\mathbb{IV}$	$\mathbb{IV} - [\ln S, \ln S]_T$	
						Absolute	Relative
0%	0.0400	0.0000	0.0400	0.0400	0.0400	0.0000	0.00%
10%	0.0360	-0.0814	0.0400	0.0400	0.0399	-0.0001	-0.36%
20%	0.0320	-0.1215	0.0400	0.0400	0.0396	-0.0004	-0.92%
30%	0.0280	-0.1513	0.0400	0.0400	0.0393	-0.0007	-1.63%
40%	0.0240	-0.1761	0.0400	0.0400	0.0390	-0.0010	-2.44%
50%	0.0200	-0.1979	0.0400	0.0400	0.0387	-0.0013	-3.36%
60%	0.0160	-0.2174	0.0400	0.0400	0.0383	-0.0017	-4.36%
70%	0.0120	-0.2354	0.0400	0.0400	0.0378	-0.0022	-5.43%
80%	0.0080	-0.2521	0.0400	0.0400	0.0374	-0.0026	-6.58%
90%	0.0040	-0.2677	0.0400	0.0400	0.0369	-0.0031	-7.80%

**Panel B: SVJ model**

$\frac{\text{var}_{Jump}}{\text{var}}$	$\sigma_0^2$	$\alpha$	$E^Q[\ln S, \ln S]_T$	$\mathbb{V}$	$\mathbb{IV}$	$\mathbb{IV} - [\ln S, \ln S]_T$		$\mathbb{V} - [\ln S, \ln S]_T$	
						Absolute	Relative	Absolute	Relative
0%	0.0415	0.0000	0.0400	0.0402	0.0400	0.0000	0.00%	0.0002	0.61%
10%	0.0369	-0.0868	0.0400	0.0402	0.0399	-0.0001	-0.37%	0.0002	0.61%
20%	0.0323	-0.1372	0.0400	0.0402	0.0396	-0.0004	-1.01%	0.0002	0.60%
30%	0.0278	-0.1826	0.0400	0.0402	0.0392	-0.0008	-1.89%	0.0002	0.59%
40%	0.0232	-0.2296	0.0400	0.0402	0.0388	-0.0012	-3.04%	0.0002	0.58%
50%	0.0186	-0.2825	0.0400	0.0402	0.0382	-0.0018	-4.54%	0.0002	0.57%
60%	0.0141	-0.3471	0.0400	0.0402	0.0374	-0.0026	-6.52%	0.0002	0.55%
70%	0.0095	-0.4338	0.0400	0.0402	0.0363	-0.0037	-9.24%	0.0002	0.52%
80%	0.0049	-0.5690	0.0400	0.0402	0.0347	-0.0053	-13.34%	0.0002	0.47%
90%	0.0004	-0.8545	0.0400	0.0401	0.0316	-0.0084	-21.04%	0.0001	0.36%

**Table 2. Description of Option Data**

*Implied volatility* is the Black-Scholes implied volatility. *Range of Moneyness* on a certain date for a given underlying asset is defined as  $(K^{\max} - K^{\min})/K^{atm}$ , where  $K^{\max}$ ,  $K^{\min}$ , and  $K^{atm}$  are the maximum, minimum, and the at-the-money strikes, respectively.

Panel A. SPX Index Options						
	30-day	23-day	16-day	9-day	End of Month	
					Near Term	Next Term
Number of Dates	166	164	161	165	166	166
Total # of Options	7632	7666	7025	6011	7297	7061
Average # of Options Per Date	45.98	46.74	43.63	36.43	43.96	42.54
Average # of Puts Per Date	28.75	29.20	27.27	22.53	27.27	25.90
Average # of Calls Per Date	17.22	17.54	16.36	13.90	16.69	16.63
Average Range of Moneyness	42.54%	38.87%	33.78%	26.37%	34.19%	46.19%
Average Option Price	7.21	5.84	4.75	3.34	5.25	10.49
Average Implied Volatility	26.82%	26.95%	27.90%	29.41%	27.42%	26.00%
Average Trading Volume	2633	1960	1844	2257	1881	811
Average Open Interest	15512	19276	20806	22121	20243	11255

Panel B. Individual Equity Options				
	30-day	23-day	16-day	9-day
Number of Dates	166	164	161	165
Number of Firms	179	175	173	162
Total Number of Dates	8430	7411	6113	4268
Total # of Options	58105	49373	39086	25957
Average # of Options Per Date	6.89	6.66	6.39	6.08
Average # of Puts Per Date	3.65	3.57	3.41	3.22
Average # of Calls Per Date	3.24	3.09	2.98	2.86
Average Range of Moneyness	39.94%	36.35%	33.11%	28.65%
Average Option Price	1.07	0.92	0.82	0.73
Average Implied Volatility	48.55%	48.87%	52.04%	58.47%
Average Trading Volume	665	857	929	1231
Average Open Interest	5439	6333	7017	7567

**Table 3. Comparison Between  $\mathbb{V}$  and  $\mathbb{IV}$** 

The table reports the summary statistics of  $\sqrt{\mathbb{V}}$ ,  $\sqrt{\mathbb{IV}}$ ,  $\sqrt{\mathbb{V}} - \sqrt{\mathbb{IV}}$ ,  $JTIX = \mathbb{V} - \mathbb{IV}$ , and  $\sqrt{\mathbb{RV}}$ . Panel A shows the results for SPX index options. Panel B shows the results for individual options. Comparisons in full period, recession period and non-recession period are shown in each Panel. The recession period, defined by NBER, is: March 2001-November 2001 and December 2007- June 2009.  $\Delta$  refers to the difference between the two volatility measures:  $\sqrt{\mathbb{V}} - \sqrt{\mathbb{IV}}$ .  $\sqrt{\mathbb{RV}}$  is the average realized volatility calculated using the formula:  $\sqrt{\frac{252}{n} \sum_{i=1}^n \ln^2 \frac{S_i}{S_{i-1}}}$ , where  $S_i$  is the underlying security's closing price on the  $i$ 'th trading day of the option's remaining maturity, and  $n$  is the total number of trading days corresponding to the options's remaining maturity. The statistics from index options are time series average, and the statistics from individual options are pooled averages (across firms and dates).  $T$  is the remaining maturity of the option. "FIN" denotes financial firms and "Non-FIN" denotes non-financial firms. All numbers are scaled up by a factor of 100.

Panel A. SPX Index Options																			
$T$		Full Period					Recession					Non-Recession							
		Obs	$\sqrt{\mathbb{V}}$	$\sqrt{\mathbb{IV}}$	$\Delta$	$JTIX$	$\sqrt{\mathbb{RV}}$	Obs	$\sqrt{\mathbb{V}}$	$\sqrt{\mathbb{IV}}$	$\Delta$	$JTIX$	$\sqrt{\mathbb{RV}}$	Obs	$\sqrt{\mathbb{V}}$	$\sqrt{\mathbb{IV}}$	$\Delta$	$JTIX$	$\sqrt{\mathbb{RV}}$
30	Mean	166	22.71	22.22	0.49	0.32	18.11	28	34.04	33.04	1.00	0.97	29.38	138	20.41	20.03	0.38	0.18	15.82
	Std		10.08	9.58	0.56	0.73	10.61		15.60	14.57	1.08	1.59	16.65		6.55	6.32	0.28	0.21	7.05
	Min		10.41	10.28	0.07	0.02	6.07		19.24	18.97	0.25	0.10	13.31		10.41	10.28	0.07	0.02	6.07
	Max		78.90	75.32	4.56	6.71	77.93		78.90	75.52	4.56	6.71	77.93		40.41	39.16	2.16	1.66	44.92
23	Mean	164	22.51	22.09	0.42	0.27	18.10	28	32.39	31.56	0.84	0.79	29.21	136	20.47	20.14	0.34	0.16	15.82
	Std		9.55	9.12	0.50	0.71	10.71		14.15	13.21	1.02	1.60	16.87		6.75	6.57	0.22	0.16	7.11
	Min		9.85	9.73	0.08	0.02	5.26		18.53	18.29	0.20	0.08	12.45		9.85	9.73	0.08	0.02	5.26
	Max		81.82	76.47	5.36	8.48	84.26		81.82	76.47	5.36	8.48	84.26		42.74	41.93	1.13	0.76	46.59
16	Mean	161	22.94	22.58	0.36	0.22	18.04	26	33.41	32.71	0.71	0.62	30.31	135	20.92	20.62	0.30	0.15	15.67
	Std		9.41	9.09	0.39	0.42	11.18		12.94	12.31	0.68	0.85	18.17		7.01	6.82	0.25	0.19	7.27
	Min		10.14	10.06	0.07	0.01	5.45		20.86	20.61	0.16	0.07	13.64		10.14	10.06	0.07	0.01	5.45
	Max		66.89	64.83	2.94	3.53	87.23		66.89	64.83	2.94	3.53	87.23		42.80	41.57	2.29	1.64	41.01
9	Mean	165	23.72	23.45	0.28	0.18	17.73	27	34.71	34.18	0.53	0.52	29.36	138	21.57	21.35	0.23	0.12	15.45
	Std		10.71	10.46	0.28	0.37	12.29		16.12	15.63	0.53	0.79	21.34		7.71	7.58	0.15	0.14	7.90
	Min		10.46	10.36	0.05	0.01	4.96		20.04	19.94	0.10	0.04	10.52		10.46	10.36	0.05	0.01	4.96
	Max		80.07	78.53	2.13	3.27	102.51		80.07	78.53	2.13	3.27	102.51		49.28	48.43	1.04	0.89	48.11

Panel B. Individual Options																				
$T$	Firm Type	Full Period					Recession					Non-Recession								
		Obs	$\sqrt{\mathbb{V}}$	$\sqrt{\mathbb{IV}}$	$\Delta$	$JTIX$	$\sqrt{\mathbb{RV}}$	Obs	$\sqrt{\mathbb{V}}$	$\sqrt{\mathbb{IV}}$	$\Delta$	$JTIX$	$\sqrt{\mathbb{RV}}$	Obs	$\sqrt{\mathbb{V}}$	$\sqrt{\mathbb{IV}}$	$\Delta$	$JTIX$	$\sqrt{\mathbb{RV}}$	
30	All	Mean	8430	44.04	43.32	0.72	1.08	43.38	1919	54.70	53.31	1.39	2.71	51.37	6511	40.90	40.38	0.52	0.61	41.02
		Std		21.25	20.14	1.58	5.22	37.23		28.78	26.61	2.75	9.98	41.89		17.24	16.69	0.90	2.22	35.40
		Min		13.19	13.16	-2.05	-2.42	7.05		18.16	18.01	-0.48	-0.64	9.61		13.19	13.16	-2.05	-2.42	7.05
		Max		252.82	224.39	39.60	153.72	1134.79		252.82	224.39	39.60	153.72	1134.79		198.87	188.90	34.26	102.35	480.13
	FIN	Mean	1541	47.76	46.58	1.18	2.37	47.81	365	72.35	69.11	3.23	8.18	72.42	1176	40.13	39.59	0.54	0.56	40.17
		Std		28.48	26.18	2.87	10.39	47.89		42.67	38.19	5.28	20.19	73.73		15.91	15.48	0.68	1.15	32.80
		Min		13.19	13.16	-2.05	-2.42	7.59		18.16	18.01	-0.42	-0.64	12.41		13.19	13.16	-2.05	-2.42	7.59
		Max		241.94	211.06	39.60	153.72	1134.79		241.94	211.06	39.60	153.72	1134.79		131.54	124.61	6.93	17.75	384.27
	Non-FIN	Mean	6889	43.21	42.59	0.61	0.80	42.39	1554	50.56	49.60	0.96	1.42	46.42	5335	41.07	40.55	0.51	0.62	41.21
		Std		19.17	18.45	1.07	2.96	34.32		22.50	21.46	1.36	4.33	27.65		17.52	16.94	0.94	2.39	35.95
		Min		14.66	14.61	-1.23	-2.05	7.05		19.65	19.46	-0.48	-0.60	9.61		14.66	14.61	-1.23	-2.05	7.05
		Max		252.82	224.39	34.26	135.65	480.13		252.82	224.39	28.43	135.65	416.89		198.87	188.90	34.26	102.35	480.13
23	All	Mean	7411	44.15	43.52	0.63	0.88	45.37	1826	51.73	50.60	1.12	1.89	52.38	5585	41.67	41.20	0.47	0.56	43.08
		Std		19.71	18.85	1.23	3.44	42.50		24.68	23.13	1.96	6.02	50.08		17.08	16.58	0.80	1.84	39.44
		Min		15.57	15.47	-1.06	-1.31	6.87		16.33	16.28	-1.06	-1.13	8.80		15.57	15.47	-1.00	-1.31	6.87
		Max		224.22	202.33	28.71	93.36	1321.19		224.22	202.33	25.33	93.36	1321.19		198.03	183.34	28.71	64.39	448.78
	FIN	Mean	1410	47.70	46.72	0.98	1.71	50.78	360	67.31	64.90	2.41	5.18	76.10	1050	40.98	40.48	0.49	0.52	42.10
		Std		25.08	23.47	2.04	6.23	57.85		34.80	31.80	3.54	11.54	90.71		15.92	15.54	0.58	1.03	37.19
		Min		15.57	15.47	-1.06	-1.13	8.32		21.28	21.11	-1.06	-1.13	13.34		15.57	15.47	-0.60	-0.65	8.32
		Max		224.22	202.33	25.33	93.36	1321.19		224.22	202.33	25.33	93.36	1321.19		147.50	143.54	5.44	14.16	438.48
	Non-FIN	Mean	6001	43.31	42.76	0.55	0.69	44.10	1466	47.90	47.10	0.81	1.08	46.56	4535	41.83	41.37	0.47	0.56	43.30
		Std		18.12	17.51	0.93	2.29	37.90		19.69	18.87	1.10	3.02	30.60		17.33	16.81	0.84	1.98	39.95
		Min		15.63	15.57	-1.00	-1.31	6.87		16.33	16.28	-0.55	-0.84	8.80		15.63	15.57	-1.00	-1.31	6.87
		Max		206.29	183.34	28.71	91.46	463.18		206.29	182.78	23.51	91.46	463.18		198.03	183.34	28.71	64.39	448.78
16	All	Mean	6113	46.99	46.42	0.57	0.83	47.00	1571	53.43	52.48	0.95	1.68	53.65	4542	44.77	44.33	0.44	0.53	44.70
		Std		20.51	19.77	1.05	2.91	46.78		25.98	24.65	1.69	5.05	55.09		17.70	17.30	0.65	1.50	43.30
		Min		16.33	16.24	-2.47	-4.16	6.35		16.85	16.80	-0.68	-1.07	8.16		16.33	16.24	-2.47	-4.16	6.35
		Max		231.36	222.45	20.01	67.19	1562.64		231.36	222.45	20.01	67.19	1562.64		215.85	211.04	11.30	35.70	539.78
	FIN	Mean	1259	50.70	49.82	0.88	1.61	51.61	334	70.92	68.83	2.09	4.65	78.95	925	43.41	42.96	0.45	0.51	41.76
		Std		26.23	24.81	1.77	5.41	62.60		35.79	33.28	2.98	9.57	99.76		16.55	16.17	0.59	1.49	37.25
		Min		16.91	16.82	-0.81	-1.67	9.25		24.88	24.76	-0.51	-0.64	15.32		16.91	16.82	-0.81	-1.67	9.25
		Max		231.36	222.45	20.01	67.19	1562.64		231.36	222.45	20.01	67.19	1562.64		215.85	211.04	9.96	34.75	539.78
	Non-FIN	Mean	4854	46.03	45.54	0.49	0.63	45.81	1237	48.71	48.06	0.64	0.88	46.84	3617	45.11	44.68	0.44	0.54	45.46
		Std		18.62	18.14	0.74	1.71	41.63		20.18	19.53	0.88	2.16	30.98		17.97	17.56	0.67	1.51	44.69
		Min		16.33	16.24	-2.47	-4.16	6.35		16.85	16.80	-0.68	-1.07	8.16		16.33	16.24	-2.47	-4.16	6.35
		Max		191.74	182.88	13.61	43.82	523.03		191.74	182.88	13.61	43.82	523.03		189.81	180.65	11.30	35.70	523.03
9	All	Mean	4268	52.86	52.35	0.51	0.81	48.89	1226	58.84	58.09	0.75	1.43	54.83	3042	50.45	50.04	0.41	0.56	46.50
		Std		23.10	22.49	0.85	2.56	47.71		28.88	27.91	1.21	4.07	46.44		19.82	19.43	0.63	1.51	48.01
		Min		18.58	18.59	-2.83	-5.15	2.11		19.30	19.24	-1.23	-1.76	8.24		18.58	18.59	-2.83	-5.15	2.11
		Max		254.20	241.45	13.64	63.22	767.53		254.20	241.45	13.64	63.22	767.53		209.13	204.41	8.74	26.18	658.65
	FIN	Mean	972	57.82	57.06	0.75	1.49	53.85	301	76.58	75.10	1.48	3.52	77.						



**Table 5. Predictive Regression: Summary Statistics**

This table reports summary statistics of the variables used in predictive regressions over the sample period from January 1996 to October 2009. Panel A reports the basic statistics. Panel B reports the cross-correlation matrix. Panel C reports the autocorrelations up to order six. Panel D reports results for the Phillips-Perron unit root test.  $R_t - r_t^f$  is the monthly excess return on the S&P 500 index.  $\mathbb{V}$  is the annualized holding period variance.  $\mathbb{W}$  is the annualized integrated variance.  $\mathbb{R}\mathbb{V}$  is the annualized realized variance from Hao Zhou's website.  $\log(P/E)$  and  $\log(P/D)$  are the logarithm of price-earnings ratio and price-dividend ratio on S&P 500 index, respectively.  $RREL$  is the detrended annualized risk-free rate.  $CAY$  is the consumption-wealth ratio of Lettau and Ludvigson (2001).  $TERM$  is the difference between the ten-year and three-month Treasury yield.  $DEF$  is the difference between Moody's BAA and AAA bond yield.  $SKEW$  and  $KURT$  are the skew and kurtosis of the risk-neutral distribution.

	$R_t - r_t^f$	$\mathbb{V}_t$	$\mathbb{W}_t$	$\mathbb{R}\mathbb{V}_t$	$\mathbb{V}_t - \mathbb{R}\mathbb{V}_{t-1}$	$\mathbb{W}_t - \mathbb{R}\mathbb{V}_{t-1}$	$\mathbb{V}_t - \mathbb{R}\mathbb{V}_{t-1}$	$\mathbb{W}_t - \mathbb{R}\mathbb{V}_{t-1}$	$SKEW$	$KURT$	$\log(P_t/E_t)$	$\log(P_t/D_t)$	$TERM_t$	$DEF_t$	$RREL_t$	$CAY_t$
Mean	0.0020	0.0622	0.0589	0.0033	0.0392	0.0329	0.0293	0.0260	-1.9345	11.0024	3.2680	4.0773	0.0144	0.0100	-0.0021	-0.0060
Median	0.0474	0.0602	0.0541	0.0065	0.0173	0.0547	0.0290	0.0287	0.5199	4.1919	0.4181	0.2408	0.0122	0.0051	0.0091	0.0171
Std. Dev.	0.0085	0.0502	0.0487	0.0016	0.0347	0.0193	0.0213	0.0202	-1.8949	9.7910	3.2156	4.0671	0.0124	0.0086	-0.0021	-0.0092
Skewness	-0.85	3.46	3.19	6.46	2.10	6.82	1.21	-0.90	-0.10	0.88	1.92	-0.54	0.21	2.78	-0.47	0.24
Kurtosis	4.31	18.88	16.36	55.27	10.67	62.66	12.67	22.24	2.34	3.38	7.36	3.57	1.84	11.38	2.80	2.04
AR(1)	0.13	0.80	0.80	0.65	0.71	0.64	0.37	0.27	0.50	0.46	0.97	0.97	0.98	0.97	0.85	0.96

Panel B. Correlation Matrix

	$R_t - r_t^f$	$V_t$	$IV_t$	$JTIX$	$JTIX/V$	$\mathbb{R}V_{t-1}$	$V_t - \mathbb{R}V_{t-1}$	$IV_t - \mathbb{R}V_{t-1}$	$SKEW$	$KURT$	$\log(P_t/E_t)$	$\log(P_t/D_t)$	$TERM_t$	$DEF_t$	$RREL_t$	$CAY_t$
$R_t - r_t^f$	1.00	-0.43	-0.43	-0.39	-0.21	-0.40	-0.13	-0.04	-0.19	0.23	0.03	-0.05	-0.04	-0.16	0.14	-0.11
$V_t$	1.00	1.00	0.93	0.82	0.88	0.42	0.21	0.06	0.06	-0.11	0.43	-0.30	0.25	0.67	-0.38	0.40
$IV_t$	1.00	1.00	0.92	0.82	0.86	0.45	0.25	0.09	0.09	-0.14	0.44	-0.29	0.25	0.67	-0.39	0.41
$JTIX$	1.00	1.00	1.00	0.83	0.94	0.16	-0.06	-0.11	0.07	0.29	0.29	-0.31	0.21	0.62	-0.29	0.28
$JTIX/V$	1.00	1.00	1.00	1.00	0.71	0.36	0.18	-0.43	0.31	0.50	0.50	-0.24	0.17	0.58	-0.34	0.25
$\mathbb{R}V_{t-1}$	1.00	1.00	1.00	1.00	1.00	-0.07	-0.28	0.00	-0.01	0.24	0.24	-0.27	0.22	0.61	-0.29	0.26
$V_t - \mathbb{R}V_{t-1}$	1.00	1.00	1.00	1.00	1.00	1.00	0.97	0.13	-0.21	0.43	0.43	-0.11	0.10	0.23	-0.25	0.34
$IV_t - \mathbb{R}V_{t-1}$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.15	-0.23	0.37	0.37	-0.04	0.05	0.09	-0.18	0.28
$SKEW$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-0.92	-0.01	0.08	0.14	-0.05	-0.16	0.35	0.35
$KURT$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-0.09	-0.05	0.10	0.20	-0.42
$\log(P_t/E_t)$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-0.19	0.47	0.60	-0.41	0.19
$\log(P_t/D_t)$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-0.36	-0.62	0.22	-0.18
$TERM_t$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.45	-0.47	0.15
$DEF_t$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-0.43	0.08
$RREL_t$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-0.33
$CAY_t$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Panel C. Autocorrelations

	$V$	$IV$	$JTIX$	$JTIX/V$	$V_t - \mathbb{R}V_{t-1}$	$IV_t - \mathbb{R}V_{t-1}$	$SKEW$	$KURT$
ACF(1)	0.79	0.80	0.65	0.71	0.36	0.27	0.49	0.46
ACF(2)	0.55	0.57	0.38	0.56	0.32	0.23	0.39	0.39
ACF(3)	0.47	0.48	0.33	0.49	0.26	0.16	0.41	0.35
ACF(4)	0.42	0.44	0.26	0.39	0.14	0.05	0.34	0.29
ACF(5)	0.34	0.36	0.21	0.33	0.20	0.14	0.30	0.37
ACF(6)	0.24	0.25	0.12	0.29	0.13	0.08	0.28	0.33

Panel D. Phillips-Perron Unit Root Test

	$V$	$IV$	$JTIX$	$JTIX/V$	$V_t - \mathbb{R}V_{t-1}$	$IV_t - \mathbb{R}V_{t-1}$	$SKEW$	$KURT$
$Z_{t\alpha}$	-4.29	-4.19	-5.94	-5.06	-9.02	-10.02	-7.81	-8.40
$p$ -value	< 0.01	< 0.01	< 0.01	< 0.01	< 0.01	< 0.01	< 0.01	< 0.01

**Table 6. Predictability Regressions**

Regression results for horizons of one-month, three-month, six-month, one-year and two-year, respectively. The full period extends from January 1996 to October 2009. All regression are based on monthly frequency. Hodrick (1992)  $t$ -statistics are reported below the coefficient. For one-month horizon, Newey-West (1994)  $t$ -statistics are also reported. Obs is the number of observations.

		Panel A. Monthly Return Prediction										
		0.01	0.00	0.08	0.00	0.00	0.00	0.01	0.25	0.24	0.27	0.25
<i>Const</i>	Coef.	-0.01	-0.01	0.00	0.00	0.00	0.00	0.01	0.25	0.24	0.27	0.25
	Newey West $t$	(-1.54)	(-1.73*)	(1.02)	(0.66)	(0.81)	(0.24)	(1.02)	(3.55***)	(3.32***)	(3.75***)	(3.75***)
$V_t - \mathbb{R}V_{t-1}$	Hodrick $t$	(-1.53)	(-1.78*)	(1.03)	(0.70)	(1.01)	(0.83)	(1.04)	(3.29***)	(3.12***)	(3.32***)	(3.21***)
	Coef.	0.29							0.46			
$IV_t - \mathbb{R}V_{t-1}$	Newey West $t$	(2.57**)							(5.00***)			
	Hodrick $t$	(2.09**)							(3.86***)			
<i>JTIX</i>	Coef.	0.33							0.42	0.47	0.42	
	Newey West $t$	(4.05***)							(4.11***)	(5.09***)	(5.09***)	
<i>JTIX/V</i>	Hodrick $t$	(2.80***)							(3.43***)	(3.93***)	(3.78***)	
	Coef.	-0.59									0.91	
<i>JTIX/V</i>	Newey West $t$	(-0.89)									(2.00**)	
	Hodrick $t$	(-0.93)									(1.71*)	
$\log(P/E)$	Coef.	-0.14										0.21
	Newey West $t$	(-0.44)										(2.00**)
$\log(P/D)$	Hodrick $t$	(-0.49)										(0.78)
	Coef.	0.00										
<i>RREL</i>	Newey West $t$	(0.00)										
	Hodrick $t$	(-0.00)										
<i>CAY</i>	Coef.	-0.02										
	Newey West $t$	(-0.81)										
<i>TERM</i>	Hodrick $t$	(-1.00)										
	Coef.	0.56							0.40	0.41	0.38	0.43
<i>DEF</i>	Newey West $t$	(1.43)							(1.00)	(1.01)	(0.96)	(0.96)
	Hodrick $t$	(1.46)							(0.94)	(0.94)	(0.90)	(1.02)
$\log(P/E)$	Coef.	-0.08							-0.36	-0.29	-0.41	-0.34
	Newey West $t$	(-0.38)							(-2.19**)	(-1.83*)	(-2.21**)	(-2.21**)
<i>DEF</i>	Hodrick $t$	(-0.39)							(-1.75*)	(-1.42)	(-1.86*)	(-1.60)
	Coef.	-0.12							0.12	0.10	0.15	0.14
$\log(P/D)$	Newey West $t$	(-0.33)							(0.44)	(0.34)	(0.53)	(0.53)
	Hodrick $t$	(-0.42)							(0.44)	(0.34)	(0.52)	(0.48)
$\log(P/E)$	Coef.	-0.91							-2.97	-2.49	-3.42	-3.00
	Newey West $t$	(-0.73)							(-3.29***)	(-2.74***)	(-2.68**)	(-2.68**)
<i>Adj.R<sup>2</sup></i>	Hodrick $t$	(-0.84)							(-2.66***)	(-2.13**)	(-2.55**)	(-2.17**)
	Coef.	2.60%	3.38%	0.06%	-0.37%	-0.61%	0.41%	0.53%	-0.53%	8.86%	8.27%	8.49%
Obs		165	165	165	165	165	165	165	165	165	165	165
												8.02%

**Panel B. Three-month Return Prediction**

<i>Const</i>	-0.02 (-1.97*)	0.01 (0.65)	-0.01 (-0.21)	-0.03 (-0.34)	0.26 (1.10)	0.01 (0.89)	0.01 (0.53)	0.01 (0.67)	0.02 (0.71)	0.64 (2.58**)	0.60 (2.41**)	0.68 (2.73***)	0.68 (2.74***)
$V_t - \mathbb{R}V_{t-1}$	1.02 (3.23***)									1.41 (4.76***)			
$IV_t - \mathbb{R}V_{t-1}$	1.08 (3.80***)	-0.66 (-0.33)									1.29 (4.53***)	1.45 (4.79***)	1.27 (4.68***)
<i>JTIX</i>												3.08 (1.85*)	
<i>JTIX/V</i>			0.28 (0.38)										1.45 (2.04**)
$\log(P/E)$				0.01 (0.38)									
$\log(P/D)$					-0.06 (-1.09)					-0.15 (-2.64***)	-0.14 (-2.48**)	-0.16 (-2.77***)	-0.17 (-2.91***)
<i>RREL</i>						1.96 (1.73*)				2.47 (2.03**)	2.49 (2.04**)	2.42 (2.00**)	2.69 (2.20**)
<i>CAY</i>							0.32 (0.56)			-0.36 (-0.60)	-0.15 (-0.25)	-0.56 (-0.85)	-0.49 (-0.78)
<i>TERM</i>								-0.37 (-0.42)		0.52 (0.62)	0.42 (0.50)	0.61 (0.74)	0.79 (0.94)
<i>DEF</i>									-1.71 (-0.52)	-6.43 (-1.78*)	-4.93 (-1.34)	-8.13 (-1.99**)	-8.66 (-2.06**)
<i>Adj.R</i> <sup>2</sup>	10.72% 163	11.75% 163	-0.32% 163	-0.39% 163	2.24% 163	3.50% 163	-0.24% 163	-0.36% 163	0.36% 163	27.80% 163	25.98% 163	28.13% 163	30.04% 163
Obs	163	163	163	163	163	163	163	163	163	163	163	163	163

**Panel C. Six-month Return Prediction**

<i>Const</i>	-0.04 (-1.51)	0.00 (0.07)	-0.03 (-0.68)	0.02 (0.13)	0.54 (1.15)	0.02 (0.78)	0.02 (0.67)	0.01 (0.42)	0.02 (0.36)	0.84 (1.59)	0.78 (1.48)	0.96 (1.80*)	0.97 (1.81*)
$V_t - \mathbb{R}V_{t-1}$	1.51 (4.13***)									1.85 (4.42***)			
$IV_t - \mathbb{R}V_{t-1}$	1.44 (4.04***)	2.00 (0.51)									1.61 (4.12***)	1.95 (4.48***)	1.58 (4.21***)
<i>JTIX</i>												6.90 (2.18**)	
<i>JTIX/V</i>			1.11 (0.81)										2.76 (2.17**)
$\log(P/E)$				0.00 (-0.08)									
$\log(P/D)$					-0.13 (-1.14)					-0.20 (-1.66*)	-0.19 (-1.55)	-0.23 (-1.83*)	-0.25 (-1.97*)
<i>RREL</i>						4.15 (1.77*)				6.86 (2.69***)	6.86 (2.67***)	6.68 (2.65***)	7.25 (2.84***)
<i>CAY</i>							1.54 (1.48)			1.11 (0.91)	1.42 (1.18)	0.52 (0.42)	0.72 (0.58)
<i>TERM</i>								-0.36 (-0.20)		1.31 (0.79)	1.18 (0.71)	1.58 (0.97)	1.91 (1.16)
<i>DEF</i>									-1.34 (-0.22)	-5.36 (-0.81)	-3.33 (-0.51)	-10.57 (-1.65)	-10.28 (-1.49)
<i>Adj.R</i> <sup>2</sup>	10.19% 160	8.96% 160	0.34% 160	-0.62% 160	4.22% 160	7.42% 160	3.32% 160	-0.53% 160	-0.39% 160	34.73% 160	32.18% 160	37.47% 160	38.77% 160
Obs	160	160	160	160	160	160	160	160	160	160	160	160	160

**Panel D. One-year Return Prediction**

<i>Const</i>	-0.03 (-0.61)	-0.02 (0.05)	0.55 (1.08)	1.30 (1.36)	0.02 (0.54)	0.03 (0.67)	-0.01 (-0.13)	0.19 (1.45)	1.51 (1.48)	1.46 (1.43)	1.67 (1.62)	1.70 (1.65)
$V_t - \mathbb{R}V_{t-1}$	1.28 (1.61)								1.54 (2.05**)			
$\mathbb{I}V_t - \mathbb{R}V_{t-1}$	1.09 (1.60)								1.22 (1.73*)	1.58 (2.09**)	1.11 (1.61)	
<i>JTIX</i>		2.16 (1.23)								8.85 (1.83*)		3.81 (1.85*)
<i>JTIX/V</i>		0.63 (0.39)										
$\log(P/E)$			-0.17 (-1.04)									
$\log(P/D)$				-0.31 (-1.35)								
<i>RREL</i>					9.03 (1.99**)							
<i>CAY</i>						3.77 (1.97*)						
<i>TERM</i>							1.19 (0.38)					
<i>DEF</i>							-20.84 (-1.43)					
<i>Adj. R</i> <sup>2</sup>	2.18% 154	1.57% 154	-0.43% 154	4.41% 154	8.65% 154	16.32% 154	10.05% 154	-0.16% 154	7.84% 154	55.91% 154	59.60% 154	61.93% 154
Obs.												

**Panel E. Two-year Return Prediction**

<i>Const</i>	-0.05 (-0.70)	-0.05 (-0.73)	0.02 (0.15)	0.33 (0.44)	4.09 (2.45**)	0.03 (0.32)	0.08 (0.89)	-0.13 (-1.21)	0.11 (0.39)	3.26 (1.93*)	3.24 (1.91*)	3.35 (2.01**)	3.34 (1.97*)
$V_t - \mathbb{R}V_{t-1}$	2.67 (1.75*)									2.62 (1.86*)			
$\mathbb{I}V_t - \mathbb{R}V_{t-1}$	2.99 (1.82*)									2.78 (1.84*)	1.54 (1.16)	1.56 (1.91*)	
<i>JTIX</i>		14.92 (0.95)									14.68 (1.19)		4.19 (1.40)
<i>JTIX/V</i>			0.26 (0.08)										
$\log(P/E)$				-0.10 (-0.38)									
$\log(P/D)$					-0.98 (-2.38**)								
<i>RREL</i>						-2.59 (-0.40)							
<i>CAY</i>							8.50 (2.77***)						
<i>TERM</i>							11.93 (2.75***)						
<i>DEF</i>							-10.02 (-0.33)						
<i>Adj. R</i> <sup>2</sup>	4.15% 142	4.43% 142	0.82% 142	-0.70% 142	0.05% 142	36.26% 142	-0.18% 142	25.18% 142	22.95% 142	76.44% 142	76.61% 142	77.99% 142	77.99% 142
Obs.													

**Table 7. Predicted Excess Return by  $\mathbb{V} - \mathbb{R}\mathbb{V}$  and  $\mathbb{I}\mathbb{V} - \mathbb{R}\mathbb{V}$** 

The predicted excess return by  $\mathbb{V} - \mathbb{R}\mathbb{V}$  or  $\mathbb{I}\mathbb{V} - \mathbb{R}\mathbb{V}$  for each observation date is computed as the the estimated coefficient in the multivariate regression (from Panel D of Table 6) multiplied by the variable value. “Subprime recession” refers to the recent NBER recession: December 2007 - June 2009.

Panel A. Summary Statistics of $VRPs$					
		Full	NBER	NBER	Subprime
		Period	Non-recession	Recession	Recession
$\mathbb{V} - \mathbb{R}\mathbb{V}$	Mean	0.0293	0.0275	0.0381	0.0393
$\mathbb{I}\mathbb{V} - \mathbb{R}\mathbb{V}$	Mean	0.0260	0.0254	0.0290	0.0275

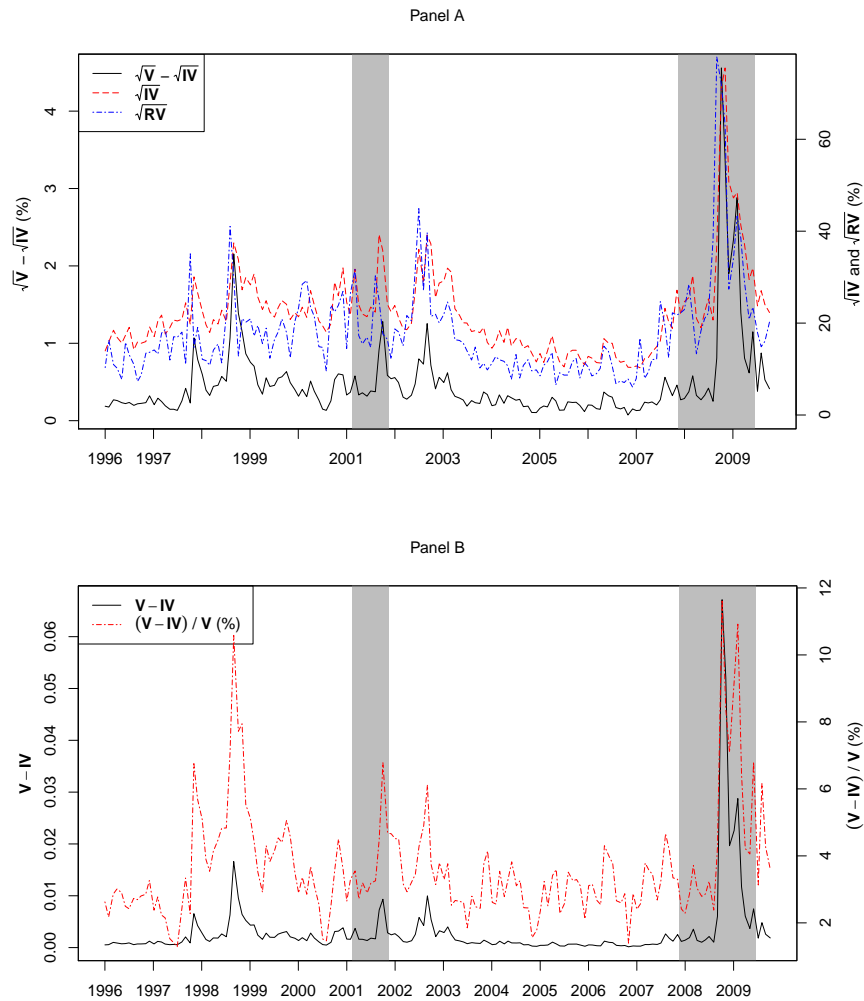
  

Panel B. Summary Statistics of Predicted Return					
		Full	NBER	NBER	Subprime
		Period	Non-recession	Recession	Recession
$\mathbb{V} - \mathbb{R}\mathbb{V}$	Mean	4.51%	4.23%	5.87%	6.06%
$\mathbb{I}\mathbb{V} - \mathbb{R}\mathbb{V}$	Mean	3.17%	3.09%	3.53%	3.35%
Difference	Mean	1.35%	1.14%	2.34%	2.71%

**Table 8. Prediction with  $\mathbb{V} - \mathbb{R}\mathbb{V}$  and  $JTIX$** 

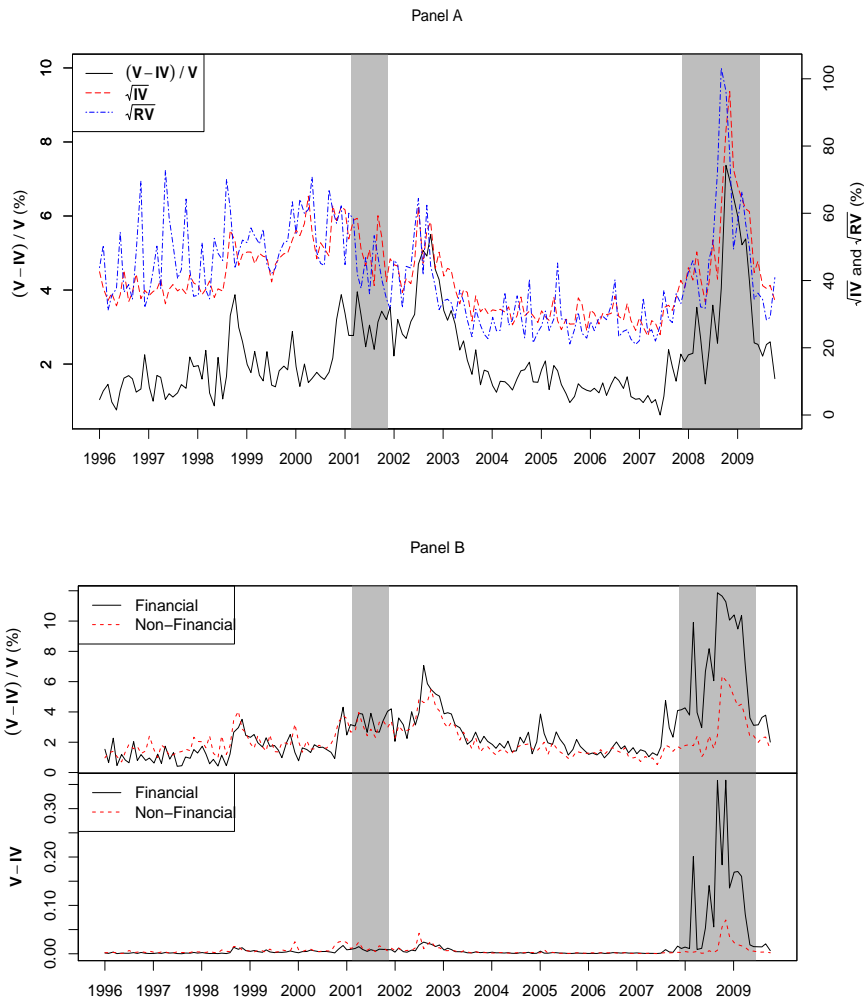
Regression results for horizons of one-month, three-month, six-month, one-year and two-year. Hodrick (1992)  $t$ -statistics are reported in parentheses. Obs is the number of observations.

	$JTIX$						$JTIX/\mathbb{V}$				
	1m	3m	6m	12m	24m		1m	3m	6m	12m	24m
$Const$	0.27 (3.32***)	0.68 (2.73***)	0.96 (1.80*)	1.67 (1.62)	3.35 (2.01**)	$Const$	0.26 (3.30***)	0.7 (2.81***)	0.98 (1.83*)	1.71 (1.66*)	3.34 (1.98**)
$\mathbb{V}_t - \mathbb{R}\mathbb{V}_{t-1}$	0.47 (3.93***)	1.45 (4.79***)	1.95 (4.48***)	1.58 (2.09**)	1.54 (1.16)	$\mathbb{V}_t - \mathbb{R}\mathbb{V}_{t-1}$	0.45 (3.91***)	1.32 (4.74***)	1.67 (4.46***)	1.14 (1.70*)	1.44 (1.93*)
$JTIX$	0.44 (0.95)	1.63 (1.08)	4.95 (1.68*)	7.27 (1.58)	13.14 (1.04)	$JTIX/\mathbb{V}$	0.09 (0.33)	1.09 (1.58)	2.29 (1.85*)	3.50 (1.72*)	4.07 (1.37)
$\log(P/D)$	-0.06 (-3.28***)	-0.16 (-2.77***)	-0.23 (-1.83*)	-0.38 (-1.58)	-0.80 (-2.01**)	$\log(P/D)$	-0.06 (-3.24***)	-0.17 (-2.93***)	-0.25 (-1.97*)	-0.42 (-1.71*)	-0.83 (-2.04**)
$RREL$	0.38 (0.90)	2.42 (2.00**)	6.68 (2.65***)	13.76 (2.96***)	9.77 (2.06**)	$RREL$	0.41 (0.96)	2.60 (2.13**)	7.14 (2.79***)	14.61 (3.10***)	11.12 (2.29**)
$CAY$	-0.41 (-1.86*)	-0.56 (-0.85)	0.52 (0.42)	3.21 (1.43)	5.83 (1.97*)	$CAY$	-0.37 (-1.77*)	-0.58 (-0.91)	0.61 (0.49)	3.37 (1.53)	6.10 (2.10**)
$TERM$	0.15 (0.52)	0.61 (0.74)	1.58 (0.97)	5.64 (1.82*)	13.6 (3.91***)	$TERM$	0.14 (0.49)	0.78 (0.93)	1.89 (1.15)	6.05 (1.91*)	14.12 (4.10***)
$DEF$	-3.42 (-2.55**)	-8.13 (-1.99**)	-10.57 (-1.65)	-20.62 (-1.17)	-23.7 (-0.94)	$DEF$	-3.17 (-2.33**)	-9.11 (-2.18**)	-10.86 (-1.57)	-20.53 (-1.25)	-23.38 (-0.89)
$Adj.R^2$	8.49%	28.13%	37.47%	59.60%	76.61%	$Adj.R^2$	8.34%	29.86%	39.03%	61.80%	77.96%
Obs	165	163	160	154	142	Obs	165	163	160	154	142



**Figure 1. Time Series Plot of the Comparison from SPX 30-day Options**

Panel A plots the time series difference between 30-day holding period volatility ( $\sqrt{V}$ ) and the integrated volatility ( $\sqrt{IV}$ ) for S&P 500 during the sample period January 1996- October 2009. Plots of  $\sqrt{IV}$  and the realized volatility,  $\sqrt{RV}$ , are also shown for comparison. Panel B plots  $JTIX = V - IV$ , and  $JTIX/V$ . Shaded areas represent NBER defined recessions (in our sample: March 2001- November 2001 and December 2007- June 2009). Plots are based on monthly observations.



**Figure 2. Time Series Plot of the Comparison from Individual Options**

Plots of  $JTIX = \mathbb{V} - \mathbb{IV}$  and  $JTIX/\mathbb{V} = (\mathbb{V} - \mathbb{IV})/\mathbb{V}$ . Panel A plots  $JTIX/\mathbb{V}$  for individual firms during the sample period January 1996- October 2009. Panel B plots  $JTIX$  and  $JTIX/\mathbb{V}$  for financial firms and non-financial firms, respectively. Shaded areas represent NBER defined recession period (in our sample: March 2001- November 2001 and December 2007- June 2009). Plots are based on monthly observations.