Tail and Volatility Indices from Option Prices

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Abstract

Both volatility and the tail of stock return distributions are impacted by discontinuities or large jumps in the stock price process. In this paper, we construct a model-free jump and tail index by measuring the impact of jumps on the Chicago Board Options Exchange's VIX index. Our jump and tail index is constructed from a portfolio of risk-reversals using 30-day index options, and measures time variations in the intensity of return jumps. Using the index, we document a 50-fold increase in jump fears during the financial crisis, and that jump fears predict index returns after controlling for stock return variability.

Keywords: VIX, quadratic variation, tail risk, jump intensity, return predictability

JEL classification: G1, G12, G13

1 Introduction

Understanding time variation in volatility is important in asset pricing since it impacts the pricing of both equities and options. It is also of interest to understand whether time variation in tail risk —the possibility of an extreme return from a discontinuity or large jump in the stock price process– should be considered an additional channel of risk.[1] However, jumps in the stock price process not only determine the tail of the distribution but also impact stock return variability. Before one can determine whether there are potentially distinct roles for stock return variability and tail risk, respectively, it is essential to differentiate one from the other. In this paper, we address this issue by constructing model-free volatility and tail indices from option prices that allow researchers to distinguish between the two channels.

Volatility and jump/tail indices already exist in the literature. The most widely used option-based measure of stock return variability is the Chicago Board Options Exchange's VIX. However, as elaborated below, the VIX is *not* model free and is biased in the presence of discontinuities, making it difficult to distinguish between volatility and tail risk. As formalized by Carr and Wu (2003), a model-free measure of jump risk can be constructed from the pricing of extreme returns using close-to-maturity, deep out-of-the-money (OTM) options. By combining this theory with extreme value statistics, Bollerslev and Todorov (2011) construct an "investor [jump] fear index."

Our first contribution is to show that the Bakshi-Kapadia-Madan (2003; BKM hereafter) measure of the variance of the holding period return is more accurate than the VIX for measuring quadratic variation—a measure of stock return variability—when there is significant jump risk, as, for example, for the *entire* class of Lévy models (e.g., Merton, 1976), and the stochastic volatility and jump model of Bates (2000). Moreover, if the stock price process has no discontinuities (e.g., Hull and White, 1987; Heston, 1993), it is about as accurate when measured from short-maturity options even though the VIX is designed specifically to measure the quadratic variation of a jump-free process ("integrated variance").[2]

[1] A number of papers have related jump or tail risk to asset risk premia. For example, Naik and Lee (1990), Longstaff and Piazzesi (2004), and Liu, Pan, and Wang (2005) model jump risk premia in equity prices, while Gabaix (2012) and Wachter (2012), extending initial work of Rietz (1988) and Barro (2006), relate equity risk premia to time-varying consumption disaster risk.

[2] The VIX was constructed to measure the integrated variance using the log-contract based on the analysis of Carr and Madan (1998), Demeterfi, Derman, Kamal, and Zou (1999a, 1999b), and Britten-Jones and Neuberger (2000) and earlier work by Neuberger (1994) and Dupire (1996). Jiang and Tian (2005) argue in their Proposition 1 that the VIX is an accurate measure of quadratic variation. Carr and Wu (2009) show that the VIX is biased but use numerical simulations to argue that the bias is negligible. Consequently, a number of recent papers have used the VIX as a model-free measure of quadratic variation (e.g., Bollerslev, Gibson, and Zhou, 2011; Drechsler and Yaron, 2011; Wu, 2011).

Next, we construct a jump and tail index based on the fact that the accuracy of the VIX deteriorates rapidly when a larger proportion of stock return variability is determined by fears of jumps. Building on analysis by Carr and Wu (2009), we show that the bias in the VIX is proportional to the jump intensity. By comparing the integrated variance to the BKM variance, we can measure the jump-induced bias and construct a model-free tail index. In contrast to the existing literature, we do not need short-dated options to infer tail risk; we construct our index from standard 30-day maturity index options.

Technically, our jump and tail index measures time variation in the jump intensity process. It is determined by higher-order moments of the jump distribution and is therefore statistically distinguishable from the quadratic variation. Economically, the difference between these two measures of stock return variability maps into a short position in an option portfolio of *risk reversals*. This option portfolio constitutes the hedge that a dealer in variance swaps (with payoff defined in terms of the sum of squared returns) should engage in to immunize a short position from risks of discontinuities. When downside jumps dominate, the price of the risk reversal portfolio is negative and the integrated variance underestimates the quadratic variation.

We construct our volatility and tail indices from the Standard & Poor's 500 Index option data over 1996–2010. We document that the time variation in tail risk is driven primarily by downside jump fears. At the peak of the financial crisis, fears of jumps in the market were an extraordinary 50-fold those of the median month. The tail index allows us to precisely compare the severity of crises over our sample period and reveals that the Long Term Capital Management (LTCM) crisis had more jump risk than the Asian currency crisis, the 2001–2002 recession, or the prelude to the Iraq war.

Finally, using the volatility and tail indices, we examine the two potential channels of risk. To do so, we employ predictability regressions following the setups of Bollerslev, Tauchen, and Zhou (2009) (BTZ hereafter), and Bakshi, Panayotov and Skoulakis (2011) (BPS hereafter).[3]

BTZ demonstrate that the spread between the VIX and the historical variance predicts index returns. We first use their framework to examine the economic impact of the jump-induced bias in the integrated variance. We document that the integrated variance underestimates expected one-year returns by 1.50% because of the jump-induced bias. Using the VIX to construct the spread results in anomalous findings, including one where the predicted return in the financial crisis of 2007–2009 is *lower* than in the previous recession of 2001–2002. The

[3] There is a long-standing tradition of using predictability regressions (e.g., see Cochrane, 2007, and references therein). More closely related papers to our motivation which also use predictability regressions are Bollerslev, Gibson and Zhou (2011), Drechsler and Yaron (2011), and Kelly (2010).

use of the BKM variance eliminates these anomalies. Our results underscore the importance of correctly accounting for jumps when estimating stock return variability.

Should the jump-induced tail of the distribution be considered an additional channel of risk? We document that the tail index is significant after controlling for either the BTZ variance spread or the forward variance of BPS. Over the entire sample period that includes the financial crisis, the tail index is significant over medium horizons of six months and above. Importantly, in the relatively quiet period between the collapse of LTCM and the failure of Lehman Brothers, the tail index is highly significant at a one-month horizon, indicating that investors' fears of downside tail events are incorporated into equity prices even in times of relative tranquility. The tail risk is also economically significant. Over the entire sample period, while a one standard deviation increase in the BTZ variance spread predicts an increased excess return of 5.4%, a similar increase in the tail index predicts an increased excess return of over 7%. The sum total of the evidence indicates that fears of jumps operate through two distinct channels of stock return volatility and tail risk, respectively.

In related literature, Bakshi, Cao, and Chen (1997), Bates (2000), Pan (2002), and Broadie, Chernov, and Johannes (2007), among others, demonstrate the significance of jump risk using parametric option pricing models. Naik and Lee (1990), Longstaff and Piazessi (2004) and Liu, Pan, and Wang (2005) develop equilibrium option pricing models that specifically focus on jump risk. Broadie and Jain (2008), Cont and Kokholm (2010), and Carr, Lee, and Wu (2011) observe, as we do, that jumps bias the VIX. Bakshi and Madan (2006) note the importance of downside risk in determining volatility spreads.

Empirical evidence that jump risk contributes significantly to the variability of observed stock log returns under the physical measure has been found by Aït-Sahalia and Jacod (2008, 2009a, 2009b, 2010), Lee and Mykland (2008), and Lee and Hannig (2010), amongst others. Bakshi, Madan, and Panayotov (2010) use a jump model to model tail risk in index returns and find that downside jumps dominate upside jumps. Kelly (2010) argues that tail-related information can be inferred from the cross sections of returns. We add to this literature by providing, under a risk-neutral measure, a model-free methodology for measuring quadratic variation and time variation in jump intensity.

Finally, an important segment of the literature focuses on consumption disasters. This literature, starting with Rietz (1988) and further elaborated by Barro (2006) and Gabaix (2012), focuses on how infrequent consumption disasters affect asset risk premiums, potentially providing a channel through which returns are predictable. By assuming that dividends and consumption are driven by the same shock and by modeling tail risk as a jump process, Wachter

(2012) shows that the equity premium will depend on time-varying jump risk. Although a precise correspondence between consumption tail risk and the tail risk inferred from index options is yet incomplete (see Backus, Chernov and Martin, 2011, for a recent effort), our findings regarding the importance of downside jump risk broadly support this literature.

The remainder of the paper is organized as follows. Section 2 illustrates our approach using the Merton jump diffusion model. Section 3 collects our primary theoretical results. Section 4 describes the time-variation in the indices. Sections 5 present our empirical results. The last section concludes the study.

2 An illustration using the Merton jump diffusion model

We use the Merton (1976) jump diffusion model to motivate the construction of our jump and tail index. Let the stock price S_T at time T be specified by a jump diffusion model under the risk-neutral measure $\mathbb{Q}$,

$$S_T = S_0 + \int_0^T (r - \lambda\mu_J) S_{t^-}\, dt + \int_0^T \sigma S_{t^-}\, dW_t + \int_0^T \int_{R^0} S_{t^-}(e^x - 1)\mu[dx, dt], \tag{1}$$

where r is the constant risk-free rate, σ is the volatility, W_t is standard Brownian motion, R^0 is the real line excluding zero, and $\mu[dx, dt]$ is the Poisson random measure for the compound Poisson process with compensator equal to $\lambda \frac{1}{\sqrt{2\pi\sigma_J^2}} e^{-\frac{1}{2}(x-\alpha)^2}$, with λ as the jump intensity. From Ito's lemma, the log of the stock price is

$$\begin{aligned} \ln S_T &= \ln S_0 + \int_0^T \frac{1}{S_{t^-}} dS_t - \int_0^T \frac{1}{2}\sigma^2 dt + \int_0^T \int_{\mathbb{R}^0} (1 + x - e^x)\ \mu[dx, dt], &(2) \\ &= \ln S_0 + \int_0^T (r - \frac{1}{2}\sigma^2 - \lambda\mu_J)\, dt + \int_0^T \sigma\, dW_t + \int_0^T \int_{\mathbb{R}^0} x\, \mu[dx, dt]. &(3) \end{aligned}$$

Denote the quadratic variation over the period $[0, T]$ as $[\ln S, \ln S]_T$. The quadratic variation of the jump diffusion process is (e.g., Cont and Tankov, 2003)

$$[\ln S, \ln S]_T = \int_0^T \sigma^2 dt + \int_0^T \int_{\mathbb{R}^0} x^2 \mu[dx, dt]. \tag{4}$$

First, we derive a relation between $\mathrm{E}_0^{\mathbb{Q}}[\ln S, \ln S]_T$ and $\mathrm{var}_0^{\mathbb{Q}}(\ln S_T/S_0)$. From Ito's lemma, the square of the log return, $(\ln S_t/S_0)^2$ is,

$$(\ln S_T/S_0)^2 = \int_0^T 2\ln(S_{t-}/S_0)\, d\ln S_t + [\ln S, \ln S]_T. \tag{5}$$

The expected value of the stochastic integral in equation (5) is (see Appendix A),

$$\begin{aligned}\mathrm{E}_0^{\mathbb{Q}} \int_0^T 2\ln(S_{t-}/S_0)\, d\ln S_t &= \left((r - \frac{1}{2}\sigma^2 - \lambda\mu_J) + \lambda\alpha\right)^2 T^2, \\ &= (\mathrm{E}_0^{\mathbb{Q}} \ln(S_{t-}/S_0))^2. \end{aligned} \tag{6}$$

Substituting equation (6) in equation (5), and rearranging,

$$\begin{aligned}\mathrm{E}_0^{\mathbb{Q}}[\ln S, \ln S]_T &= \mathrm{E}_0^{\mathbb{Q}}(\ln S_T/S_0)^2 - \left(\mathrm{E}_0^{\mathbb{Q}}(\ln S_T/S_0)\right)^2, \\ &= \mathrm{var}_0^{\mathbb{Q}}(\ln S_T/S_0). \end{aligned} \tag{7}$$

Equation (7) states the quadratic variation is equal to the variance of the holding period return for the Merton model. From Bakshi, Kapadia and Madan (2003), $\mathrm{var}_0^{\mathbb{Q}}(\ln S_T/S_0)$ can be estimated model-free from option prices, and, therefore, so can the quadratic variation.

Next, with some abuse of notation, denote $\mathrm{E}_0^{\mathbb{Q}}[\ln S, \ln S]_T^c$ as the estimate of the expectation of integrated variance, $\mathrm{E}_0^{\mathbb{Q}} \int_0^T \sigma^2 dt$, under the assumption that the stock return process has no discontinuities. Carr and Madan (1998), Demeterfi, Derman, Kamal and Zou (1999a, 1999b), and Britten-Jones and Neuberger (2000) demonstrate that, for a purely continuous process,

$$\mathrm{E}_0^{\mathbb{Q}}[\ln S, \ln S]_T^c = 2\,\mathrm{E}_0^{\mathbb{Q}}\left[\int_0^T \frac{1}{S_t} dS_t - \ln S_T/S_0\right]. \tag{8}$$

The RHS of equation (8) can be replicated using options and, therefore, the integrated variance can be estimated. The VIX is based on this analysis.

But, in the presence of discontinuities, from equation (2),

$$\begin{aligned}2\mathrm{E}_0^{\mathbb{Q}}\left[\int_0^T \frac{1}{S_{t-}} dS_t - \ln S_T/S_0\right] &= \mathrm{E}_0^{\mathbb{Q}} \int_0^T \sigma^2 dt - 2\mathrm{E}_0^{\mathbb{Q}} \int_0^T \int_{R^0} (1 + x - e^x)\,\mu[dx, dt], \\ &= \mathrm{E}_0^{\mathbb{Q}}[\ln S, \ln S]_T - 2\mathrm{E}_0^{\mathbb{Q}} \int_0^T \int_{R^0} \left(1 + x + \frac{x^2}{2} - e^x\right) \mu[dx, dt]. \end{aligned} \tag{9}$$

Thus, $\mathrm{E}_0^{\mathbb{Q}}[\ln S, \ln S]_T^c \equiv 2\,\mathrm{E}_0^{\mathbb{Q}}\left[\int_0^T \frac{1}{S_{t-}} dS_t - \ln S_T/S_0\right]$ is a biased estimator of both the quadratic variation and the integrated variance.[4]

But the BKM variance does measure the quadratic variation. Replacing $\mathrm{E}_0^{\mathbb{Q}}[\ln S, \ln S]_T$ by $\mathrm{var}_0^{\mathbb{Q}}(\ln S_T/S_0)$ in equation (9), we obtain,

$$\mathrm{var}_0^{\mathbb{Q}}(\ln S_T/S_0) - 2\,\mathrm{E}_0^{\mathbb{Q}}\left[\int_0^T \frac{1}{S_{t-}} dS_t - \ln S_T/S_0\right] = 2\mathrm{E}_0^{\mathbb{Q}}\int_0^T\int_{R^0}\left(1 + x + \frac{x^2}{2} - e^x\right)\mu[dx, dt]. \tag{10}$$

Equation (10) states that the difference between the variance of the holding period return and the integrated variance measure is determined solely by discontinuities in the stock price process. We use the time-variation in this difference to construct a model-free jump and tail index.

3 Measuring stock return variability and tail risk

3.1 Quadratic variation and variance of the holding period return

Let the log stock price $\ln S_t$ at time t, $t \geq 0$, be a semimartingale defined over a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathbb{Q})$, with $S_0 = 1$. Denote the quadratic variation over a horizon $T > 0$ as $[\ln S, \ln S]_T$ and the variance of the holding period return as $\mathrm{var}_0^{\mathbb{Q}}(\ln S_T/S_0)$. Our first result characterizes the relation between these two measures of stock return variability.

Proposition 1 *Let $E_0^{\mathbb{Q}}[\ln S, \ln S]_T$ and $var_0^{\mathbb{Q}}(\ln S_T/S_0)$ be the expected quadratic variation and variance of the holding period return, respectively, over a horizon $T < \infty$. Denote the difference between the two measures of variability as $\mathbb{D}(T) = var_0^{\mathbb{Q}}(\ln S_T/S_0) - E_0^{\mathbb{Q}}[\ln S, \ln S]_T$. Then,*

$$\mathbb{D}(T) = E_0^{\mathbb{Q}}\left[\int_0^T 2\ln S_{t-}/S_0\, d\ln S_t\right] - \left(E_0^{\mathbb{Q}}(\ln S_T/S_0)\right)^2. \tag{11}$$

$\mathbb{D}(T)$ can be further characterized as follows:

i. Suppose the log return process decomposes as $\ln S_t/S_0 = A_t + M_t$, where A_t is a continuous finite variation process with $A_0 = 0$, and M_t is a square-integrable martingale with

[4] Our analysis differs from that of Carr and Madan (1998), Demeterfi, Derman, Kamal, and Zou (1999a, 1999b), and Britten-Jones and Neuberger (2000) because we consider a contract that pays the *square* of the log return (equation 5), as opposed to a contract that pays the log return (equation 8). Earlier literature focused on the log-contract because, under the assumption of no discontinuities, the analysis indicated how a variance swap could be replicated. Unfortunately, as we see, the log-contract does not hedge nor price the variance swap in the presence of discontinuities.

$M_0 = 0$. Then, if A_t is deterministic, $\mathbb{D}(T) = 0$ and the two measures of variability are equivalent.

ii. *Suppose the log return process is a two-dimensional diffusion with stochastic volatility,*

$$
\begin{aligned}
d\ln S_t &= (r - \frac{1}{2}\sigma_t^2)\,dt + \sigma_t\,dW_{1,t}, && (12)\\
d\sigma_t^2 &= \theta[\sigma_t^2]\,dt + \eta[\sigma_t^2]\,dW_{2,t}, && (13)
\end{aligned}
$$

where $dW_{1,t}$ and $dW_{2,t}$ are standard Brownian motions with correlation ρ. Then, as $T \to 0$, $\frac{1}{T}\,\mathbb{D}(T) = O(T)$. Moreover, if $\rho = 0$, then $\frac{1}{T}\,\mathbb{D}(T) = O(T^2)$.

Proof: See Appendix A.

Proposition 1 fully characterizes the relation between the two measures of stock return variability. First, the proposition indicates that the two measures of variability are equivalent when the log return process can be decomposed into a martingale and a deterministic drift. Intuitively, this is because the deterministic drift adds neither to the quadratic variation (because it is a continuous finite variation process) nor to the variance (because it is deterministic).[5]

A Lévy process with a characteristic function given by the Lévy–Khintchine theorem can be decomposed into a deterministic drift and a martingale. Therefore, quadratic variation and the variance of the holding period return are equivalent for the entire class of Lévy processes.[6]

Example 1 *Merton (1976) model:*
Continuing the illustration of Section 2, observe that the log return for the Merton model can be decomposed as

$$
\begin{aligned}
\ln S_t/S_0 &= \left((r - \frac{1}{2}\sigma^2) + \lambda\alpha\right) t + M_t,\\
&= A_t + M_t, && (14)
\end{aligned}
$$

where M_t is the sum of a continuous and pure jump martingale. The drift A_t is deterministic for this model and therefore the quadratic variation and variance are equivalent. We now arrive at the conclusion without explicitly evaluating the stochastic integral in equation (6).

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[5] We thank G. Lowther for this intuition.

[6] This class includes many commonly used models, including the geometric Brownian motion (Black and Scholes, 1973), jump diffusion models (e.g., Merton, 1976; Kou, 2002), and infinite activity Lévy processes, such as the variance gamma process of Madan and Seneta (1990), the normal inverse Gaussian process of Barndorff-Nielson (1998), and the finite-moment stable process of Carr, Geman, Madan, and Yor (2002). For these models, by the Lévy-Khintchine theorem, the characteristic function is determined by (γ, σ^2, ν), so that $\mathrm{E}_0 e^{iu \ln S_t/S_0} = e^{t\psi(u)}$, where $\psi(u) = i\gamma u - \frac{1}{2}\sigma^2 u^2 + \int_{R^0}\left(e^{iux} - 1 - iux\mathbb{1}_{[|x|<1]}\right)\nu[dx]$.

The drift of the log return is stochastic in a stochastic volatility model such as Heston's (1993). Because the variance of the holding period return accounts for the stochasticity of the drift, it differs from the quadratic variation by $\mathbb{D}(T)$. The second part of the proposition shows that the annualized $\mathbb{D}(T)$ is $O(T)$ for small T for stochastic volatility models. In the special case when the volatility process is uncorrelated with the log return process, as in Hull and White (1987), $\rho = 0$, $\mathbb{D}(T)$ reduces with T even faster. Indeed, for $\rho = 0$, we can further characterize $D(T)$ (see Appendix A) as

$$\mathbb{D}(T) = \frac{1}{4}\text{var}_0^{\mathbb{Q}} \int_0^T \sigma_t^2 dt. \tag{15}$$

When an analytical solution is available for $\text{var}_0^{\mathbb{Q}} \int_0^T \sigma_t^2 dt$, $\mathbb{D}(T)$ can be estimated precisely.

Example 2 *Heston (1993) model:*
For the Heston model, $d\sigma_t^2 = \kappa(\theta - \sigma_t^2) + \eta\sigma_t dW_{2,t}$, Bollerslev and Zhou (2002) demonstrate in equation (A.5) in their appendix that $var_0^{\mathbb{Q}} \int_0^T \sigma_t^2 dt = A(T)\,\sigma_0^2 + B(T)$, where

$$A(T) = \frac{\eta^2}{\kappa^2}\left(\frac{1}{\kappa} - 2e^{-\kappa T}T - \frac{1}{\kappa}e^{-2\kappa T}\right), \tag{16}$$

$$B(T) = \frac{\eta^2}{\kappa^2}\left(\theta T\left(1 + 2e^{-\kappa T}\right) + \frac{\theta}{2\kappa}\left(e^{-\kappa T} + 5\right)\left(e^{-\kappa T} - 1\right)\right). \tag{17}$$

Expanding around $T = 0$ and simplifying yields

$$var_0^{\mathbb{Q}} \int_0^T \sigma_t^2 dt = \eta^2(\theta - \sigma_0^2)T^3 + O(T^4).$$

For $\rho = 0$ and $\sigma_0^2 = \theta$, $\frac{1}{T}\mathbb{D}(T) = O(T^3)$.

The example illustrates that, depending on parameter values, $\mathbb{D}(T)$ may reduce with T even faster than noted in Proposition 1.

When $\rho \neq 0$ (because we can write $W_{2,t} = \rho W_{1,t} + \sqrt{1-\rho}W_{\sigma,t}$, where $W_{\sigma,t}$ is independent of $W_{1,t}$), $\mathbb{D}(T)$ has an additional component proportional to ρ. Below we use numerical experiments to estimate the magnitude of $\mathbb{D}(T)$ for $\rho \neq 0$ and show that it is negligible for typical parameter values.

In their Proposition 1, BKM demonstrate that the variance of the holding period return can be estimated model free from option prices. Thus, using the BKM methodology, we can precisely estimate the quadratic variation for Lévy processes and, to a very good approximation, using short-maturity options, for stochastic volatility diffusions.

3.2 Quadratic variation, integrated variance, and jump risk

To derive a measure of jump risk, we put more structure on the stock return process. Let the log of the stock price be a general diffusion with jumps:

$$\ln S_T = \ln S_0 + \int_0^T (a_t - \frac{1}{2}\sigma_t^2)\, dt + \int_0^T \sigma_t\, dW_t + \int_0^T \int_{\mathbb{R}^0} x\, \mu[dx, dt], \tag{18}$$

where the time variation in σ_t is left unspecified while a_t is restricted to ensure that the discounted stock price is a martingale. Let the Poisson random measure have an intensity measure $\mu[dx, dt] \equiv \nu_t[dx]dt$. Given our focus on tail risk and because we have already included a diffusion component, we assume that $\int_{\mathbb{R}^0} \nu[dx] < \infty$ and that all moments $\int_{\mathbb{R}^0} x^n \nu[dx]$, $n = 1, 2, ...,$ exist.

As in Carr and Madan (1998), Demeterfi, Derman, Kamal and Zou (1999a, 1999b), and Britten-Jones and Neuberger (2000), we define $\mathrm{E}_0^{\mathbb{Q}}[\ln S, \ln S]_T^c$ as the (VIX) measure of integrated variance under the assumption that the process is continuous:

$$\mathrm{E}_0^{\mathbb{Q}}[\ln S, \ln S]_T^c \equiv \mathrm{E}_0^{\mathbb{Q}}\left[2\left(\int_0^T \frac{dS_t}{S_t} - \ln \frac{S_T}{S_0}\right)\right]. \tag{19}$$

In the absence of discontinuities in the stock return process, $\mathrm{E}_0^{\mathbb{Q}}\left[2\left(\int_0^T \frac{dS_t}{S_t} - \ln \frac{S_T}{S_0}\right)\right] = \mathrm{E}_0^{\mathbb{Q}} \int_0^T \sigma_t^2 dt$. If there are discontinuities, Carr and Wu (2009) show in their Proposition 1 that the difference between $\mathrm{E}_0^{\mathbb{Q}}[\ln S, \ln S]_T$ and $\mathrm{E}_0^{\mathbb{Q}}[\ln S, \ln S]_T^c$ is determined by the jump distribution,

$$\mathrm{E}_0^{Q}[\ln S, \ln S]_T - \mathrm{E}_0^{\mathbb{Q}}[\ln S, \ln S]_T^c = 2\mathrm{E}_0^{\mathbb{Q}} \int_0^T \int_{R^0} \psi(x)\, \mu[dx, dt], \tag{20}$$

where $\psi(x) = 1 + x + \frac{1}{2}x^2 - e^x$.

From iterated expectations, the right-hand side of equation (20) is determined by the compensator of the jump process. Tail risk is also determined by the expectation of jump intensity because in the presence of jumps, the tail of the stock return distribution is determined by (large) jumps.

Proposition 2 *Let the log price process be specified as in equation (18). Let the intensity measure $\nu_t[dx]$ be of the form*

$$\nu_t[dx] = \lambda_t f(x) dx,$$

where λ_t is the jump arrival intensity of a jump of any size with jump size distribution $f(x)$. Then,

$$E_0^Q[\ln S, \ln S]_T - E_0^{\mathbb{Q}}[\ln S, \ln S]_T^c = 2\,\Psi(f(x))\,\Lambda_{0,T}, \tag{21}$$

where

$$\Lambda_{0,T} = E_0^{\mathbb{Q}} \int_0^T \lambda_t dt, \tag{22}$$

with $\Psi(f(x)) = \int_{R^0} \psi(x) f(x) dx$ and $\psi(x) = 1 + x + \frac{1}{2}x^2 - e^x$.

Proof: See Appendix A.

Proposition 2 states that the difference between quadratic variation and the measure of integrated variance over an interval T is proportional to the expectation of the number of jumps over that interval. Because $\Psi(\cdot)$ is determined by higher-order moments ($n \geq 3$) of the jump distribution, the difference is clearly distinguished from quadratic variation. Dividing equation (21) by $\mathrm{E}_0^{\mathbb{Q}}[\ln S, \ln S]_T$, we can measure the time variation in the contribution of discontinuities to the total quadratic variation (e.g., Bollerslev and Todorov, 2011).

Proposition 2 can be generalized to allow for upside and downside jumps. Defining the intensity measures for upside and downside jumps as $\nu^+[x] = \lambda_t f^+(x)dx$ and $\nu^-[x] = \lambda_t f^-(x)dx$, we now obtain

$$\mathrm{E}_0^Q[\ln S, \ln S]_T - \mathrm{E}_0^{\mathbb{Q}}[\ln S, \ln S]_T^c = 2\,\left(\Psi^+ + \Psi^-\right)\,\Lambda_{0,T}, \tag{23}$$

where $\Psi^+ \equiv \int_{R^+} \psi(x) f^+(x) dx$ and $\Psi^- \equiv \int_{R^-} \psi(x) f^-(x) dx$. Under this specification, the dominance of downside versus upside jumps determines the sign of $\mathrm{E}_0^Q[\ln S, \ln S]_T - \mathrm{E}_0^{\mathbb{Q}}[\ln S, \ln S]_T^c$.

To illustrate, consider the model of Bakshi and Wu (2010) with a double exponential jump size distribution (Kou, 2002):

$$f^+(x) = \begin{cases} e^{-\beta_+|x|}, x > 0, \\ 0, x < 0; \end{cases} \tag{24}$$

$$f^-(x) = \begin{cases} 0, x > 0, \\ e^{-\beta_-|x|}, x < 0; \end{cases} \tag{25}$$

We can explicitly evaluate $\Psi(f^+(x))$ and $\Psi(f^-(x))$ to observe that $\mathrm{E}_0^Q[\ln S, \ln S]_T - \mathrm{E}_0^{\mathbb{Q}}[\ln S, \ln S]_T^c$ is positive (negative) when $\beta_+ >> \beta_-$ ($\beta_- >> \beta_+$). Intuitively, from its definition in Proposition 2, the magnitude of Ψ is determined to first order by the negative of the third moment of the jump size distribution. When downside jumps dominate, the third moment is negative and $\mathrm{E}_0^Q[\ln S, \ln S]_T - \mathrm{E}_0^{\mathbb{Q}}[\ln S, \ln S]_T^c > 0$; that is, the integrated variance underestimates the true quadratic variation.

3.3 Numerical analysis for the stochastic volatility and jump (SVJ) model

From Proposition 1, although $\mathrm{var}_0^{\mathbb{Q}}(\ln S_T/S_0)$ measures quadratic variation precisely for Lévy processes, its accuracy for stochastic volatility models depends on the maturity of options chosen to estimate the variance. In developing the tail index, we are especially interested in using options of maturity 30 days. Therefore, before proceeding further, we conduct numerical simulations to compare the accuracy with which the variance (measured by BKM) and integrated variance (measured by the VIX), respectively, estimate quadratic variation.

Let $\mathbb{V}$ be the annualized variance, $\mathbb{V} = \frac{1}{T}\mathrm{var}_0^{\mathbb{Q}}(\ln S_T/S_0) = \frac{1}{T}\left(\mathrm{E}_0^{\mathbb{Q}}\left(\ln S_T/S_0\right)^2 - \mu_{0,T}^2\right)$, where $\mu_{t,T} = \mathrm{E}_0^{\mathbb{Q}} \ln S_T/S_t$. From BKM, the price of the variance contract is estimated from OTM calls and puts of maturity T. Denoting $C(S_t;K,T)$ and $P(S_t;K,T)$ as the call and put of strike K and T as the remaining time to expiration, BKM demonstrate that

$$e^{-rT}\mathbb{V} = \frac{1}{T}\left[\int_{K>S_0} \frac{2(1-\ln(K/S_0))}{K^2}C(S_0;K,T)dK + \int_{K<S_0} \frac{2(1+\ln(S_0/K))}{K^2}P(S_0;K,T)dK - e^{-rT}\mu_{0,T}^2\right], \tag{26}$$

where r is the constant risk-free rate.

Similarly, let the annualized integrated variance be denoted $\mathbb{IV} = \frac{1}{T}\mathrm{E}_0^{\mathbb{Q}}[\ln S, \ln S]_T^c$. For a continuous stochastic process, $\mathbb{IV}$ can also be estimated from OTM calls and puts as

$$e^{-rT}\mathbb{IV} = \frac{2}{T}\left[\int_{K>S_0} \frac{1}{K^2}C(S_0;K,T)dK + \int_{K<S_0} \frac{1}{K^2}P(S_0;K,T)dK - e^{-rT}\left(e^{rT} - 1 - rT\right)\right]. \tag{27}$$

Demeterfi, Derman, Kamal, and Zou (1999b) use this particular formulation (their equation (26)), and Carr and Madan (1998) and Britten-Jones and Neuberger (2000) use equivalent but slightly different formulations.

In Table 1, we compare the theoretical quadratic variation, $\mathrm{E}_0^{\mathbb{Q}}[\ln S, \ln S]_T$, with those estimated from option prices using $\mathbb{V}$ and $\mathbb{IV}$, respectively, for the Merton (1974) jump diffusion model and the SVJ model of Bates (2000). Except for the initial variance (σ_0^2) and the mean of the jump size distribution (α), we calibrate the parameters to those empirically estimated by Pan (2002). The initial volatility and the mean of the jump size distribution are adjusted to vary the contribution of the variability from jumps to the total variance from zero to 90%.

Panel A provides the comparison for the Merton model. For the Merton jump diffusion model, $\mathbb{V}$ measures the quadratic variation perfectly, but $\mathbb{IV}$ does so with error because of jump risk. The error increases as jumps contribute a larger fraction to the total variance.

Panel B provides numerical results for the SVJ model. Because the drift is stochastic, $\mathbb{V}$ measures the quadratic variation with a (small) error. The maximum error is when there are no jumps, and is only 0.61% in relative terms. That is, while the true $\sqrt{[\ln S, \ln S]_T}$ is 20%, the BKM volatility estimate is 20.05%. As the contribution of jumps increases, $\mathbb{V}$ is even more accurate. In contrast, $\mathbb{IV}$ becomes less accurate as the contribution of jumps to return variability increases. When the contribution of jumps to the variance is below 20%, the relative error is less than 1% of the variance, but it increases manifold as jump risk increases. For example, when jumps contribute over 70% to the total variance, $\sqrt{\mathbb{IV}}$ is 19% instead of the correct 20%, an economically significance bias.

Proposition 1 notes that an increase in the magnitude of ρ makes $\mathbb{V}$ less accurate. To check, we consider a correlation of $\rho = -0.90$. Even for this extreme case, the error is negligible. At worst, $\sqrt{\mathbb{V}}$ gives an estimate of 20.09% instead of the correct 20%. The bias is well within the bounds of accuracy with which we can estimate risk-neutral densities using option prices.

In summary, in the presence of jumps, the BKM variance measures quadratic variation more accurately than the integrated variance since it correctly accounts for jumps, while the stochasticity of the drift adds negligible error at short maturities. When jumps contribute less than 20% to the variance, both $\mathbb{V}$ and $\mathbb{IV}$ accurately estimate the quadratic variation.[7] However, with increasing jump risk, $\mathbb{IV}$ gets progressively less accurate.

3.4 Formalizing the jump and tail index, JTIX

From Propositions 1 and 2, the difference between the variance and the integrated variance is the sum of two components, the first determined by the stochasticity of the drift and the second determined by jump risk. On an annualized basis,

$$\frac{1}{T}\left(\mathrm{var}_0^{\mathbb{Q}}(\ln S_T/S_0) - \mathrm{E}_0^{\mathbb{Q}}[\ln S, \ln S]_T^c\right) = \frac{1}{T}\left(\mathrm{E}_0^{\mathbb{Q}}\int_0^T \ln S_T/S_0 \, d\ln S_t - \mathrm{E}_0^{\mathbb{Q}}\left(\ln S_T/S_0\right)^2\right) + \frac{2}{T}\Psi\Lambda_{0,T}. \tag{28}$$

Proposition 1 combined with our simulation evidence indicates that the impact of the stochasticity of the drift (the first term) can be neglected for standard jump diffusion models for

[7] Here, our analysis concurs with that of Jiang and Tian (2005) and Carr and Wu (2009): The parameterizations chosen in their numerical experiments correspond to (low) jump contributions of 14% and 11.7% to the quadratic variation, respectively. When jump risk is of low economic importance, we can accurately measure quadratic variation using either the VIX or the BKM variance.

short-maturity options, that is,

$$\frac{1}{T}\left(\mathrm{var}_0^{\mathbb{Q}}(\ln S_T/S_0) - \mathrm{E}_0^{\mathbb{Q}}[\ln S, \ln S]_T^c\right) \approx \frac{2}{T}\Psi\Lambda_{0,T}. \tag{29}$$

Equation (29) states that the time variation in the difference between the variance and integrated variance is determined by the time variation in jump intensity. Our *jump and tail index*, JTIX, is motivated by equation (29). Defining $\widehat{\mathrm{JTIX}} = \mathbb{V} - \mathbb{IV}$ using equations (26) and (27), we obtain

$$\widehat{\mathrm{JTIX}} = \mathbb{V} - \mathbb{IV} = \frac{2}{T}e^{rT}\left[\int_{K<S_0}\frac{\ln(S_0/K)}{K^2}P(S_0;K,T)\,dK - \int_{K>S_0}\frac{\ln(K/S_0)}{K^2}C(S_0;K,T)\,dK\right] + \bar{\alpha}, \tag{30}$$

where $\bar{\alpha} = \frac{2}{T}(e^{rT} - 1 - rT) - \frac{1}{T}\mu_{0,T}^2$. Because $\bar{\alpha}$ is small, $\widehat{\mathrm{JTIX}}$ is primarily determined by the OTM option portfolio represented by the first two terms of equation (30). Economically, the option portfolio comprising the tail index is a short position in a *risk reversal* and the hedge that a dealer in (short) variance swaps would buy to protect against the risk of discontinuities.

Our jump and tail index, JTIX, is constructed as the 22-day moving average

$$\mathrm{JTIX}_t = \frac{1}{22}\sum_{i=1}^{22}\widehat{\mathrm{JTIX}}_{t-i+1}. \tag{31}$$

As in Bollerslev and Todorov (2011), we use a 22-day moving average to reduce estimation errors. To validate and investigate the economic significance of JTIX, we pose the following questions.

1. **Is** JTIX $= 0$ **($\mathbb{V} \approx \mathbb{IV}$)?**

The first question we consider is whether jump risk is significant. This question is equivalent to asking whether the VIX is an accurate estimator of quadratic variation. When jump risk is negligible, $\mathbb{V}$ and $\mathbb{IV}$ are approximately equal and JTIX is close to zero.

2. Is there time variation in JTIX **and is the time variation related to jump risk?**

If the difference between $\mathbb{V}$ and $\mathbb{IV}$ is related to λ_t, then it should increase in periods when fears of jumps are higher. If downside jumps are more prevalent than upside jumps, we expect the time variation in the jump and tail index to be countercyclical and JTIX to be especially high in times of severe market stress.

3. What are the channels for jump risk?

Our primary question centers on whether jump risk is important for predicting market returns. Assuming it is, we are interested in understanding the channel through which jump risk is significant. Does jump risk impact market returns through the contribution of jumps to volatility or through the impact of jumps on the tail of the distribution?

In investigating these questions, we follow BTZ and BPS. BTZ demonstrate that the *variance spread*, $\mathbb{VS}$, defined as the difference between the integrated variance and the past realized variance, $\mathbb{VS}_t = \mathbb{IV}_t - \mathbb{RV}_{t-1}$, predicts index returns.[8] BPS find that the forward variance also predicts short-horizon returns. Both set of results indicate that there is useful information in option-based estimates of stock return variability.

We proceed as follows. First, following BTZ, we investigate whether the contribution of jumps to stock return variability is economically important. If it is important to correctly account for jump risk, then $\mathbb{V} - \mathbb{RV}$ will be more significant than $\mathbb{IV} - \mathbb{RV}$ in predicting index returns. Second, we consider whether tail risk is important in addition to stock return variability for predicting index returns. If tail risk is a separate channel, then JTIX should be significant in addition to the variance spread of BTZ or the forward variance of BPS. Together, the exercises allow us to understand whether jump risk is significant through one or both channels of volatility and tail risk.

4 The jump and tail index

To construct the volatility indices and the tail index, we use option prices on the S&P 500 (SPX) over the sample period of January 1996 to October 2010. The options data, dividend yield for the index, and zero coupon yield are from OptionMetrics. We clean the data with the usual filters, the details of which are provided in Appendix B.

We construct the volatility and tail indices as follows. First, we obtain option prices across a continuum of strikes. Following Jiang and Tian (2005) and Carr and Wu (2009), we interpolate the Black–Scholes implied volatility across the range of observed strikes using a cubic spline, assuming the smile to be flat beyond the observed range of strikes. Next, we linearly interpolate the smiles of the two near-month maturities to construct a 30-day implied volatility curve for each day. The interpolated implied volatility curve is converted back to option prices using the Black–Scholes formula. The daily volatility indices, $\mathbb{V}$ and $\mathbb{IV}$, are computed using the BKM and VIX formulas, equations (26) and (27), respectively. Finally, the tail index JTIX is constructed as the 22-day moving average of $\mathbb{V} - \mathbb{IV}$, as noted in equation (31).

[8] BTZ suggest that the variance spread is important in predicting index returns because it measures the variance risk premium. Technically, the variance risk premium (Bakshi and Kapadia, 2003; Carr and Wu, 2009) is the negative of the difference between the risk-neutral variance and the variance realized over the remaining maturity of the option, that is, $-(\mathbb{V}_t - \mathbb{RV}_t)$ or -($\mathbb{IV}_t - \mathbb{RV}_t$). To avoid confusion, we call it a variance spread.

Figure 1 compares the two constant maturity volatility indices, the holding period volatility, and the integrated volatility by plotting $\sqrt{\mathbb{V}} - \sqrt{\mathbb{IV}}$ over our sample period of January 1996 to October 2010 with a daily frequency. The plot demonstrates that $\mathbb{V}$ is always greater than $\mathbb{IV}$. On average, over this period, $\sqrt{\mathbb{V}}$ is 23.4%, which is 0.5% (volatility points) higher than $\sqrt{\mathbb{IV}}$ on an annualized basis. The results are consistent with our expectation that when downside jumps dominate, the integrated variance will underestimate the quadratic variation.

Figure 2 plots the tail indices, JTIX, and JTIX/$\mathbb{V}$. The plot of JTIX shows that jump risk is intimately associated with times of crisis and economic downturns, with a manifold increase in jump risk. The most prominent spike occurs in October of 2008, with a more than 50-fold increase in jump risk compared with that of the median day over the sample period. Additional spikes occur in the period leading up to the Iraq war, the dot-com bust of 2001, the Russian bond and LTCM crises in 1998, and the Asian currency crisis in 1997. Unlike the two volatility indices (especially the integrated variance measure), the jump and tail index clearly differentiates between the 2001 recession and the LTCM crisis: The LTCM crisis has twice the tail risk as the aftermath of the dot-com bubble. The tail index also captures the sharp increase in tail risk in November of 2007, coincident with the crash in the Seoul stock market during the Asian currency crisis. Interestingly, excepting the Iraq war, all the sharp increases in tail risk correspond to the handful of financial crises that occurred in our sample period. Extending the popular analogy of the Chicago Board Options Exchange VIX to a fear index, JTIX can be viewed as an index of extreme fear.

To sharpen the distinction between the holding period variance and the integrated variance, we also plot JTIX/$\mathbb{V}$. The plot indicates that $\mathbb{IV}$ underestimates market variance by over 15% at the peak of the recent financial crisis. If we use the numerical analysis of Table 1 as a guide, this magnitude of underestimation suggests that jump risk may comprise over 80% of stock return variability at the peak of the crisis.

The discussion in Section 3.2 noted that if the intensity measure differs for downside and upside jumps, $\nu^+[x] = \lambda_t f^+(x)dx$ and $\nu^-[x] = \lambda_t f^-(x)dx$, then an increase in λ_t skews the jump size distribution further to the left or right, respectively, depending on whether downside or upside jumps dominate. The jump and tail index captures (to first order) the left (right) skew through the relative pricing of OTM puts and calls—the short risk reversal option portfolio in equation (30). To further grasp the relative importance of downside and upside jumps, we decompose JTIX $\approx$ JTIX$_-$ $-$ JTIX$_+$, where JTIX$_-$ and JTIX$_+$ correspond to the put and call portfolios, respectively, of the risk reversal portfolio:

$$\text{JTIX}_{-} = \frac{2}{T} e^{rT} \int_{K<S_0} \left(\frac{\ln(S_0/K)}{K^2} \right) P(S_0; K, T)\, dK, \tag{32}$$

$$\text{JTIX}_{+} = \frac{2}{T} e^{rT} \int_{K>S_0} \left(\frac{\ln(K/S_0)}{K^2} \right) C(S_0; K, T)\, dK. \tag{33}$$

Figure 3 plots JTIX_+ and JTIX_-. Comparing Figures 3 and 2 confirms that the time variation in JTIX is driven by OTM put prices, that is, downside jump risk. Economically, the price of the risk reversal portfolio is driven by time variation in OTM put prices. Interestingly, using risk-reversals in currency options, Bakshi, Carr, and Wu (2008) also find evidence of one-sided jump risk.

In Figure 4, we compare JTIX with the (negative of) Bollerslev and Todorov's (2011) fear index (-BT).[9] The BT index measures the difference between the variance risk premiums associated with negative and positive jumps, respectively, and is estimated from short-dated deep OTM options. Not only are both indices highly correlated, but they also show similar peaks identifying the major financial crises. Not surprisingly, there is a close link between investor fears noted in the jump variance risk premium and time-varying jump intensity $\Lambda_{0,T}$.

The evidence from the 30-day volatility and tail indices is consistent with the initial two hypotheses set up in Section 3.4; $\mathbb{V}$ is consistently different from $\mathbb{IV}$ and JTIX is higher in times of economic stress and times of financial crises. To verify that these observations are robust, we present evidence in Table 2 from options of remaining maturity ranging from nine to 30 days. For this exercise, we use monthly data.

Table 2 reports statistics for $\sqrt{\mathbb{V}}$, $\sqrt{\mathbb{IV}}$, and the differences $\Delta = \sqrt{\mathbb{V}} - \sqrt{\mathbb{IV}}$ and $\mathbb{V} - \mathbb{IV}$, as well as the realized volatility, $\sqrt{\mathbb{RV}}$, for each maturity. On average, across the entire sample period, $\sqrt{\mathbb{V}}$ is larger than $\sqrt{\mathbb{IV}}$ by a statistically significant 0.49% (annualized) for 30-day options, consistent with our earlier results using the constant-maturity volatility indices of Figure 1. This difference is twice as large in recessions. The results for maturities ranging from nine to 23 days present a consistent picture, confirming that $\mathbb{V}$ is statistically greater than $\mathbb{IV}$ for all maturities.

[9] We thank Tim Bollerslev and Viktor Todorov for sharing their fear index.

5 Channels of jump risk

We undertake two exercises in this section. First, we ascertain whether it is economically important to correctly account for jump risk in estimating stock return variability. Second, we consider whether the tail should be considered an additional channel for jump risk after accounting for stock return variability.

5.1 Setup for predictability regressions

Our basic setup follows that of BTZ, who demonstrate that the spread between the integrated variance and historical variance, $\mathbb{IV}_t - \mathbb{RV}_{t-1}$, predicts market excess returns after controlling for the usual set of traditional predictors. The predictive regression is specified as

$$R_{t+j} - r^f_{t+j} = \alpha + \beta_1 \cdot (\mathbb{VS}_t) + \beta_2 \cdot (\text{Tail Index}) + \Gamma' \cdot \bar{Z}_t + \epsilon_t, \tag{34}$$

where R_{t+j} denotes the log return of the S&P 500 from the end of month t to the end of month $t+j$, r^f_{t+j} denotes the risk-free return for the same horizon, and $\bar{Z}_t$ denotes the commonly used predictors, discussed further below. We alternatively define $\mathbb{VS}_t$ as either $\mathbb{V}_t - \mathbb{RV}_{t-1}$ or $\mathbb{IV}_t - \mathbb{RV}_{t-1}$. In this base specification, we also include the tail index as either JTIX or JTIX/$\mathbb{V}$. All variables are sampled at the end of the month. We use five different horizons, ranging from one month to two years.

For common predictors $\bar{Z}_t$, we include the earnings or dividend yield, the term spread, and the default spread. The term spread and default spread are included to control for any predictable impact of the business cycle.[10] Quarterly price–earnings ratios and dividend yields for the S&P 500 are from Standard & Poor's website. When monthly data are not available, we use the most recent quarterly data. The term spread ($TERM$) is defined as the difference between 10-year T-bond and three-month T-bill yields. The default spread (DEF) is defined as the difference between Moody's Baa and Aaa corporate bond yields. Data needed to calculate the term spread and the default spread are from the website of the Federal Reserve Bank of St. Louis. To be consistent with BTZ, we download the monthly realized variance $\mathbb{RV}$ calculated from high-frequency intra-day return data from Hao Zhou's website.

Table 3 describes the data: Panel A reports the summary statistics of our variables and

[10] An earlier version of this paper included the quarterly CAY, as defined by Lettau and Ludvigson (2001) and downloaded from their website, and $RREL$, the detrended risk-free rate, defined as the one-month T-bill rate minus the preceding 12-month moving average. The results were similar to the more parsimonious regression used in the current version.

Panel B reports their correlation matrix. The tail index JTIX is contemporaneously negatively correlated with index excess returns, and positively correlated with historical realized volatility. With the variance spreads, the tail index JTIX has low correlations. The correlations of JTIX with $\mathbb{V} - \mathbb{RV}$ and $\mathbb{IV} - \mathbb{RV}$ are 0.20 and 0.00, respectively. The results are similar for JTIX/$\mathbb{V}$. In contrast, JTIX and JTIX/$\mathbb{V}$ are more highly correlated with the default spread DEF, with correlations of 0.70 and 0.58, respectively.

The last row of Panel A of Table 3 reports first-order autocorrelations for the control variables. As noted in previous literature, autocorrelations are extremely high for many of the control predictors—0.97 for the price-to-dividend ratio and 0.98 for the term spread—raising concerns about correct inference. In contrast, Panel C documents that $\mathbb{V} - \mathbb{RV}$, $\mathbb{IV} - \mathbb{RV}$, JTIX, and JTIX/$\mathbb{V}$ have much lower autocorrelations, and, in Panel D, the Phillips–Perron unit root test rejects the null hypothesis of a unit root in the tail index and variance spreads.

5.2 Jump-induced bias in volatility

5.2.1 Statistical significance

Table 4 reports the regression results for each horizon. We report t-statistics based on Hodrick's (1992) 1B standard errors under the null of no predictability.

In univariate regressions, both $\mathbb{V} - \mathbb{RV}$ and $\mathbb{IV} - \mathbb{RV}$ are consistently significant at horizons up to one year. As noted by BTZ, the highest R-squared values are at the three- to six-month horizons. Control variables have lower significance on a univariate basis. Corroborating the results of BTZ, on a univariate basis, the variance spread has higher predictive power than variables used in the previous literature and is often the only variable that is significant.

To examine the importance of accounting for jump risk in the estimation of stock return variability, we focus on the multivariate specifications. First, we compare the results when $\mathbb{VS}$ is defined as $\mathbb{V} - \mathbb{RV}$ with those when the variance spread is defined as $\mathbb{IV} - \mathbb{RV}$. The Hodrick t-statistics and adjusted R-squared values in the multivariate regressions are higher for $\mathbb{V} - \mathbb{RV}$ for all horizons. For example, for the one-year horizon, the adjusted R-squared value increases from 24.4% when $\mathbb{IV}$ is used to define the variance spread to 26.5% when $\mathbb{V}$ is used to define the variance spread. The coefficient of $\mathbb{V} - \mathbb{RV}$ at 1.97 is almost 20% higher than the estimated coefficient of 1.67 for $\mathbb{IV} - \mathbb{RV}$.

In specifications [10] and [11] of Table 4, we add JTIX and JTIX/$\mathbb{V}$ to the regression specification with $\mathbb{IV} - \mathbb{RV}$. Here JTIX is significant at the 90% level for the six-month horizon and significant at the 95% levels for one- and two-year horizons. The results are slightly more significant for JTIX/$\mathbb{V}$. We find JTIX/$\mathbb{V}$ to be significant at the 95% level for the six-month and two-year horizons and at the 90% level for the one-month and one-year horizons. The sign of the coefficients for both JTIX and JTIX/$\mathbb{V}$ is positive, indicating that an increase in the tail index predicts higher expected excess returns.

It is worth emphasizing that the significance of the tail indices in specifications [10] and [11] with $\mathbb{IV} - \mathbb{RV}$ does not definitively indicate that both channels of stock return variability and tail risk are significant because $\mathbb{IV}$ is biased in the presence of jumps. Nevertheless, as we shall discuss below, the coefficients of JTIX and JTIX/$\mathbb{V}$ are too large to be explained solely a jump-induced bias in $\mathbb{IV}$.

5.2.2 Economic significance

To understand the economic significance of the jump-induced bias in $\mathbb{IV}$, we consider the return predicted by each measure of variance spread. Panel A of Table 5 tabulates the variance spread in terms of both $\mathbb{V} - \mathbb{RV}$ and $\mathbb{IV} - \mathbb{RV}$. Over the entire sample period, on average, $\mathbb{V} - \mathbb{RV}$ is 0.34% (variance points) higher than $\mathbb{IV} - \mathbb{RV}$ and higher by 0.91% (variance points) in recession months. The measure of integrated variance particularly underestimates the variance spread in the recent financial crisis because of the extraordinarily high degree of jump risk in this period; $\mathbb{V} - \mathbb{RV}$ is 48% greater than $\mathbb{IV} - \mathbb{RV}$.

Panel B of Table 5 reports the magnitude of the excess return predicted by $\mathbb{V} - \mathbb{RV}$ and $\mathbb{IV} - \mathbb{RV}$, respectively. The returns are predicted using the coefficients estimated in Panel D of Table 4 for each of the variance spreads for the one-year horizon (specifications [8] and [9]). Over the entire sample period, the one-year excess return predicted by $\mathbb{V} - \mathbb{RV}$ is 6.1%, compared with 4.6% as predicted by $\mathbb{IV} - \mathbb{RV}$. As noted earlier, jump risk reached its highest level in the most current recession. The excess return predicted by $\mathbb{V} - \mathbb{RV}$ in the most recent recession is 7.9%, compared with 4.7% as predicted by $\mathbb{IV} - \mathbb{RV}$. The return predicted by $\mathbb{IV} - \mathbb{RV}$ during the financial crisis is *lower* than that predicted for the prior recession. This last set of results underscores the economic importance of jump risk. Not correctly accounting for jump risk in the measurement of stock return variability leads to the extraordinary conclusion that the financial crisis period had lower risk than the 2001–2002 recession.

Finally, we compare the difference between the excess returns predicted by $\mathbb{V} - \mathbb{RV}$ and $\mathbb{IV} - \mathbb{RV}$ with that predicted by JTIX. As noted earlier, the annualized return predicted by $\mathbb{V} - \mathbb{RV}$ is 1.50% higher than the return predicted by $\mathbb{IV} - \mathbb{RV}$. In contrast, the annualized return predicted by JTIX over the entire sample period (using the coefficient estimated in specification [10] of Panel D of Table 5) is about 3.5%. The economic significance of the tail index is too high to be solely explained by a jump-induced bias in $\mathbb{IV}$. The evidence suggests that there is an additional channel through which jump risk is important.

5.3 Tail risk as an additional channel

Having ascertained the significance of jumps in its contribution to stock return variability, we next consider whether the jump-induced tail should be considered an additional channel for jump risk. As our first exercise, we include the tail index in the regression specification of equation (34) with the variance spread computed using the BKM variance. Given that the BKM variance accounts for discontinuities, the tail index should not be significant if the only channel for jump risk is its contribution to volatility.

We report the results in Table 6. As in previous tables, $\mathbb{V} - \mathbb{RV}$ is statistically significant for all horizons. Including the tail index does not impact the economic significance of $\mathbb{V} - \mathbb{RV}$; the signs and magnitudes of the estimated coefficients remain about the same.

How do the results with respect to the tail index compare with those previously reported in Table 4? The magnitude of the coefficients for JTIX and JTIX/$\mathbb{V}$ reduces slightly, reflecting absence of jump-bias in the estimate of stock return variability. Nevertheless, JTIX remains significant at the 95% level for the one-year horizon and at the 90% level for the two-year horizon. The results for JTIX/$\mathbb{V}$ are similar: JTIX/$\mathbb{V}$ is significant at the 95% level for the six-month horizon and at the 90% level for the one- and two-year horizons. The coefficients are economically significant. Using the one-year horizon as an illustration, a one standard deviation increase in JTIX increases annualized expected excess return by over 7%. In comparison, a one standard deviation in $\mathbb{V} - \mathbb{RV}$ increases annual expected return by 5.4%.

BTZ observe that, in contrast to the variance spread, the VIX by itself does *not* predict returns. We confirm their analysis by including $\mathbb{IV}$ in our regression; the coefficient of $\mathbb{IV}$ included in the regression with $\mathbb{VS}$ and JTIX or JTIX/$\mathbb{V}$ is insignificant. Nevertheless, the question arises as to whether it is possible to use information in $\mathbb{IV}$ to improve upon JTIX. Although the tail index is economically different from the VIX —JTIX is approximately a short position in a portfolio of risk reversals while the integrated variance is a position in a portfolio of strangles— economic stress affects both indices similarly. In times of stress, the prices of all OTM options increase (increasing $\mathbb{IV}$) and OTM puts increase more than OTM calls (increasing JTIX).

To investigate, we extract the principal components from the daily time series of $\mathbb{IV}$ and JTIX and find the component that is more highly correlated with JTIX. For simplicity, we call this principal component the jump factor, even though it is a linear combination of $\mathbb{IV}$ and JTIX. We include the end-of-month values of the jump factor in the regression instead of JTIX. The results are reported in Table 7. The significance levels for the jump factor are not much different from those for JTIX. The jump factor is significant at the 90% level for the six-month and two-year horizons and at the 95% level for the one-year horizon. Overall, this exercise suggests that $\mathbb{IV}$ does not have additional information useful to improve upon JTIX.

5.4 Tail risk, forward variance, and the quiet period

As our second exercise, we consider the importance of the tail index within the setup of BPS. They find that the forward integrated variance implied from option prices predicts short-horizon returns over horizons of one to six months. Their result is significant because, as noted earlier, the VIX itself is not significant. We investigate whether the tail index adds additional predictive power to the forward variance. Although the forward variance measures are also constructed from OTM option prices, there is no mechanical relation between JTIX and the forward variances such as between JTIX, $\mathbb{V}$, and $\mathbb{IV}$. The predictive regression is specified as

$$R_{t+j} - r^f_{t+j} = \alpha + \beta_1 \cdot \mathrm{JTIX}_t/\mathbb{V}_t + \beta_2 y_t^{(1)} + \beta_3 f_t^{(2)} + \Gamma' \cdot \bar{Z}_t + \epsilon_t, \tag{35}$$

where $y_t^{(1)}$ and $f_t^{(2)}$ are the end-of-month forward variances proposed by BPS.[11] Regarding control variables $\bar{Z}_t$, we follow BPS in including the earnings yield $(E/P)_t$ and the term spread $(TERM)$. We also include the default spread (DEF) as in earlier specifications for the variance spread. All variables are sampled at the end of the month.

In this exercise, we follow BPS in focusing on the sample period of September 1998 to September 2008. This sample period is interesting because (see Figure 2) it is a relatively quiet period bookended by the LTCM and the post-Lehman peaks in the tail index. There are two reasons to focus on this period. First, from an economic standpoint, we are especially interested in understanding the compensation for tail risk in a non-crisis period but when investors are well aware of the possibility of a tail event. This is the motivation underlying the literature on consumption disasters (e.g., Rietz, 1988; Barro, 2006; Gabaix, 2012; Wachter, 2012). Second, discontinuities during a crisis may well be very different from those in quiet

[11] We thank George Panayotov for sharing the data on forward variances. BPS define $y_t^{(1)} = -\ln H_t^{(t,1)}$, $f_t^{(2)} = \ln H_t^{(t,1)} - \ln H_t^{(1,2)}$, where $H_t^{(t,n)} = e^{-r\tau_n} \mathrm{E}_t^{\mathbb{Q}} \exp\left(-\int_t^{t+\tau_n} \sigma_u^2 du\right)$. The terms τ_1 and τ_2 are the remaining time to expiration for options expiring in the next two months, respectively.

times. In particular, our Figure 3 shows that in the immediate aftermath of the crisis, there is also risk of *upside* discontinuities. The quiet period puts the spotlight on downside jump fears.

Panel A of Table 8 reports the results for JTIX/$\mathbb{V}$ (the results for JTIX lead to the same conclusion). As in BPS, we report results for short horizons up to six months. Consistent with the findings of BPS, the forward variance $f_t^{(2)}$ significantly predicts returns over short horizons up to three months. But, in addition, the jump and tail index is significant at 99% for the one- and three-month horizons and at 90% for the six-month horizon. There is extremely strong evidence that investors are willing to pay insurance over short horizons in quiet times.

As noted by BPS, the forward integrated variances are constructed under the assumption of the absence of jumps (see Appendix A for further details on the relation between the forward variance and jumps). Could it be that the high significance of JTIX is due to a jump-induced bias in the forward variance? To check, we redo the regressions with the variance spread constructed using the BKM variance for the period corresponding to the BPS study. The results are reported in Panel B of Table 8. The significance and magnitude of the coefficients are very similar to those estimated with the BPS forward variance in Panel A. Interestingly, JTIX/$\mathbb{V}$ is significant at the one-month horizon even though $\mathbb{V} - \mathbb{RV}$ is not significant for this horizon in the quiet period. Thus, the economic significance of the tail index is not related to any potential jump-induced bias in the BPS forward variance. Instead, the evidence is consistent with investors' fears of a downside tail event.

In summary, the evidence indicates that even in relatively quiet times, investors are concerned about the possibility of a downside tail event over very short horizons. Investors fear tail risk after accounting for stock return variability.

6 Conclusion

When the risk-neutral stock return process incorporates fears of discontinuities, both stock return variability and the tail of the return distribution are determined by fears of jumps. To distinguish between the two channels, it is important to have model-free volatility and tail indices that clearly distinguish between the contribution of jumps to stock return volatility and the impact of jumps on the tail of the distribution. This paper provides a novel way of constructing a tail index that achieves this objective; time variation in volatility and tail indexes can be distinguished, even though jumps impact the former and determine the latter.

Our jump and tail index is easily constructed from a portfolio of OTM options of 30-day maturity, and can be economically interpreted as a short portfolio of risk reversals.[12] The tail index demonstrates extreme time variation in jump risk, with the intensity of jumps increasing 50-fold in times of crisis. Consequently, it is important to correctly account for jump risk in estimating stock return volatility. In times of crisis, using the integrated variance as the measure of stock return variability can underestimate market variance by as much as 15%, the variance spread by over 30%, and the predicted equity return by 40%. The tail index predicts index returns after controlling for the variance spread or forward variance. Importantly, the tail index is also significant at horizons as short as one-month in the relatively quiet period between between the LTCM and the Lehman financial crises. The evidence indicates that investors' fear of downside tail events are incorporated into equity prices.

Our conclusion that the BKM measure of holding period variance is a more accurate measure of quadratic variation than the VIX in the presence of jumps has implications for volatility derivative markets such as those of variance swaps. Besides indicating how the variance swap can be hedged against the risk of discontinuities using a portfolio of risk-reversals, our analysis suggests that payoffs of volatility derivatives should be based on the second moment of the holding period return (square of summed log returns) as opposed to the quadratic variation (sum of squared log returns).

Our empirical results indicate a role for both volatility and tail risk. It would be of interest for future research to develop a model that provides a single framework for both these risks. It would also be interesting to understand whether the risk premium associated with downside jump risk is best understood as arising from risk of consumption disasters, or in terms of wealth disasters resulting in a higher marginal utility of aggregate wealth.

[12] Interestingly, our analysis supports the popular practice of using the difference in OTM put and call implied volatilities of similar (out) moneyness or the difference between the OTM and ATM puts as a measure of the option's "skew". Although they do not capture the skewness of the risk-neutral distribution, from their (approximate) correspondence to the price of a risk-reversal, these measures may be interpreted as rough measures of jump risk.

7 Appendix A: Proofs of results

Proof of Proposition 1

Given that $\ln S_t$ is a semimartingale, the quadratic variation exists and is defined by stochastic integration by parts:

$$(\ln S_T/S_0)^2 = 2\int_0^T \ln S_{t-}/S_0 \, d\ln S_t + [\ln S, \ln S]_T. \tag{A-1}$$

From the definition of the variance,

$$\mathrm{var}_0^{\mathbb{Q}}(\ln S_T/S_0) \equiv \mathrm{E}_0^{\mathbb{Q}}(\ln S_T/S_0)^2 - (\mathrm{E}_0^{\mathbb{Q}}\ln S_T/S_0)^2. \tag{A-2}$$

From equations (A-1) and (A-2), it follows that $\mathbb{D}(T) = \left(\mathrm{E}_0^{\mathbb{Q}}(\ln S_T/S_0)\right)^2 - \mathrm{E}_0^{\mathbb{Q}}\left[\int_0^T 2\ln S_{t-}/S_0 \, d\ln S_t\right]$ and that $\mathbb{D} = 0$ if and only if $2\mathrm{E}_0^{\mathbb{Q}}\int_0^T \ln S_{t-}/S_0 \, d\ln S_t = \left(\mathrm{E}_0^{\mathbb{Q}}\ln S_T/S_0\right)^2$.

Proof of Proposition 1, part i.

Next, when $\ln S_t/S_0 = A_t + M_t$ with A_t deterministic and M_t a martingale,

$$\begin{aligned}
\mathrm{E}_0^{\mathbb{Q}}\int_0^T 2\ln S_{t-}/S_0 \, d\ln S_t &= \mathrm{E}_0^{\mathbb{Q}}\int_0^T \ln S_{t-}/S_0 \, \mathrm{E}_t \, d\ln S_t, && \text{(A-3)}\\
&= \int_0^T 2A_t \, dA_t, && \text{(A-4)}\\
&= (A_T)^2 = \left(\mathrm{E}_0^{\mathbb{Q}}\ln S_T/S_0\right)^2. && \text{(A-5)}
\end{aligned}$$

Therefore, $\mathbb{D} = 0$. In the above equations, the first equality follows from the law of iterative expectations and the second because M_t is a martingale. The third equality follows because the drift is of finite variation with continuous paths (Theorem I.53 of Protter, 2004). This proves Proposition 1, part i. □

Proof of Proposition 1, part ii.

For the stochastic volatility diffusion model defined by equations (12) and (13),

$$\begin{aligned}
\mathbb{D}(T) &= \mathrm{E}_0^{\mathbb{Q}}\left[\int_0^T 2\ln S_t/S_0 \, d\ln S_t\right] - \left(\mathrm{E}_0^{\mathbb{Q}}(\ln S_T/S_0)\right)^2 && \text{(A-6)}\\
&= I + II.
\end{aligned}$$

To study the speed of convergence of $\mathbb{D}(T)$ when $T \to 0$, we apply the Ito–Taylor expansion (Milstein, 1995) to each term of $\mathbb{D}(T)$ in equation (A-6). We define the operators $\mathcal{L} \equiv \frac{\partial}{\partial t} + (r - \frac{1}{2}\sigma_t^2)\frac{\partial}{\partial(\ln S_t)} + \theta[\sigma_t^2]\frac{\partial}{\partial(\sigma_t^2)} + \frac{1}{2}\sigma_t^2\frac{\partial^2}{\partial(\ln S_t)^2} + \frac{1}{2}\eta^2[\sigma_t^2]\frac{\partial^2}{\partial(\sigma_t^2)^2} + \rho\sigma_t\eta[\sigma_t^2]\frac{\partial^2}{\partial(\ln S_t)\partial(\sigma_t^2)}$, $\Gamma_1 \equiv \sigma_t\frac{\partial}{\partial(\ln S_t)}$, and $\Gamma_2 \equiv \eta[\sigma_t^2]\frac{\partial}{\partial(\sigma_t^2)}$. Noting that applying the Ito–Taylor expansion on a deterministic function $g(\ln S_t, \sigma_t^2, t)$ yields $g(\ln S_t, \sigma_t^2, t) = g(\ln S_0, \sigma_0^2, 0) + \int_0^t (\mathcal{L}[g]du + \Gamma_1[g]dW_{1,u} + \Gamma_2[g]dW_{2,u})$, applying the expansion twice to the integral in term I of equation (A-6) yields

$$\begin{aligned}
\int_0^T \ln S_t/S_0 \, d\ln S_t &= \int_0^T \ln\frac{S_t}{S_0}(r - \frac{1}{2}\sigma_t^2)dt + \int_0^T \ln\frac{S_t}{S_0}\sigma_t dW_{1,t}, \\
&= \int_0^T \int_0^t \left[(r - \frac{1}{2}\sigma_u^2)^2 - \frac{1}{2}\ln\frac{S_u}{S_0}\theta[\sigma_u^2] - \frac{1}{2}\sigma_u\eta[\sigma_u^2]\rho\right] du\, dt \\
&\quad + \int_0^T \int_0^t (r - \frac{1}{2}\sigma_u^2)\sigma_u dW_{1,u}dt - \frac{1}{2}\int_0^T \int_0^t \ln\frac{S_u}{S_0}\eta[\sigma_u^2]dW_{2,u}dt + \int_0^T \ln\frac{S_t}{S_0}\sigma_t dW_{1,t}, \\
&= \int_0^T \int_0^t \left[(r - \frac{1}{2}\sigma_0^2)^2 - \frac{1}{2}\sigma_0\eta[\sigma_0^2]\rho\right] du\, dt \\
&\quad + \int_0^T \int_0^t (r - \frac{1}{2}\sigma_0^2)\sigma_0 dW_{1,u}\, dt - \frac{1}{2}\int_0^T \int_0^t \ln\frac{S_0}{S_0}\eta[\sigma_0^2]dW_{2,u}\, dt + \int_0^T \ln\frac{S_t}{S_0}\sigma_t dW_{1,t} \\
&\quad + A_0, \qquad \text{(A-7)}
\end{aligned}$$

where A_0 consists of terms such as $\int_0^T \int_0^t \int_0^v \mathcal{L}^2[\cdot]dv\, du\, dt$, $\int_0^T \int_0^t \int_0^v \Gamma_1[\mathcal{L}[\cdot]]\, dW_v^1\, du\, dt$, and so on. With repeated applications of the Ito-Taylor expansion and using the martingale property of the Ito integral, $\mathrm{E}_0^{\mathbb{Q}}(A_0) = \sum_{n=0}^{\infty} h^n(\sigma_0^2)\frac{T^{n+3}}{(n+3)!}$, for deterministic functions h^n, $n \in \{0, 1, 2...\}$, and, therefore, $\mathrm{E}_0^{\mathbb{Q}}[A_0] = O(T^3)$. Taking expectations and integrating, it follows that

$$\mathrm{E}_0^{\mathbb{Q}} \int_0^T 2\ln S_t/S_0 \, d\ln S_t = \left[(r - \frac{1}{2}\sigma_0^2)^2 - \frac{1}{2}\sigma_0\eta[\sigma_0^2]\rho\right] T^2 + O(T^3). \qquad \text{(A-8)}$$

We can similarly proceed to evaluate term II of equation (A-6) by applying the stochastic Taylor expansion to the integral defining the log return process:

$$\begin{aligned}
\ln\frac{S_t}{S_0} &= \int_0^T (r - \frac{1}{2}\sigma_t^2)dt + \int_0^T \sqrt{\sigma_t^2}dW_t^1, \\
&= (r - \frac{1}{2}\sigma_0^2)T - \int_0^T \int_0^t \frac{1}{2}\,\theta[\sigma_u^2]du\, dt + \int_0^T \int_0^t -\frac{1}{2}\,\eta[\sigma_u^2]dW_{2,u}\, dt + \int_0^T \sqrt{\sigma_t^2}dW_t^1, \\
&= (r - \frac{1}{2}\sigma_0^2)T - \frac{T^2}{4}\theta[\sigma_0^2] + \int_0^T \int_0^t -\frac{1}{2}\,\eta[\sigma_u^2]dW_{2,u}\, dt + \int_0^T \sqrt{\sigma_t^2}dW_t^1 + A_1, \qquad \text{(A-9)}
\end{aligned}$$

where $A_1 = O(T^3)$. Taking expectations,

$$\mathrm{E}_0^{\mathbb{Q}}(\ln S_T/S_0) = (r - \frac{1}{2}\sigma_0^2)T - \frac{T^2}{4}\theta[\sigma_0^2] + O(T^3). \qquad \text{(A-10)}$$

Combining equations (A-8) and (A-10), we have

$$\mathbb{D}(T) = \mathrm{E}_0^{\mathbb{Q}} \int_0^T 2\ln S_t/S_0 \, d\ln S_t - \left(\mathrm{E}_0^{\mathbb{Q}} \ln\frac{S_t}{S_0}\right)^2 = -\frac{1}{2}\sigma_0\eta[\sigma_0^2]\rho T^2 + O(T^3). \qquad \text{(A-11)}$$

It follows that $\frac{1}{T}D(T)$ is $O(T)$ and, when $\rho = 0$, $\frac{1}{T}D(T) = O(T^2)$.

$\square$

Proof of Proposition 2
Given that jumps of all sizes have the same arrival intensity, it follows that $\mathrm{E}_0^{\mathbb{Q}} 2\int_0^T \int_{R^0} \psi(x)\, \mu[dx,dt] = 2\int_0^T \mathrm{E}_0^{\mathbb{Q}} \left[\int_{R^0} \psi(x)\, f(x) dx\right] \lambda_t dt = 2\Psi(f(x)) \mathrm{E}_0^{\mathbb{Q}} \int_0^T \lambda_t dt$, where $\Psi(\cdot)$ is determined by the jump size distribution. $\square$

Proof of Equation 6
To evaluate the integral, first observe from equation (3) that

$$\mathrm{E}_0^{\mathbb{Q}} \ln(S_{t-}/S_0) = (r - \frac{1}{2}\sigma^2 - \lambda\mu_J)t + \lambda\alpha t. \tag{A-12}$$

Therefore,

$$\begin{aligned}
\mathrm{E}_0^{\mathbb{Q}} \int_0^T 2\ln(S_{t-}/S_0)\, d\ln S_t &= 2\,\mathrm{E}_0^{\mathbb{Q}} \left[\int_0^T \ln(S_{t-}/S_0)\,(r - \frac{1}{2}\sigma^2 - \lambda\mu_J)\, dt + \int_0^T \int_{R^0} \ln(S_{t-}/S_0) x\, \mu[dx,dt] \right], \\
&= 2 \left[\int_0^T \mathrm{E}_0^{\mathbb{Q}} \left(\ln(S_{t-}/S_0)\right)\, (r - \frac{1}{2}\sigma^2 - \lambda\mu_J)\, dt \right. \\
&+ \left. \left[. \int_0^T \int_{R^0} \mathrm{E}_0^{\mathbb{Q}} \left(\ln(S_{t-}/S_0)\right) x\, \frac{\lambda}{\sqrt{2\pi\sigma_J^2}} e^{-\frac{(x-\alpha)^2}{2}} dx dt \right] \right], \\
&= \left((r - \frac{1}{2}\sigma^2 - \lambda\mu_J) + \lambda\alpha \right)^2 T^2, \\
&= \left(\mathrm{E}_0^{\mathbb{Q}} \ln(S_{t-}/S_0) \right)^2 .
\end{aligned} \tag{A-13}$$

Therefore, for the Merton model, $\mathrm{E}_0^{\mathbb{Q}} \int_0^T 2\ln(S_{t-}/S_0)\, d\ln S_t = \left(\mathrm{E}_0^{\mathbb{Q}} \ln(S_{t-}/S_0) \right)^2$. $\square$

Proof of equation 15, $\mathbb{D}(T) = \frac{1}{4} var \int_0^T \sigma_t^2 dt$ for a stochastic volatility diffusion with $\rho = 0$
Let $\ln S_T/S_0$ be a continuous semimartingale,

$$\ln S_T = \ln S_0 + \int_0^T (r - \frac{1}{2}\sigma_t^2) dt + \int_0^T \sigma_t dW_{1,t}, \tag{A-14}$$

where σ_t^2 is another continuous semimartingale, orthogonal to $W_{1,t}$. By the law of total variance,

$$\mathrm{var}_0^{\mathbb{Q}} (\ln S_T/S_0) = \mathrm{var}_0^{\mathbb{Q}} \left(\mathrm{E}_0^{\mathbb{Q}} \ln S_T/S_0 \mid \{\sigma_t^2\}_{0 \le t \le T} \right) + \mathrm{E}_0^{\mathbb{Q}} \left(\mathrm{var}_0^{\mathbb{Q}} (\ln S_T/S_0) \mid \{\sigma_t^2\}_{0 \le t \le T} \right). \tag{A-15}$$

Now, from equation(A-14),

$$\mathrm{var}_0^{\mathbb{Q}} \left(\mathrm{E}_0^{\mathbb{Q}} \ln S_T/S_0 \mid \{\sigma_t^2\}_{(0 \le t \le T)} \right) = \frac{1}{4} \mathrm{var}_0^{\mathbb{Q}} \int_0^T \sigma_t^2 dt, \tag{A-16}$$

and

$$\mathrm{E}_0^{\mathbb{Q}}\left(\mathrm{var}_0^{\mathbb{Q}}(\ln S_T/S_0) \mid \{\sigma_t^2\}\right) = \mathrm{E}_0^{\mathbb{Q}} \int_0^T \sigma_t^2 dt \tag{A-17}$$

from Ito isometry. Putting this together into (A-15) and given that the quadratic variation for the diffusion process is equal to the integrated variance, $\int_0^T \sigma_t^2 dt$, we obtain

$$\mathbb{D}(T) = \mathrm{var}_0^{\mathbb{Q}}(\ln S_T/S_0) - \mathrm{E}_0^{\mathbb{Q}}[\ln S, \ln S]_T = \frac{1}{4}\mathrm{var}_0^{\mathbb{Q}} \int_0^T \sigma_t^2 dt. \tag{A-18}$$

Thus, if the stochastic volatility process is independent of $W_{1,t}$, then $\mathbb{D}(T)$ is proportional to the variance of the integrated variance. □

Power claim of Carr and Lee (2008) in presence of jumps

First, following Carr and Lee (2008), assume there are no discontinuities and that the volatility process, σ_t, is independent of the diffusion determining the log stock process, $W_{1,t}$. Without loss of generality, we can also assume that the risk-free rate is zero. If so,

$$\ln S_T/S_0 = \int_0^T -\frac{1}{2}\sigma_t^2 dt + \int_0^T \sigma_t dW_{1,t} \tag{A-19}$$

and, therefore, the power claim,

$$\begin{aligned} \mathrm{E}_0^{\mathbb{Q}} \exp(p \ln S_T/S_0) &= \mathrm{E}_0^{\mathbb{Q}} \exp\left(\int_0^T -\frac{p}{2}\sigma_t^2 dt + \int_0^T p\sigma_t dW_{1,t}\right), & \text{(A-20)} \\ &= \mathrm{E}_0^{\mathbb{Q}} \exp\left((\frac{p^2}{2} - \frac{p}{2}) \int_0^T \sigma_t^2 dt\right). & \text{(A-21)} \end{aligned}$$

BPS use this theory to estimate $\mathrm{E}_0^{\mathbb{Q}}\left(\exp(\int_0^T \sigma_t^2 dt\right)$ from option prices.[13] Now relax the assumption of no discontinuities by adding Merton-style jumps, following Section 2,

$$\mathrm{E}_0^{\mathbb{Q}} \exp\left(p \ln S_T/S_0\right) = \mathrm{E}_0^{\mathbb{Q}} \exp\left(\int_0^T -\frac{p}{2}\sigma^2 dt - \int_0^T p\lambda\mu_J\, dt + \int_0^T p\sigma\, dW_t + \int_0^T \int_{\mathbb{R}^0} px\, \mu[dx, dt]\right). \tag{A-22}$$

[13] Because the volatility process is independent of $W_{1,t}$, the log return is normally distributed with mean and variance equal to $-\frac{1}{2}\int_0^T \sigma_t^2 dt$ and $\int_0^T \sigma_t^2 dt$, respectively, leading to equation (A-21). We can rewrite equation (A-21) as

$$\exp\left(\bar{\lambda} \int_0^T \sigma_t^2 dt\right) = \mathrm{E}_0^{\mathbb{Q}}(S_T/S_0)^{\frac{1}{2} \pm \sqrt{\frac{1}{4} + 2\bar{\lambda}}}.$$

Because the power claim $(S_T/S_0)^p$ can be synthesized from options, we can estimate the price of a claim paying the exponential of the integrated variance. In addition, Carr and Lee (2008) demonstrate that a properly chosen portfolio of power claims is not sensitive to a non-zero correlation. Following the Carr–Lee analysis, BPS consider the case for $\bar{\lambda} = -1$.

We can re-write equation A-22 using the definition of quadratic variation to get,

$$\mathrm{E}_0^{\mathbb{Q}} \exp\left(p \ln S_T/S_0\right) = \mathrm{E}_0^{\mathbb{Q}} \left[\exp\left(-\frac{p}{2}[\ln S, \ln S]_T + \int_0^T p\sigma\, dW_t\right) \exp\left(-\int_0^T p\lambda\mu_J\, dt + \int_0^T \int_{\mathbb{R}^0} (px + \frac{p}{2}x^2)\, \mu[dx, dt]\right)\right]. \tag{A-23}$$

A comparison of equation (A-23) with equation (A-20) demonstrates that the power claim can measure the quadratic variation without bias only if jumps are absent. □

8 Appendix B: Description of Data

In line with previous literature, we clean the data using several filters. First, we exclude options that have (i) missing implied volatility in OptionMetrics, (ii) zero open interest, and (iii) bids equal to zero or negative bid–ask spreads. Second, we verify that options do not violate the no-arbitrage bounds. For an option of strike K maturing at time T with current stock price S_t and dividend D, we require that $\max(0, S_t - PV[D] - PV[K]) \leq c(S_t; T, K) \leq S_t - PV[D]$ for European call options and $\max(0, PV[K] - (S_t - PV(D))) \leq p \leq PV[K]$ for European put options, where $PV[\cdot]$ is the present value function. Fourth, if two calls or puts with different strikes have identical mid-quotes, that is, $c(S_t; T, K_1) = c(S_t; T, K_2)$ or $p(S_t; T, K_1) = p(S_t; T, K_2)$, we discard the one furthest away from the money quote. Finally, we keep only OTM options and include observations for a given date only if there are at least two valid OTM call and put quotes. Table B1 provides summary statistics of the final sample.

Table 1. Table B1. Option data
This table reports the summary statistics of all options used to construct a 30-day JTIX with a daily frequency. The variable *Implied volatility* is the Black–Scholes implied volatility; *Range of Moneyness* on a certain date for a given underlying asset is defined as $(K^{\max} - K^{\min})/K^{atm}$, where $K^{\max}$, $K^{\min}$, and K^{atm} are the maximum, minimum, and at-the-money strikes, respectively; near term refers to options with the shortest maturity (but greater than seven days) on each date; and next term refers to options with the second shortest maturity on each date. The sample period is from January 1996 to October 2010.

	Near Term				Next Term			
	MEAN	SD	MIN	MAX	MEAN	SD	MIN	MAX
# of options per date	46	23	13	135	45	28	11	136
Range of moneyness per date	37%	15%	10%	137%	48%	18%	13%	153%
Option price	5.53	7.44	0.08	68.10	10.25	11.50	0.08	84.05
Implied volatility	28.02%	15.50%	4.88%	183.33%	26.64%	13.32%	6.69%	181.76%
Maturity	22	9	7	39	51	9	14	79
Trading volume	2,186	5,374	0	200,777	957	2,885	0	120,790
Open interest	19,465	30,566	1	366,996	119,72	22,640	1	348,442

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Table 1. Numerical comparison: $\mathbf{E}^{\mathbb{Q}}[\ln S, \ln S]_T$, $\mathbb{V}$, and $\mathbb{IV}$

Numerical comparison of annualized expected quadratic variation $\mathrm{E}^{\mathbb{Q}}[\ln S, \ln S]_T$ with $\mathbb{V}$ and $\mathbb{IV}$ for the jump diffusion model of Merton (1976) (Panel A) and the jump diffusion and stochastic volatility (SVJ) model of Bates (2000) (Panel B). The specification for the Merton model is $d\ln S_t = (r - \mu_J\lambda - \frac{1}{2}\sigma^2)\,dt + \sigma dW_t + x\,dJ_t$, where $x \sim N(\alpha, \sigma_J^2)$, and $\mu_J = e^{\alpha+\frac{1}{2}\sigma_J^2} - 1$. The specification for the SVJ model is: $d\ln S_t = (r - \frac{1}{2}\sigma_t^2 - \lambda\mu_J)dt + \sigma_t dW_t^1 + x\,dJ_t$, $d\sigma_t^2 = \kappa(\sigma_t^2 - \theta)dt + \eta\sigma_t dW_t^2$, where $corr(dW_t^1, dW_t^2) = \rho$, $x \sim N(\alpha, \sigma_J^2)$ and $\lambda = \lambda_0 + \lambda_1 \cdot \sigma_t^2$. The parameters are from Pan (2002): $\lambda_0 = 0$, $\lambda_1 = 12.3$ (for Merton's model, $\lambda = \lambda_1 \cdot 0.04$), $\sigma_J = 0.0387$, $\kappa = 3.3$, $\theta = 0.0296$, $\eta = 0.3$, and $\rho = -0.53$. In addition, the risk-free rate $r = 0.03$ and time to maturity $\tau = 30/365$. The remaining parameters are shown in the table. The first column denotes the contribution of jumps to the total variance, defined as the holding period variance of the pure jump component of the stochastic process divided by the holding period variance of the combined jump diffusion process.

Panel A. Merton model

						$\mathbb{IV} - [\ln S, \ln S]_T$	
$\frac{\mathrm{var}_{Jump}}{\mathrm{var}}$	σ_0^2	α	$\mathrm{E}^{\mathbb{Q}}[\ln S, \ln S]_T$	$\mathbb{V}$	$\mathbb{IV}$	Absolute	Relative (%)
0%	0.0400	0.0000	0.0400	0.0400	0.0400	0.0000	0.00
10%	0.0360	-0.0814	0.0400	0.0400	0.0399	-0.0001	-0.36
20%	0.0320	-0.1215	0.0400	0.0400	0.0396	-0.0004	-0.92
30%	0.0280	-0.1513	0.0400	0.0400	0.0393	-0.0007	-1.63
40%	0.0240	-0.1761	0.0400	0.0400	0.0390	-0.0010	-2.44
50%	0.0200	-0.1979	0.0400	0.0400	0.0387	-0.0013	-3.36
60%	0.0160	-0.2174	0.0400	0.0400	0.0383	-0.0017	-4.36
70%	0.0120	-0.2354	0.0400	0.0400	0.0378	-0.0022	-5.43
80%	0.0080	-0.2521	0.0400	0.0400	0.0374	-0.0026	-6.58
90%	0.0040	-0.2677	0.0400	0.0400	0.0369	-0.0031	-7.80

Panel B: SVJ model

						$\mathbb{IV} - [\ln S, \ln S]_T$		$\mathbb{V} - [\ln S, \ln S]_T$	
$\frac{\mathrm{var}_{Jump}}{\mathrm{var}}$	σ_0^2	α	$\mathrm{E}^{\mathbb{Q}}[\ln S, \ln S]_T$	$\mathbb{V}$	$\mathbb{IV}$	Absolute	Relative (%)	Absolute	Relative (%)
0%	0.0415	0.0000	0.0400	0.0402	0.0400	0.0000	0.00	0.0002	0.61
10%	0.0369	-0.0868	0.0400	0.0402	0.0399	-0.0001	-0.37	0.0002	0.61
20%	0.0323	-0.1372	0.0400	0.0402	0.0396	-0.0004	-1.01	0.0002	0.60
30%	0.0278	-0.1826	0.0400	0.0402	0.0392	-0.0008	-1.89	0.0002	0.59
40%	0.0232	-0.2296	0.0400	0.0402	0.0388	-0.0012	-3.04	0.0002	0.58
50%	0.0186	-0.2825	0.0400	0.0402	0.0382	-0.0018	-4.54	0.0002	0.57
60%	0.0141	-0.3471	0.0400	0.0402	0.0374	-0.0026	-6.52	0.0002	0.55
70%	0.0095	-0.4338	0.0400	0.0402	0.0363	-0.0037	-9.24	0.0002	0.52
80%	0.0049	-0.5690	0.0400	0.0402	0.0347	-0.0053	-13.34	0.0002	0.47
90%	0.0004	-0.8545	0.0400	0.0401	0.0316	-0.0084	-21.04	0.0001	0.36

Table 2. Comparison between $\mathbb{V}$ and $\mathbb{IV}$

This table reports the summary statistics of $\sqrt{\mathbb{V}}$, $\sqrt{\mathbb{IV}}$, $\sqrt{\mathbb{V}}-\sqrt{\mathbb{IV}}$, $\mathbb{V}-\mathbb{IV}$, and $\sqrt{\mathbb{RV}}$. We show the results for the entire sample period of January 1996 to October 2010, as well as sample periods corresponding to recessions and non-recessions. Recession periods defined by the National Bureau of Economic Research (NBER) are from March 2001 to November 2001 and from December 2007 to June 2009. The term Δ is $\sqrt{\mathbb{V}}-\sqrt{\mathbb{IV}}$ and $\sqrt{\mathbb{RV}}$ is the realized volatility calculated using the formula $\sqrt{\frac{252}{n}\sum_{i=1}^{n}\left(\ln\frac{S_i}{S_{i-1}}\right)^2}$, where S_i is the underlying security's closing price on the ith trading day of the option's remaining maturity, n is the total number of trading days corresponding to the options's remaining maturity. T is the remaining maturity of the option. All numbers are scaled up by a factor of 100.

		Full Period						Recession						Non-Recession					
T		Obs	$\sqrt{\mathbb{V}}$	$\sqrt{\mathbb{IV}}$	Δ	$\mathbb{V}-\mathbb{IV}$	$\sqrt{\mathbb{RV}}$	Obs	$\sqrt{\mathbb{V}}$	$\sqrt{\mathbb{IV}}$	Δ	$\mathbb{V}-\mathbb{IV}$	$\sqrt{\mathbb{RV}}$	Obs	$\sqrt{\mathbb{V}}$	$\sqrt{\mathbb{IV}}$	Δ	$\mathbb{V}-\mathbb{IV}$	$\sqrt{\mathbb{RV}}$
30	Mean	178	22.75	22.26	0.49	0.32	18.03	28	34.04	33.04	1.00	0.97	29.38	150	20.64	20.24	0.40	0.19	15.92
	Std. Dev.		9.83	9.34	0.55	0.71	10.40		15.60	14.58	1.08	1.59	16.65		6.51	6.27	0.29	0.22	7.04
	Min		10.41	10.28	0.07	0.02	6.07		19.24	18.97	0.25	0.10	13.31		10.41	10.28	0.07	0.02	6.07
	Max		78.90	75.52	4.56	6.71	77.93		78.90	75.52	4.56	6.71	77.93		40.41	39.16	2.15	1.65	44.92
23	Mean	176	22.58	22.15	0.43	0.27	18.05	28	32.39	31.55	0.84	0.79	29.21	148	20.73	20.37	0.35	0.17	15.94
	Std. Dev.		9.30	8.89	0.48	0.69	10.51		14.16	13.21	1.02	1.60	16.87		6.67	6.48	0.23	0.16	7.14
	Min		9.85	9.73	0.08	0.02	5.26		18.53	18.29	0.20	0.08	12.45		9.85	9.73	0.08	0.02	5.26
	Max		81.82	76.47	5.36	8.48	84.26		81.82	76.47	5.36	8.48	84.26		42.72	41.92	1.18	0.85	46.59
16	Mean	173	23.03	22.66	0.37	0.23	17.97	26	33.41	32.71	0.71	0.62	30.31	147	21.20	20.89	0.31	0.16	15.78
	Std. Dev.		9.19	8.88	0.38	0.41	10.96		12.94	12.31	0.68	0.85	18.17		6.95	6.76	0.25	0.19	7.29
	Min		10.14	10.06	0.07	0.01	5.45		20.86	20.61	0.16	0.07	13.64		10.14	10.06	0.07	0.01	5.45
	Max		66.89	64.83	2.94	3.53	87.23		66.89	64.83	2.94	3.53	87.23		42.72	41.49	2.29	1.64	41.01
9	Mean	177	23.72	23.44	0.28	0.18	17.49	27	34.69	34.15	0.53	0.52	29.36	150	21.74	21.51	0.23	0.12	15.35
	Std. Dev.		10.42	10.18	0.27	0.36	12.02		16.13	15.64	0.53	0.79	21.34		7.56	7.44	0.15	0.13	7.82
	Min		10.46	10.36	0.05	0.01	4.96		20.00	19.90	0.10	0.04	10.52		10.46	10.36	0.05	0.01	4.96
	Max		80.07	78.53	2.13	3.27	102.51		80.07	78.53	2.13	3.27	102.51		49.21	48.36	1.03	0.88	48.11

Table 3. Predictive regression: Summary statistics

This table reports the summary statistics of the variables used in predictive regressions over the sample period from January 1996 to October 2010. All variables are sampled monthly at the end of the month. Panel A reports the basic statistics. Panel B reports the cross-correlation matrix. Panel C reports the autocorrelations up to order six. Panel D reports the results for the Phillips–Perron unit root test. Here $R_t - r_t^f$ is the monthly excess log return on the S&P 500 index; $\mathbb{V}$ is the annualized holding period variance; $\mathbb{IV}$ is the annualized integrated variance; $\mathbb{RV}$ is the annualized realized variance from Hao Zhou's website; E/P and $\log(P/D)$ are the earnings–yield ratio and the logarithm of the price–dividend ratio on the S&P 500, respectively; $TERM$ is the difference between the 10-year and three-month Treasury yields; and DEF is the difference between Moody's BAA and AAA bond yields.

Panel A. Summary Statistics

	$R_t - r_t^f$	JTIX_t	$\text{JTIX}_t/\mathbb{V}_t$	$\mathbb{RV}_{t-1}$	$\mathbb{V}_t - \mathbb{RV}_{t-1}$	$\mathbb{IV}_t - \mathbb{RV}_{t-1}$	E_t/P_t	$log(P_t/D_t)$	$TERM_t$	DEF_t
Mean	0.0026	0.0035	0.0421	0.0326	0.0310	0.0276	0.0106	4.0639	0.0157	0.0101
Median	0.0092	0.0018	0.0360	0.0193	0.0239	0.0224	0.0115	4.0508	0.0142	0.0089
Std. Dev.	0.0478	0.0069	0.0224	0.0530	0.0290	0.0285	0.0059	0.2372	0.0126	0.0049
Max	0.0900	0.0581	0.1551	0.5755	0.1935	0.1744	0.0191	4.4932	0.0369	0.0338
Min	-0.1837	0.0003	0.0141	0.0047	-0.1132	-0.1788	-0.0257	3.3767	-0.0070	0.0055
Skewness	-0.79	5.80	2.67	6.97	1.17	-0.81	-3.95	-0.40	0.10	2.82
Kurtosis	4.09	41.81	12.47	65.96	11.91	21.20	25.08	3.45	1.74	11.93
AR(1)	0.11	0.77	0.64	0.63	0.36	0.27	0.77	0.97	0.98	0.96

Panel B. Correlation Matrix

	$R_t - r_t^f$	JTIX	JTIX/$\mathbb{V}$	$\mathbb{RV}_{t-1}$	$\mathbb{V}_t - \mathbb{RV}_{t-1}$	$\mathbb{IV}_t - \mathbb{RV}_{t-1}$	E_t/P_t	$\log(P_t/D_t)$	$TERM_t$	DEF_t
$R_t - r_t^f$	1.00	-0.30	-0.01	-0.41	-0.14	-0.06	0.27	-0.06	-0.02	-0.15
JTIX		1.00	0.77	0.88	0.20	0.00	-0.44	-0.34	0.21	0.70
JTIX/$\mathbb{V}$			1.00	0.57	0.19	0.05	-0.43	-0.26	0.17	0.58
$\mathbb{RV}_{t-1}$				1.00	-0.06	-0.27	-0.33	-0.26	0.19	0.61
$\mathbb{V}_t - \mathbb{RV}_{t-1}$					1.00	0.98	-0.36	-0.13	0.14	0.24
$\mathbb{IV}_t - \mathbb{RV}_{t-1}$						1.00	-0.29	-0.07	0.09	0.11
E_t/P_t							1.00	0.01	-0.07	-0.51
$\log(P_t/D_t)$								1.00	-0.40	-0.62
$TERM_t$									1.00	0.43
DEF_t										1.00

Panel C. Autocorrelations

	$\mathbb{V}$	$\mathbb{IV}$	JTIX	JTIX/$\mathbb{V}$	$\mathbb{V}_t - \mathbb{RV}_{t-1}$	$\mathbb{IV}_t - \mathbb{RV}_{t-1}$
ACF(1)	0.79	0.79	0.77	0.64	0.36	0.27
ACF(2)	0.54	0.56	0.47	0.42	0.33	0.25
ACF(3)	0.46	0.47	0.35	0.28	0.26	0.17
ACF(4)	0.41	0.42	0.27	0.18	0.14	0.05
ACF(5)	0.33	0.35	0.16	0.15	0.21	0.15
ACF(6)	0.24	0.25	0.08	0.11	0.15	0.10

Panel D. Phillips–Perron Unit Root Test

	$\mathbb{V}$	$\mathbb{IV}$	JTIX	JTIX/$\mathbb{V}$	$\mathbb{V}_t - \mathbb{RV}_{t-1}$	$\mathbb{IV}_t - \mathbb{RV}_{t-1}$
Z_{t_α}	-4.54	-4.44	-4.96	-6.31	-9.46	-10.40
p-value	< 0.01	< 0.01	< 0.01	< 0.01	< 0.01	< 0.01

Table 4. Predictability regressions with variance spread

This table presents the predictive regression results for S&P 500 excess log returns for horizons of one month, three months, six months, one year, and two years, respectively. Here JTIX is the tail index, $\mathbb{V}$ is the variance of the holding period return, $\mathbb{IV}$ is the integrated variance, and $\mathbb{RV}$ is the realized variance. The term $\log(P/D)$ is the logarithm of the price–dividend ratio on the S&P 500, *TERM* is the difference between the 10-year and three-month Treasury yields, and *DEF* is the difference between Moody's BAA and AAA bond yields. The sample period is from January 1996 to October 2010. All regressions are monthly, using the end-of-month values of the variables. The t-statistics are computed using Hodrick's (1992) 1B standard errors under the null of no predictability and are reported below the coefficients. The joint p-value—based on Hodrick's (1992) standard errors—denotes p-values for the null hypothesis that the slope coefficients are jointly equal to zero. The term $Adj.R^2$ is the adjusted coefficient of determination and Obs. is the number of observations.

Panel A. Monthly Return Prediction

	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
$Const$	-0.0078	-0.0077	0.0033	-0.0012	0.0971	0.0034	0.0108	0.2559	0.2488	0.2705	0.2649
	(-1.44)	(-1.43)	(0.89)	(-0.13)	(1.22)	(0.66)	(1.03)	(3.21)	(3.14)	(3.34)	(3.35)
$\mathbb{V}_t - \mathbb{RV}_{t-1}$	0.3299							0.3951			
	(1.98)							(2.33)			
$\mathbb{IV}_t - \mathbb{RV}_{t-1}$		0.3679							0.3887	0.4053	0.3894
		(2.11)							(2.21)	(2.35)	(2.22)
JTIX			-0.2471							0.9423	
			(-0.36)							(1.11)	
JTIX/$\mathbb{V}$				0.0863							0.3802
				(0.43)							(1.68)
$\log(P/D)$					-0.0233			-0.0575	-0.0563	-0.0600	-0.0614
					(-1.20)			(-3.08)	(-3.03)	(-3.21)	(-3.31)
$TERM$						-0.0567		-0.1068	-0.1176	-0.0728	-0.0680
						(-0.21)		(-0.37)	(-0.41)	(-0.25)	(-0.24)
DEF							-0.8308	-2.9922	-2.6232	-3.7109	-3.8300
							(-0.76)	(-2.38)	(-2.11)	(-2.37)	(-2.71)
Joint p-value	0.0472	0.0346	0.7191	0.6672	0.2291	0.8341	0.4487	0.0049	0.0060	0.0038	0.0020
$Adj.R^2$	3.5%	4.3%	-0.4%	-0.4%	0.8%	-0.6%	0.2%	8.7%	8.7%	9.0%	10.3%
Obs.	178	178	178	178	178	178	178	178	178	178	178

Panel B. Three-Month Return Prediction

	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
Const	-0.0250	-0.0231	0.0101	-0.0008	0.2990	0.0095	0.0217	0.6751	0.6540	0.6918	0.6878
	(-1.93)	(-1.82)	(0.87)	(-0.03)	(1.32)	(0.62)	(0.69)	(2.73)	(2.65)	(2.79)	(2.79)
$\mathbb{V}_t - \mathbb{RV}_{t-1}$	1.0567							1.2078			
	(3.12)							(3.65)			
$\mathbb{IV}_t - \mathbb{RV}_{t-1}$		1.1168							1.1617	1.1907	1.1633
		(3.44)							(3.56)	(3.60)	(3.56)
JTIX			-0.6822							1.6448	
			(-0.30)							(0.68)	
JTIX/$\mathbb{V}$				0.2026							0.8014
				(0.34)							(1.43)
$\log(P/D)$					-0.0717			-0.1538	-0.1501	-0.1566	-0.1608
					(-1.30)			(-2.70)	(-2.64)	(-2.75)	(-2.82)
TERM						-0.1156		-0.4225	-0.4514	-0.3736	-0.3470
						(-0.14)		(-0.49)	(-0.53)	(-0.45)	(-0.41)
DEF							-1.3892	-7.2293	-6.0893	-7.9867	-8.6323
							(-0.43)	(-1.91)	(-1.61)	(-1.98)	(-2.22)
Joint p-value	0.0018	0.0006	0.7609	0.7326	0.1948	0.8887	0.6674	0.0004	0.0007	0.0011	0.0011
$Adj.R^2$	11.8%	12.8%	-0.3%	-0.3%	3.2%	-0.5%	0.0%	23.8%	22.8%	23.2%	25.2%
Obs.	177	177	177	177	177	177	177	177	177	177	177

Panel C. Six-Month Return Prediction

	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
Const	-0.0358	-0.0284	0.0043	-0.0266	0.6182	0.0090	0.0109	1.0666	1.0383	1.2089	1.1154
	(-1.53)	(-1.19)	(0.19)	(-0.66)	(1.44)	(0.29)	(0.20)	(2.12)	(2.07)	(2.35)	(2.22)
$\mathbb{V}_t - \mathbb{RV}_{t-1}$	1.5979							1.7295			
	(3.69)							(3.75)			
$\mathbb{IV}_t - \mathbb{RV}_{t-1}$		1.5271							1.5523	1.6836	1.5546
		(3.68)							(3.50)	(3.70)	(3.50)
JTIX			2.6094							7.4887	
			(0.69)							(1.94)	
JTIX/$\mathbb{V}$				0.9523							1.7777
				(1.06)							(2.17)
$\log(P/D)$					-0.1487			-0.2478	-0.2423	-0.2715	-0.2666
					(-1.42)			(-2.14)	(-2.09)	(-2.31)	(-2.29)
TERM						0.2829		-0.5613	-0.5871	-0.2401	-0.3442
						(0.17)		(-0.35)	(-0.36)	(-0.15)	(-0.21)
DEF							0.2538	-8.9320	-7.2436	-15.8731	-12.8920
							(0.05)	(-1.33)	(-1.10)	(-2.26)	(-1.84)
Joint p-value	0.0002	0.0002	0.4911	0.2900	0.1556	0.8624	0.9639	0.0007	0.0012	0.0014	0.0025
$Adj.R^2$	11.9%	10.4%	1.3%	2.1%	6.6%	-0.5%	-0.6%	23.0%	20.5%	27.7%	26.1%
Obs.	174	174	174	174	174	174	174	174	174	174	174

Panel D. One-Year Return Prediction

	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
$Const$	-0.0347	-0.0219	0.0041	-0.0394	1.4107	-0.0131	-0.0176	1.9872	1.9544	2.2355	2.0694
	(-0.77)	(-0.48)	(0.10)	(-0.68)	(1.88)	(-0.22)	(-0.20)	(2.13)	(2.09)	(2.33)	(2.21)
$\mathbb{V}_t - \mathbb{RV}_{t-1}$	1.9402							1.9689			
	(2.81)							(2.54)			
$\mathbb{IV}_t - \mathbb{RV}_{t-1}$		1.7047							1.6654	1.8785	1.6501
		(2.60)							(2.38)	(2.51)	(2.38)
JTIX			5.8250							12.0897	
			(1.31)							(2.59)	
JTIX/$\mathbb{V}$				1.5287							2.4360
				(1.31)							(1.87)
$\log(P/D)$					-0.3404			-0.4672	-0.4602	-0.5088	-0.4955
					(-1.85)			(-2.16)	(-2.13)	(-2.31)	(-2.28)
$TERM$						2.5317		0.9446	0.9059	1.5685	1.4006
						(0.87)		(0.33)	(0.32)	(0.56)	(0.49)
DEF							4.1748	-13.3816	-11.3890	-25.4329	-19.3245
							(0.50)	(-1.35)	(-1.18)	(-2.13)	(-1.68)
Joint p-value	0.0052	0.0096	0.1902	0.1940	0.0665	0.3860	0.6210	0.0070	0.0080	0.0132	0.0157
$Adj.R^2$	7.4%	5.4%	3.6%	2.4%	16.1%	1.8%	0.5%	26.5%	24.4%	32.8%	28.9%
Obs.	168	168	168	168	168	168	168	168	168	168	168

Panel E. Two-Year Return Prediction

	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
$Const$	-0.0372	-0.0202	-0.0014	-0.0372	3.0380	-0.1351	-0.0042	3.5806	3.5157	3.8152	3.6861
	(-0.45)	(-0.25)	(-0.02)	(-0.39)	(2.00)	(-1.11)	(-0.03)	(1.97)	(1.93)	(2.10)	(2.02)
$\mathbb{V}_t - \mathbb{RV}_{t-1}$	2.0635							2.7712			
	(1.47)							(2.21)			
$\mathbb{IV}_t - \mathbb{RV}_{t-1}$		1.6132							2.2375	2.2580	2.0695
		(1.26)							(2.11)	(2.12)	(2.08)
JTIX			6.7836							19.0426	
			(2.85)							(2.00)	
JTIX/$\mathbb{V}$				1.4054							3.9458
				(1.42)							(1.98)
$\log(P/D)$					-0.7345			-0.8564	-0.8406	-0.8725	-0.8895
					(-1.95)			(-2.00)	(-1.96)	(-2.04)	(-2.07)
$TERM$						11.4335		11.7565	11.7157	13.0699	12.8273
						(2.50)		(2.32)	(2.31)	(2.49)	(2.47)
DEF							2.5274	-29.7730	-27.4260	-53.8888	-42.3842
							(0.22)	(-2.12)	(-2.01)	(-2.21)	(-2.23)
Joint p-value	0.1429	0.2090	0.0076	0.2125	0.0616	0.0125	0.8310	0.0068	0.0102	0.0022	0.0125
$Adj.R^2$	3.1%	1.7%	2.0%	0.5%	25.8%	21.6%	-0.5%	55.8%	53.5%	63.4%	59.6%
Obs.	156	156	156	156	156	156	156	156	156	156	156

Table 5. Predicted excess log return by $\mathbb{V} - \mathbb{RV}$ and $\mathbb{IV} - \mathbb{RV}$
Panel A reports the mean values of the variance spreads $\mathbb{V} - \mathbb{RV}$ and $\mathbb{IV} - \mathbb{RV}$, respectively. Panel B reports the means of the excess log returns predicted by $\mathbb{V} - \mathbb{RV}$ and $\mathbb{IV} - \mathbb{RV}$, respectively. The predicted return for each observation date is computed as the multiple of the variance spread's mean value and the coefficient in the multivariate regression reported in Panel D of Table 4. The full sample period is from January 1996 to October 2010. The NBER-defined recession periods are from March 2001 to November 2001 and from December 2007 to June 2009. Subprime recession refers to the recession of December 2007 through June 2009.

Panel A. Summary Statistics of Variance Spreads

		Full Period	NBER Non-recession	NBER Recession	Subprime Recession
$\mathbb{V} - \mathbb{RV}$	Mean	0.0310	0.0295	0.0390	0.0402
$\mathbb{IV} - \mathbb{RV}$	Mean	0.0276	0.0272	0.0299	0.0283

Panel B. Summary Statistics of Predicted Return

		Full Period	NBER Non-recession	NBER Recession	Subprime Recession
$\mathbb{V} - \mathbb{RV}$	Mean	6.10%	5.81%	7.67%	7.91%
$\mathbb{IV} - \mathbb{RV}$	Mean	4.60%	4.53%	4.97%	4.71%
Difference	Mean	1.50%	1.28%	2.70%	3.19%

Table 6. Tail risk and variance spread
This table presents the predictive regression results for the S&P 500 excess returns for horizons of one month, three months, six months, one year, and two years, respectively. Here JTIX is the tail index, $\mathbb{V}$ is the variance of the holding period return, $\mathbb{RV}$ is the realized variance, $\log(P/D)$ is the logarithm of the price–dividend ratio on the S&P 500, *TERM* is the difference between the 10-year and three-month Treasury yields, and *DEF* is the difference between Moody's BAA and AAA bond yields. The sample period is from January 1996 to October 2010. All regressions are monthly, using the end-of-month values of the variables. The t-statistics, computed using Hodrick's (1992) 1B standard errors under the null of no predictability, are reported in parentheses. The joint p-value—based on Hodrick's (1992) standard errors—denotes the p-value for the null hypothesis that slope coefficients are jointly equal to zero. Here $Adj.R^2$ is the adjusted coefficient of determination and Obs. is the number of observations.

	JTIX						JTIX/$\mathbb{V}$				
	1m	3m	6m	12m	24m		1m	3m	6m	12m	24m
Const	0.2695	0.6885	1.2044	2.2305	3.8151	*Const*	0.2703	0.7037	1.1360	2.0918	3.7257
	(3.33)	(2.78)	(2.34)	(2.33)	(2.10)		(3.41)	(2.86)	(2.26)	(2.23)	(2.04)
JTIX	0.5846	0.5782	5.9869	10.4118	16.9312	JTIX/$\mathbb{V}$	0.3408	0.6798	1.6049	2.2377	3.6686
	(0.66)	(0.24)	(1.58)	(2.44)	(1.90)		(1.49)	(1.24)	(2.01)	(1.80)	(1.94)
$\mathbb{V}_t - \mathbb{RV}_{t-1}$	0.3902	1.2029	1.6788	1.8767	2.2806	$\mathbb{V}_t - \mathbb{RV}_{t-1}$	0.3812	1.1801	1.6626	1.8596	2.4368
	(2.28)	(3.65)	(3.68)	(2.48)	(2.08)		(2.23)	(3.63)	(3.73)	(2.55)	(2.17)
$\log(P/D)$	-0.0598	-0.1561	-0.2708	-0.5080	-0.8735	$\log(P/D)$	-0.0620	-0.1628	-0.2695	-0.4990	-0.8977
	(-3.20)	(-2.74)	(-2.30)	(-2.30)	(-2.04)		(-3.34)	(-2.85)	(-2.31)	(-2.29)	(-2.09)
TERM	-0.0771	-0.3933	-0.2654	1.5407	13.0291	*TERM*	-0.0608	-0.3309	-0.3351	1.4050	12.8059
	(-0.27)	(-0.47)	(-0.17)	(0.55)	(2.48)		(-0.21)	(-0.39)	(-0.21)	(0.49)	(2.46)
DEF	-3.6550	-7.8845	-15.7045	-25.2462	-53.4272	*DEF*	-4.0540	-9.3464	-13.9340	-20.5195	-43.5458
	(-2.33)	(-1.95)	(-2.25)	(-2.12)	(-2.19)		(-2.87)	(-2.38)	(-1.95)	(-1.75)	(-2.26)
Joint p-value	0.0048	0.0011	0.0015	0.0140	0.0022	Joint p-value	0.0018	0.0010	0.0018	0.0114	0.0111
$Adj.R^2$	8.6%	23.4%	27.5%	32.8%	63.3%	$Adj.R^2$	9.9%	25.3%	27.5%	30.2%	61.0%
Obs.	178	177	174	168	156	Obs.	178	177	174	168	156

Table 7. Jump factor from principal component analysis

This table presents the predictive regression results for the S&P 500 excess returns for horizons of one month, three months, six months, one year, and two years, respectively. The jump factor is the principal component extracted from a principal component analysis of JTIX and $\mathbb{IV}$ that is the most highly correlated with JTIX, $\mathbb{V}$ is the variance of the holding period return, $\mathbb{RV}$ is the realized variance, $\log(P/D)$ is the logarithm of the price–dividend ratio on the S&P 500, *TERM* is the difference between the 10-year and three-month Treasury yields, and *DEF* is the difference between Moody's BAA and AAA bond yields. The sample period is from January 1996 to October 2010. All regressions are monthly, using the end-of-month values of the variables. The t-statistics are computed using Hodrick's (1992) 1B standard errors under the null of no predictability and are reported below the coefficients. The joint p-value—based on Hodrick's (1992) standard errors—denotes the p-value for the null hypothesis that the slope coefficients are jointly equal to zero. Here $Adj.R^2$ is the adjusted coefficient of determination and Obs. is the number of observations.

	1m	3m	6m	12m	24m
Const	0.2701	0.6869	1.1750	2.1729	3.6970
	(3.43)	(2.80)	(2.31)	(2.28)	(2.03)
Jump factor	0.0050	0.0041	0.0386	0.0647	0.0930
	(0.89)	(0.27)	(1.74)	(2.41)	(1.81)
$\mathbb{V}_t - \mathbb{RV}_{t-1}$	0.4304	1.2371	2.0020	2.4181	3.0975
	(2.59)	(3.40)	(3.83)	(2.69)	(2.26)
$\log(P/D)$	-0.0595	-0.1554	-0.2623	-0.4923	-0.8467
	(-3.23)	(-2.74)	(-2.25)	(-2.25)	(-1.98)
TERM	-0.0564	-0.3808	-0.1784	1.6769	13.0962
	(-0.20)	(-0.46)	(-0.11)	(0.59)	(2.48)
DEF	-3.8232	-7.9186	-15.3518	-24.2507	-49.3912
	(-2.44)	(-2.00)	(-2.18)	(-2.01)	(-2.13)
Joint p-value	0.0034	0.0011	0.0015	0.0139	0.0040
$Adj.R^2$	9.0%	23.5%	28.6%	33.8%	62.5%
Obs.	178	177	174	168	156

Table 8. Tail risk, forward variance, and the quiet period

This table gives the predictive regression results for the S&P 500 excess returns for horizons of one month, three months, and six months, respectively, using the setup of Bakshi, Panayotov and Skoulakis (2011). The terms $y_t^{(1)}$ and $f_t^{(2)}$ are the forward variances of BPS, E/P is the earnings yield on the S&P 500, $\log(P/D)$ is the logarithm of the price–dividend ratio on the S&P 500, $TERM$ is the difference between the 10-year and three-month Treasury yields, and DEF is the difference between Moody's BAA and AAA bond yields. The sample period is September 1998 to September 2008. All regressions are monthly, using the end-of-month values of the variables. The t-statistics are computed using Hodrick's (1992) 1B standard errors under the null of no predictability and are reported below the coefficients. The joint p-value—based on Hodrick's (1992) standard errors—denotes the p-value for the null hypothesis that slope coefficients are jointly equal to zero. Here $Adj.R^2$ is the adjusted coefficient of determination and Obs. is the number of observations.

Panel A: JTIX **and forward variance**

	1m	3m	6m
$Const$	-0.0844	-0.1320	-0.1097
	(-2.34)	(-1.22)	(-0.62)
JTIX/$\mathbb{V}$	0.6316	1.1785	1.4226
	(3.90)	(2.77)	(1.87)
$y_t^{(1)}$	-3.7747	1.6240	-0.9592
	(-1.30)	(0.40)	(-0.16)
$f_t^{(2)}$	8.2915	9.7426	15.3071
	(2.62)	(2.16)	(2.15)
(E/P)	5.3461	10.5793	15.1138
	(3.58)	(2.61)	(2.42)
$TERM$	0.8413	2.1704	3.7770
	(1.84)	(1.70)	(1.68)
DEF	-4.2335	-14.1601	-28.5277
	(-2.13)	(-2.49)	(-2.77)
Joint p-value	0.0000	0.0000	0.0001
$Adj.R^2$	19.9%	24.9%	29.0%
Obs.	121	121	121

Panel B: JTIX **and variance spread**

	1m	3m	6m
$Const$	0.2969	0.7453	1.3032
	(-2.42)	(-2.17)	(-1.83)
JTIX/$\mathbb{V}$	0.6242	1.0533	1.2447
	(2.05)	(1.99)	(1.91)
$\mathbb{V} - \mathbb{RV}$	0.3078	1.0786	1.4422
	(1.09)	(2.58)	(2.38)
$\log(P/D)$	-0.0632	-0.1507	-0.2519
	(-2.17)	(-1.89)	(-1.55)
$TERM$	0.0601	0.3918	1.4148
	(0.22)	(0.48)	(0.90)
DEF	-7.2941	-21.1581	-40.4298
	(-2.62)	(-2.72)	(-2.86)
Joint p-value	0.0283	0.0294	0.0364
$Adj.R^2$	11.8	25.9	30.9
Obs.	121	121	121

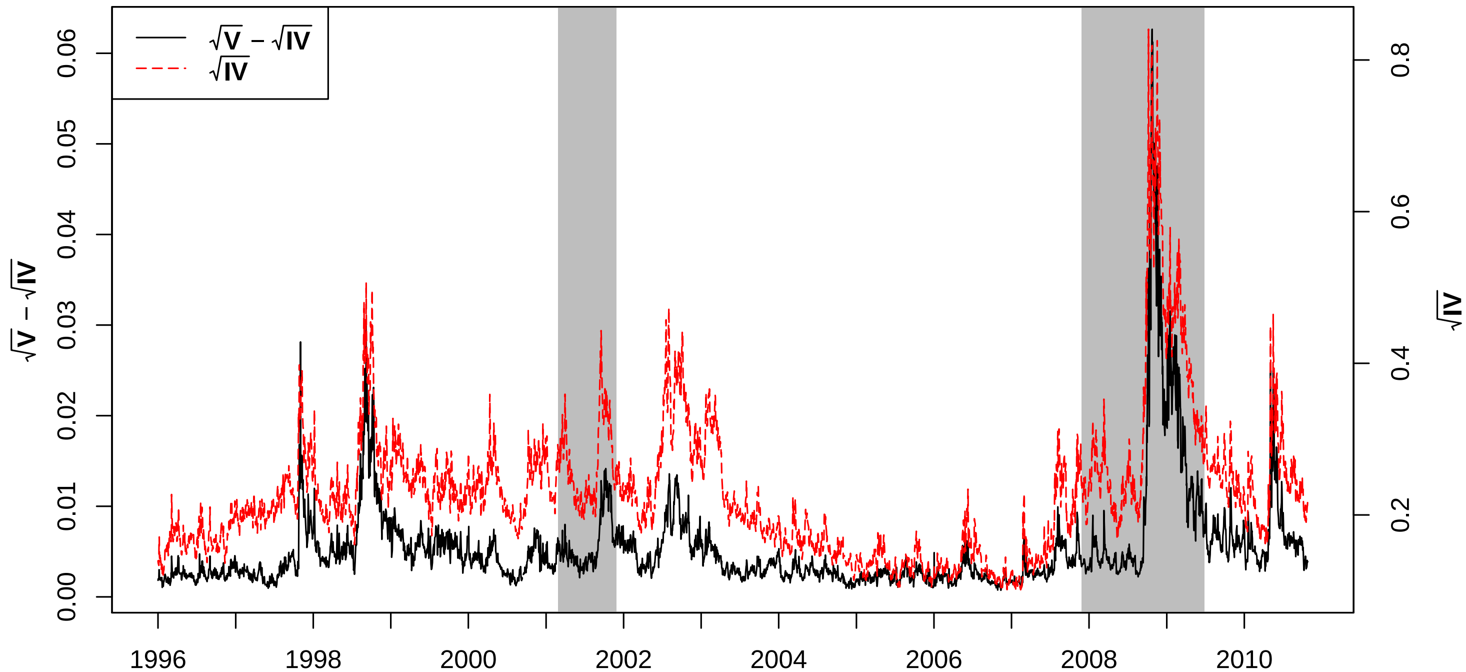

Figure 1. $\sqrt{\mathbb{IV}}$ **and** $\sqrt{\mathbb{V}} - \sqrt{\mathbb{IV}}$

This figure is a time series plot of the difference between the 30-day volatility of the holding period return ($\sqrt{\mathbb{V}}$) and the integrated volatility ($\sqrt{\mathbb{IV}}$). Here $\mathbb{V}$ and $\mathbb{IV}$ are estimated from OTM options on the S&P 500 of maturity 30 days. The shaded areas represent NBER-defined recessions, corresponding to March 2001 to November 2001 and December 2007 to June 2009. The sample period is from January 1996 to October 2010.

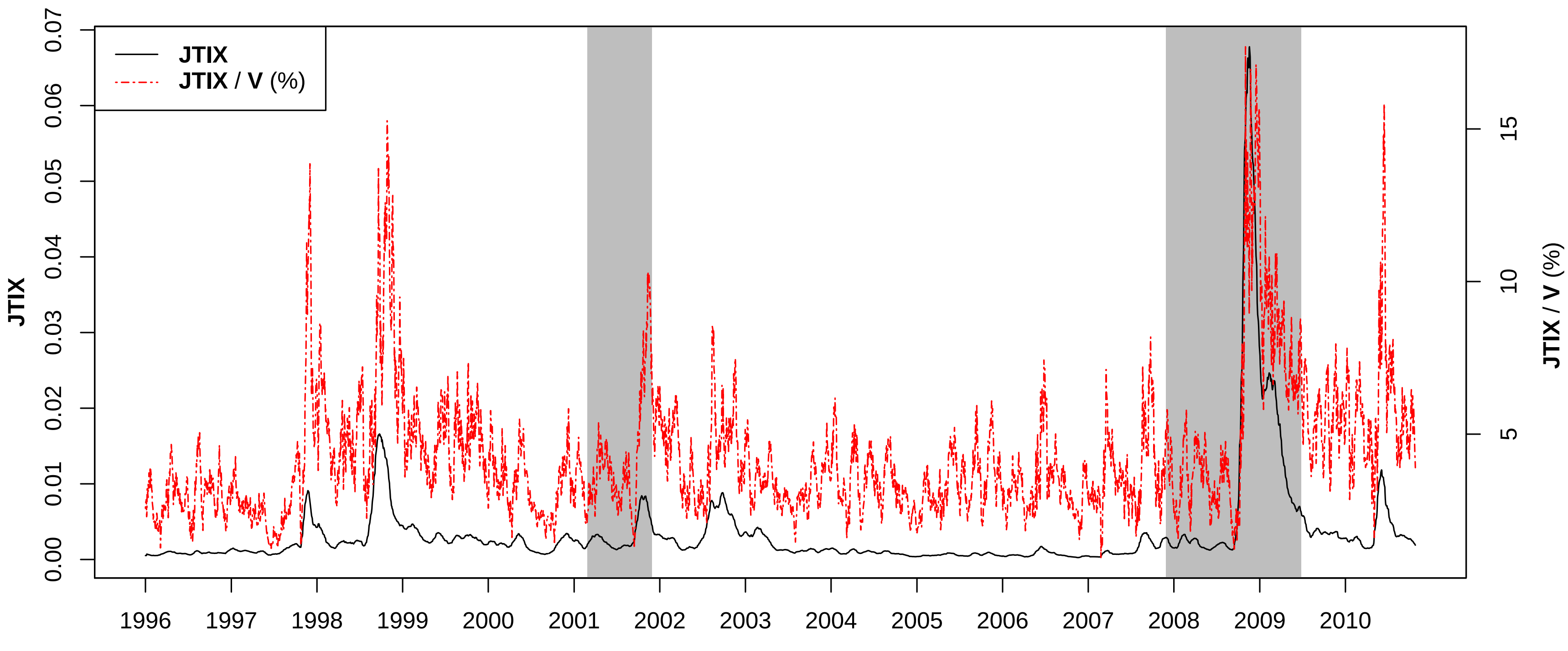

Figure 2. JTIX **and** JTIX/$\mathbb{V}$

This figure is a time series plot of the jump and tail index, JTIX, and the ratio of JTIX and the variance of the holding period return, JTIX/$\mathbb{V}$. Here JTIX and $\mathbb{V}$ are estimated from OTM options on the S&P 500 of maturity 30 days. The shaded areas represent NBER-defined recessions corresponding to March 2001 to November 2001 and December 2007 to June 2009. The sample period is from January 1996 to October 2010.

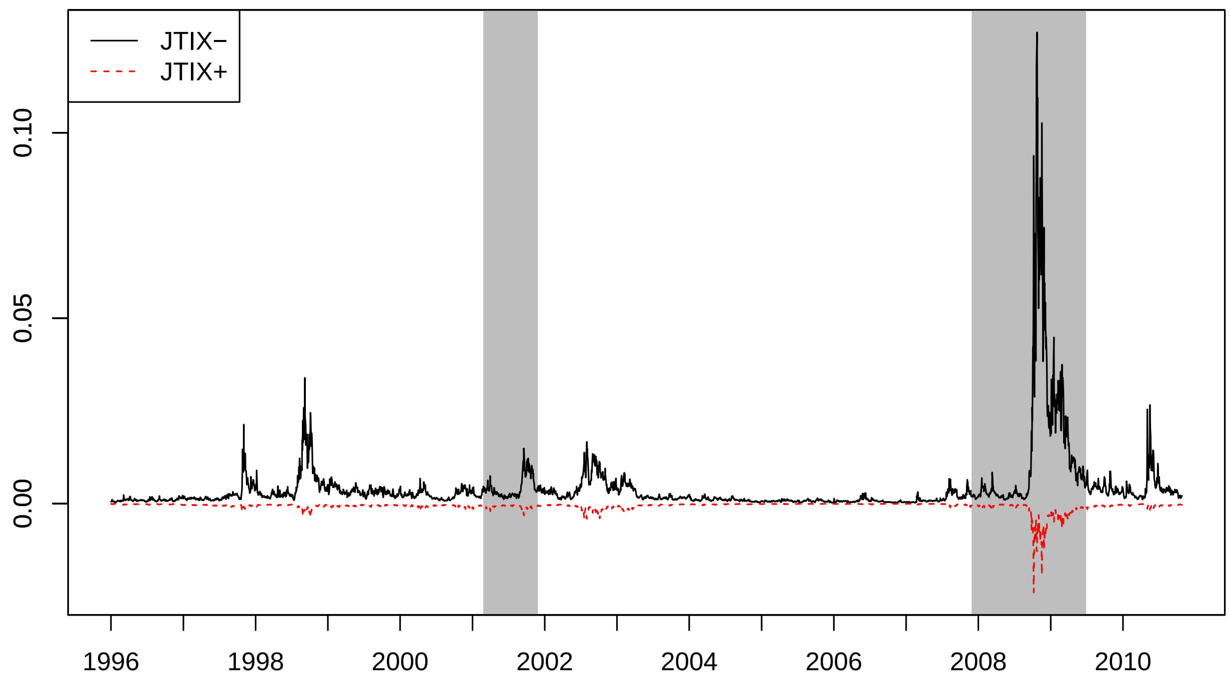

Figure 3. $JTIX_{-}$ **and** $JTIX_{+}$
This figure is a time series plot of $JTIX_{+}$ and $JTIX_{-}$. Here $JTIX_{-}$ and $JTIX_{+}$ are estimated from OTM puts and calls, respectively, on the S&P 500 of maturity 30 days. The shaded areas represent NBER-defined recessions corresponding to March 2001 to November 2001 and December 2007 to June 2009. The sample period is from January 1996 to October 2010.

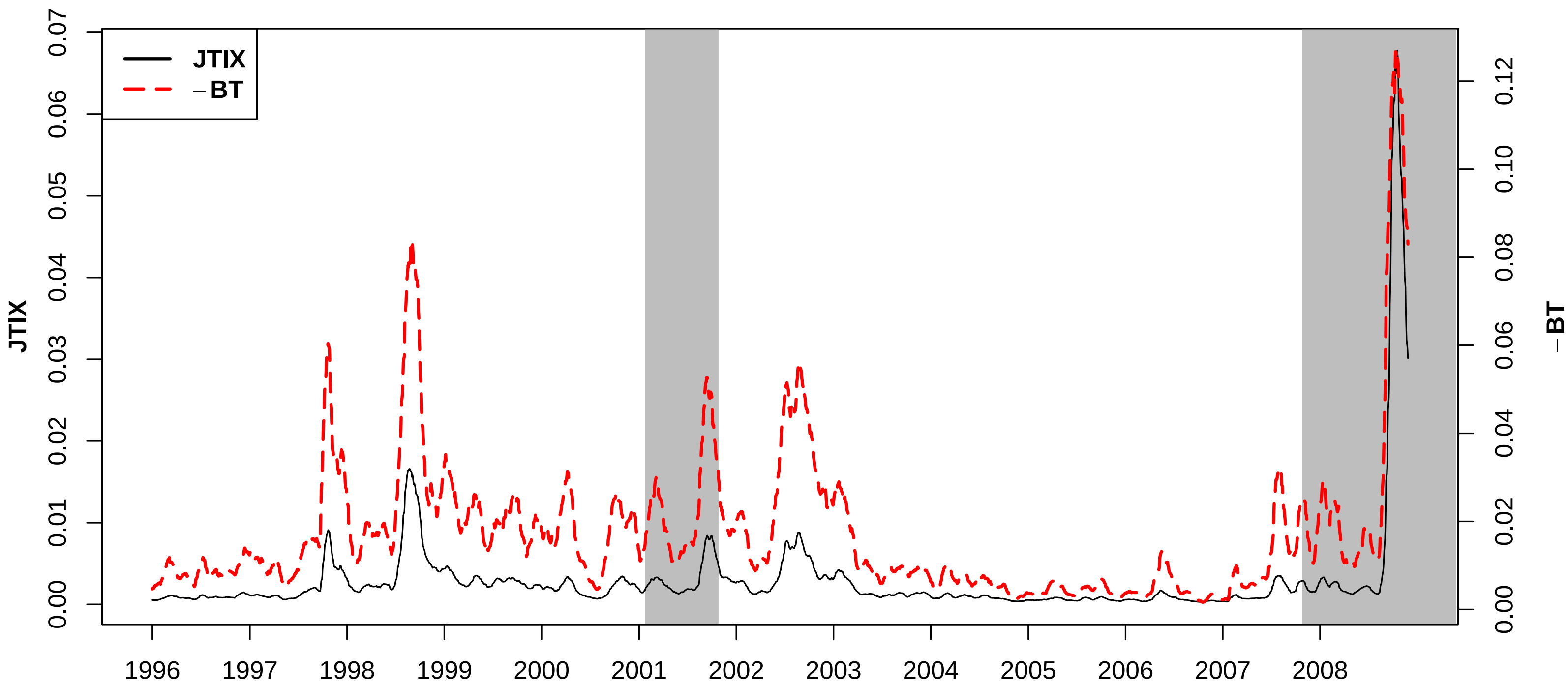

Figure 4. JTIX **and the Bollerslev–Todorov index**
This figure shows a time series plot of JTIX and the (negative of) the Bollerslev and Todorov fear (-BT) index. The shaded areas represent NBER-defined recessions corresponding to March 2001 to November 2001 and December 2007 to June 2009, respectively. The sample period is from January 1996 to December 2008.