Week 3

The Foreign Exchange Markets

Spot quotes, bid-ask spreads, triangular arbitrage. Forward rates
Spot and Forward Markets

• Foreign exchange trading occurs in two separate, but connected, markets: Spot and the Forward market.
• Spot market: Buying and selling of f/x with settlement in 2 business days from today.
• Forward market: Settlement occurs at some future date.
  – That is, you may enter into a forward contract to buy 10 million Yen 90 days from today.
F/X Rate Quotes

• We can view F/X rates as quotes in direct or indirect terms.
• **Direct**: a direct f/x quote is in units of domestic currency per unit of foreign currency [DC/FC].
  – For example, a quote of the USD-British Pound is direct for the US$ if it is quoted as 1.9631US$/BP.
• **Indirect**: an indirect f/x quote is in units of foreign currency per unit of domestic currency [FC/DC].
  – Example: 106.86 Yen/US$ is an indirect quote for the US$ (but direct quote for the Yen).
• **We will interpret every quote as being “direct”: DC/FC.** Therefore, we will always think of whichever country is in the denominator as being the “foreign” country. Please always remember this convention.
Appreciation/Depreciation

• A currency is said to appreciate (depreciate) against a foreign currency if you can buy more (less) foreign currency per unit of domestic currency.

• Examples: The WSJ reported the following on Wednesday
  - “Yesterday afternoon in New York, the euro was at $1.4635 from $1.4827 late Monday”. The Euro buys less dollars on Tuesday as opposed to Monday; therefore, the Euro depreciated (and the $ appreciated).
  - If the Yen-$ rate changed from 106.86 Yen/$ to 106.74 Y/$, has the Yen appreciated or depreciated? [Answer: Depreciated]

• When the currency is quoted in direct terms then an increase (decrease) in the quote is an appreciation (depreciation) of the foreign currency.

• When its quoted in indirect terms, then an increase in the rate is an appreciation of the domestic currency.
Cross Rates

• The cross rate is the exchange rate for converting one foreign currency to another. For example, the rate for Yen/BP would be called a cross rate. If we know the exchange rate for US$-Yen and US$-BP, we can easily calculate the cross rate.

• Example: Friday, Sept 14, 2007
  • What is the Yen/BP cross rate? (Answer: 231.38 Yen/BP)
  • What is the Euro/BP cross rate? (Answer: 1.4460 Euro/BP)

• Qt: Given the rates of the previous week did the Yen appreciate or depreciate against the BP?
Arbitraging Cross Rates (1/2)

- Suppose a bank quotes you the following rates: US$/BP = 1.8193, Yen/US$ = 110.27, and 199 Yen/BP.
- The implied Yen/BP cross rate (from the $/BP, Yen/$ quotes) is 200.61 Yen/BP.
- If the implied cross rate does not equal the quoted cross rate, there exists an arbitrage opportunity (in this case, the arbitrage opportunity has a specific name - "triangular arbitrage").
- How would you actually implement such an arbitrage? Buy low, sell high.
- In other words, you buy the currency where it is cheaper, and sell where it is more expensive.
Arbitraging Cross Rates (1/2)

• The Yen is cheaper at 200.61 Y/BP and more expensive at 199 Yen/BP.
• So you buy Yen @ 200.61 (sell BP) and sell Yen @ 199 (buy BP).
• How do you implement the arbitrage trade? You simultaneously make the following trades:
• Buy Yen for US$ @110.27, Sell Yen for BP @199, Sell BP for US$ @ 1.8193.
  – US$ 1 => 110.27 Yen (@110.27Yen/$)
  – Yen 110.27 => (110.27/199) = 0.5541 BP (@ 199 Yen/BP)
  – BP 0.5541 => 1.0081 US$ (@1.8193$/BP)
• You make gains of $8,100 for every $1 million in capital.

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Summarizing Steps in Triangular Arbitrage

- Assume that the three currencies are the US$ and two other foreign currencies and that you want to make your arbitrage in US$.
- 1. Identify the weaker non-US currency between implied cross rate and quoted cross rate.
   - Yen is weaker at implied cross rate of 200.61 Yen/BP as compared with actual quote of 199 Yen/BP.
- 2. Buy the weak currency low.
   - Buy Yen @ 110.27 Y/US$.
   - $1 => 110.27 Yen
- 3. Sell the currency you bought in (2.) high at the cross-rate to buy the other currency cheap
   - Sell Yen @ 199 Yen/BP.
   - 110.27 Yen => 0.5541 BP
- 4. Sell the currency high to buy the US$ cheap.
   - Sell BP @ 1.8193 US$/BP
   - 0.5541 BP => $1.0081
Bid-Ask Spreads

• As the dealer who trades foreign currency with you has to make money, there is a bid-ask spread associated with the quote, i.e. the price for buying the foreign currency is different from the price for selling the currency.

• **Bid**: the price at which the dealer is willing to **buy** the foreign currency

• **Ask or Offer**: the price at which the dealer is willing to **sell** the foreign currency.
Examples of Bid-Ask Spreads

• BP-US$: 1.8220-1.8229 $/BP
• The quote of 1.8220 is the bid or the price at which the dealer will buy the BP (foreign currency) and 1.8229 is the ask price.
• Qt: On a $1,000,000 round-trip transaction with the BP, what is the cost that you incur because of the bid-ask spread on BP? Ans: $494
Bid-Ask Rates and Indirect Quotes

• Suppose the rate is quoted in indirect terms. What is the bid (ask)?
• Example: Yen/USD 107.66 – 107.72
• We have to be careful of how the currency is quoted to figure out the bid and ask (using the principle that the dealer will buy foreign currency low and sell FC high).
• Thus: the dealer will buy (bid) Yen at 107.72 and sell (offer) at 107.66 (and the dealer will buy USD at 107.66 and sell at 107.72.)
• If the currency is quoted in direct (indirect) then the lower (higher) number is the bid.
• A word of warning: Its easy to get confused unless you always use one convention.
Bid-Ask and Cross Rates (1/2)

• Suppose a US bank quotes 1.7019-36 $/BP, and 0.9850-67 $/Euro. What would be the cross rate for Euro/BP in Frankfurt?

• In Frankfurt, the dealer will buy BP at the lower rate and sell BP at a higher rate (in terms of the Euro). So the cross rate will reflect this.

• Bid: the dealer buys BP at $1.7019 (lower price, bid), and sells Euro at 0.9867 (higher price, ask) = 1.7019/0.9867=1.7250 Euro/BP.
Bid-Ask and Cross Rates (2/2)

- Similarly, to get the offer rate: the dealer will sell BP at the higher rate (in terms of Euro).
- So: Dealer sells BP for USD at offer or ask rate of US$1.7036/BP, and buys Euro at bid rate of $0.9850. So the offer rate for Euro/BP is: $1.7036/0.9850 = 1.7295 Euro/BP.
- Therefore: the cross rate is 1.7250-1.7295 Euro/BP.
Triangular Arbitrage with Bid-Ask Spreads (1/2)

- 1. 1.7019-36 $/BP
- 2. 0.9850-67$/Euro
- 3. 1.7200-1.7300 Euro/BP
- The implied cross rate is 1.7248-95 Euro/BP.
- Does this constitute a triangular arbitrage?
Triangular Arbitrage with Bid-Ask Spreads (2/2)

- For there to be a triangular arbitrage, you have to be able to buy low and sell high.
- Dealer 1 (implied cross rate): 1.7248-95
- Dealer 2: 1.7200-1.7300.
- Can you buy the BP low and sell it high?
  - No. Because one dealer sells you BP @ 1.7290 while the second buys @ 1.7200. You cannot reverse the transaction also because the second dealer will sell you at 1.7300, and the first dealer buys at 1.7248. In each case, you lose money.
- Qt: Can you give examples of a quote that would allow for arbitrage? Provide two examples, one where the BP is priced too low, and one where it is priced too high.
Forward Rate Basics (1/2)

• *What is a forward rate agreement?*
• The forward exchange contract is an agreement to exchange currencies in the future at a fixed exchange rate.
• *How does one determine the forward exchange rate?*
• Answer: by the basic pricing principle that the forward exchange rate should be such that it does not allow for arbitrage.
Basics (2/2)

• According to the WSJ, on 2/6/2008, the spot for the Japanese Yen was at 106.86, the 3 month forward was at 106.24, the 6-month forward at 105.76.

• Thus, the Yen traded at a premium (was stronger) in the futures market.

  – *Does the futures prices indicate that the market expects the Yen to appreciate over the next year?*
What determines the Forward Rate

- Expectations do not determine the forward exchange rate. It does not matter that people think or feel that the currency is going to depreciate or appreciate.
- What determines the forward exchange rate? The forward exchange rate only depends on the relative interest rates.
The Forward Exchange Rate

- The forward exchange rate only depends on the relative interest rates.
- Here are the Euro-currency interest rates as of 2/6/2008 (from the CME):
  - June Eurodollar = 2.43%
  - June Euro-yen = 0.67%

We shall see that the Yen forward is at a premium to the spot (Yen is stronger) because the interest rates in Yen are lower than the interest rates in US$. 
An example to motivate the pricing of the forward/future

- Suppose, as an importer of Japanese goods, you need to make a payment in Yen exactly one year from today. However, you don’t want to take any exchange rate risk: how can you eliminate exchange risk? You have two options:
  - 1. Enter into a forward contract today - this will guarantee you an exchange rate of F, where F is the forward exchange rate.
  - 2. Buy Yen today at the spot rate, S, and hold the Yen until you need it in the future.
Determining the forward rate

• Which option will you prefer?

• Answer: you should be indifferent between the two, because if they are priced such that you prefer one over the other, you can make an arbitrage profit.

• Consider the first option, when you enter into a forward contract today at F Y/$. If you start off with $1 today, then this will guarantee you $1(1 + r(US)) F Yen/USD. This assumes that you invest your $1 in an US bank and earn the US interest rate.
• Alternatively, you can use your $1 to buy S yen today, and invest these S yen in a Japanese bank, earning an interest rate of r(JP). This will guarantee you an amount of S(1 + r(JP)) after a year.

• It must be that in either case you have the same amount of money, so that:
  • F (1 + r(US)) = S(1+r(JP)) Yen/US$
  • F = S(1+r(JP))/(1 + r(US)) Yen/US$
The Forward Rate when Exchange Rates are quoted in Direct Terms

- It is important to note the units: is it DC/FC or FC/DC.
- Suppose the rate is quoted in direct terms: $/Yen.
- Then the 1-year forward price would be:
  \[ F = S\frac{1 + r(US)}{1 + r(JP)} \] $/Yen
- In general for n days (when we use the Eurocurrency interest rates):
  \[ F = S\frac{1 + r(US) \left( \frac{n}{360} \right)}{1 + r(JP) \left( \frac{n}{360} \right)} \]
- We will use the notation that * represents the interest rate in the foreign currency, so that we can also write:
  \[ F = S\frac{1 + r(n/360)}{1 + r* (n/360)} \]
CME Eurocurrency Quotes

- Note: The CME quotes the interest rate contracts as \((100-r)\)%, where \(r\) is the interest rate.
  - If the December Eurodollar rate is 2.43%, the futures contract will be quoted at \(100 - 2.43 = 97.57\).

- Thus, to get the interest rate from the futures contract price, subtract the futures price from 100.
  - If the futures price is 95.285, then the interest rate is \((100-95.285)=4.715\)%.
Pricing the Currency Future

• The spot on 2/6/2008 is 106.57 Yen/US$. The June Euro-yen contract is at =99.33, and the June Euro-dollar contract is at 97.57.

• From the price of the futures contract, the interest rates are:
  – US$ interest rate = (100 – 97.570)=2.43%.
  – Yen interest rate = (100 – 99.330) = 0.67%

• What would be the price of the June‘08 $-Yen futures contract?

• The contract expires on the second business day before the third Wed of the month. Thus, the expiration date is June 16. Therefore, there are 131 calendar days left to expiration.

• The June Futures price is computed as:

  
  $F = 106.57 \times (1+0.0067 \times 131/360)/(1+ 0.0243 \times 131/360) = 105.89 \text{ Yen/}$

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An approximation for the forward premium

• Suppose n=360 (1-year). Then with some algebraic manipulation, we can write the forward premium as (where F, S are quoted in direct terms):
  
  \[ \frac{F - S}{S} = \frac{r - r^*}{1 + r^*} \]

• For low levels of interest rates, we can approximate this as:
  
  \[ \frac{F-S}{S} \sim r - r^* \]

• That is, the one-year forward premium is approximately equal to the difference in interest rates.

• If the forward is quotes for n days, we can annualize it:
  
  \[ \frac{360}{n} \frac{F-S}{S} \sim r - r^* \]

• Thus, if the Japanese interest rates are 2% lower than US interest rates, then the 1-year Yen forward will be (approximately) at a premium of 2% over the spot.
Forward Premium/Discount

• If $F > S$, then we say that the foreign currency is trading at a premium.
• $F < S$, then we say that the foreign currency is trading at a discount.
• *Note that $F > S$ ($F < S$) also implies that $r > r^*$ ($r < r^*$).*
Forward Contracts and Arbitrage in the Money Markets

• If the forward contract is not correctly priced, then you may be able to make arbitrage profits from this: this is called covered interest arbitrage.
Covered Interest Arbitrage

- You have the following data:
- 90 day interest rates:
  - 1. BP (r*) : 4.20-4.30% (lending-borrowing rates)
  - 2. Dollar (r): 1.70-1.85% (lending-borrowing rates)
- Exchange Rates:
  - Spot (S) : 1.5200-1.5300 $/BP
  - 90 day forward (F) : 1.5150-1.5200 - $/BP
- Is there an arbitrage?
The Mechanics of the Arbitrage

• 1. Borrow 1US$ at 1.85% for three months - so you need to repay $1 + 0.0185(90/360) = $1.004625 after three months.
• 2. Buy BP at offer price to get 1/1.53 = 0.6536 BP.
• 3. Lend BP for three months at lending rate of 4.20% - so at end of three months you have 0.6536(1 + 0.042*90/360)=0.660458 BP.
• 4. Sell BP 0.660458 in the forward market at bid of $1.515/BP to get $1.000593.
• Net gain: $(1.000593-1.0045) < 0.
• So there is no arbitrage.
• Now construct an example to demonstrate an arbitrage.
Summarizing the conditions for absence of arbitrage

- To ensure that there is no arbitrage in either direction, it must be that:
  - 1. \( \frac{F_{\text{bid}}}{S_{\text{ask}}} \frac{(1 + r*_{l} n/360)}{(1+r_{b} n/360)} < 1 \)
  - 2. \( \frac{S_{\text{bid}}}{F_{\text{ask}}} \frac{(1+r_{l} n/360)}{(1 +r*_{b} n/360)} < 1 \)

- Note that “_l” represents the lending interest rate, and “_b” represents the borrowing interest rate.
Creating a synthetic interest rate

- By borrowing/lending in a currency and then hedging your exposure with a forward contract can effectively allow you to get different effective interest rates.

- The synthetic net cost of lending or borrowing can be quickly calculated precisely in the following manner. We can re-write the relation between F, S, r and r*, as follows:
  - \((1 + \frac{[F-S]}{S})(1 + r*) = 1 + r\)
  - \([ or 1 + (\text{swap points}) /S)(1 + r*) = 1 + r . ]\)
  - So \(r \text{ (synthetic)} = (1 + (F-S)/S)(1+r*)-1\)
  - Note that you again have to be careful of bid/ask, borrowing/lending rates

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Forward Quotation in terms of Swap Spreads

• Often the forward market quotation is provided in terms of a “swap” spread: F-S.
• A swap is an exchange - in this case, it is an exchange of the spot for the forward (or vice versa.)
• Example: Spot Yen: $0.007540/Yen
• 6 Month Swap Rate: 0.00020 premium
• Forward = (0.007540+0.00020)=0.007740
• The foreign currency (Yen) is quoted at a premium.
Swap Rate with Bid-Ask Spreads

- Suppose the spot for the BP is quoted at $1.5235-1.5340/BP, with the one month swap spread at 0.0041-0.0039, and the three month swap spread at 0.0114-0.0119.

- How do we get the bid-ask spread for the forward? We use the rule that the bid-ask spreads should increase in the forward market.

- So we *subtract* the swap spread if the bid is higher than the offer, and *add* if the bid is lower than the offer. Thus: one month forward: 1.5194-1.5301 and the three month forward is: 1.5449-1.5459.
Using Futures : How many contracts? (1/4)

- Suppose you need to make a payment of 100,000,000 Yen in a few days.
- Today you decide to hedge against exchange rate risk by buying futures contract on the CME.
- Qt: how many contracts will you buy?
- You look up the product specifications on the CME, and find that each futures contact is equal to 12,500,000 Yen.
- So you need to buy 100/12.5=8 contracts.
Using Futures : Marking to market (2/4)

- The futures contract is marked to market on a daily basis. Thus, if you have to book any gains or loss at settlement on a daily basis.
- Suppose the futures price increases by “36” from the previous day. Recall that the Yen contract is quoted in 1/1000000, so that the change in the price is equal to $0.000036.
- How much do you gain per contract?
- For each “1” = 0.000001 change in the contract, the value of the contract changes by $12,500,000x0.000001 = $12.50.
- Thus, for a change of “36”, the value of the contract changes by $450. In this case, as the value of the futures increases (the Yen appreciates) you make $450 per contract.
Using Futures : The hedge (3/4)

- The Yen futures on Monday, Tue and Wed settles at 8110, 8146, 8155. You own 8 contracts.
- On Tuesday, you make $450 \times 8 = $3600.
- On Wed, you make $112.50 \times 8 = $900.
- Suppose the futures matures also on Wed, so you take delivery of the $12,500,000 \times 8 = 100,000,000$ Yen at the rate of $0.008155$/Yen. You pay $815,500 for the Yen.
- What is the net rate you get?
- Your total net cash flow is $12,500,000-3600-900 =$811,000.
- As expected, the net rate you get for your 100 million Yen is $0.008110$/Y, because you hedged on Monday at that price.
Using Futures : Other Issues (4/4)

• It is very likely that you may not be able to match either the maturity or the amount with a futures contract. You have to be careful you don’t end up increasing risk, instead of reducing risk.

• Because futures are marked to market, you have to be careful that you have the liquidity to make your margin calls, in case the market moves against you. Thus, if you don’t have the cash to may your payments, you may be forced to close out your position prematurely.

  – To see the effect of liquidity, assume that to hedge your Yen liabilities, you have to hold a (long) 100 million Yen futures position for 1 year. Suppose now the Yen depreciates by 50%. Examine how your cash flow changes.