VEHICLE LONGITUDINAL CONTROL
AND TRAFFIC STREAM MODELING

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Abstract

A simple yet efficient traffic flow model, in particular one that
describes vehicle longitudinal operational control and further characterizes traffic flow fundamental diagram, is always desirable. Though many models have been proposed in the past with each having its own merits, research in this area is far from conclusive. This paper contributes a new model, i.e., the longitudinal control model (LCM), to the arsenal with a unique set of properties. The model is suited for a variety of transportation applications, among which a concrete example is provided in this paper.

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1 Introduction

A simple yet efficient traffic flow model, in particular one that describes vehicle longitudinal operational control and further characterizes traffic flow fundamental diagram, is always desirable. For example, researchers can use such a model to study traffic flow phenomena, system analysts need the model to predict system utilization and congestion, accident investigators find the model handy to reconstruct accidents, software developers may implement the model to enable computerized simulation, and practitioners can devise strategies to improve traffic flow using such a simulation package.

Past research has resulted in many traffic flow models including microscopic car-following models and macroscopic steady-state models, each of which has its own merits and is applicable in a certain context with varying constraints. A highlight of these historical efforts will be provided later in Section 6. Nevertheless, research on traffic flow modeling is far from conclusive, and a quest for better models is constantly occurring. Joining such a journey, this paper presents a new model, the longitudinal control model (LCM), as a result of modeling from a combined perspective of Physics and Human Factors (Section 2). The model seems to possess a unique set of properties:

- The model is \textit{physically meaningful} because it captures the essentials of longitudinal vehicle control and motion on roadways with the presence of other vehicles (Subsection 2.1)
• The model is simple because it uses one equation to handle all driving situations in the longitudinal direction (Equation 2.1), and this microscopic equation aggregates to a steady-state macroscopic equivalent that characterizes traffic stream in the entire density range (Equation 5)

• The model is flexible because the microscopic equation provides the mechanism to admit different safety rules that govern vehicle driving (Subsection 2.1) and the macroscopic equation has the flexibility to fit empirical traffic flow data from a variety of sources which exhibit varying flow-density relationships including an inverse-lambda type (Subsection 3.2 and Figures 3 through 8)

• The model is consistent because the microscopic equation aggregates to its macroscopic equivalent so that the micro-macro coupling is well defined (Subsection 2.2). As a result, traffic flow modeling and simulation based on the microscopic model aggregates to predictable macroscopic behavior (Section 5, see how results of microscopic and macroscopic approaches match)

• The model is valid as verified using field observations from a variety of locations (Section 4), and the model is realistic as demonstrated in an example application (Section 5)
The unique set of properties possessed by LCM lend itself to various transportation applications including those mentioned above. An example of such applications is elaborated in Section 5 where LCM is applied to analyze traffic congestion macroscopically and microscopically. Research findings are summed up in Section 7.

2 The Longitudinal Control Model

Vehicle operational control in the longitudinal direction concerns a driver’s response in terms of acceleration and deceleration on a highway without worrying about steering including lane changing. Rather than car-following as it is conventionally termed, vehicle longitudinal control involves more driving regimes than simply car-following (e.g. free flow, approaching, stopping, etc.). A field theory was previously proposed in Ni (2011c,a), which represents the environment (e.g. the roadway and other vehicles) perceived by a driver with ID $i$ as an overall field $U_i$. As such, the driver is subject to forces as a result of the field. These forces, which impinge upon the driver’s mentality, are motivated as driving force $G_i$, roadway resistance $R_i$, and vehicle interaction $F_{ij}$ with the leading vehicle $j$, see an illustration in Figure 1. Hence, the driver’s response is the result of the net force $\sum F_i$ acting on the vehicle according to Newton’s second law of motion:
\[ \sum F_i = G_i - R_i - F_i^j \] (1)

### 2.1 Microscopic model

If the functional forms of the terms in Equation 1 are carefully chosen (mainly by experimenting with empirical data), a special case called the Longitudinal Control Model (LCM) can be explicitly derived from Equation 1 as:

\[ \ddot{x}_i(t + \tau_i) = A_i \left[ 1 - \left( \frac{\dot{x}_i(t)}{v_i} \right) - e^{1 - \frac{s_{ij}(t)}{s^*_{ij}(t)}} \right] \] (2)

where \( \ddot{x}_i(t + \tau_i) \) is the operational control (acceleration or deceleration) of driver \( i \) executed after a perception-reaction time \( \tau_i \) from the current moment \( t \). \( A_i \) is the maximum acceleration desired by driver \( i \) when starting from standing still, \( \dot{x}_i \) is vehicle \( i \)'s speed, \( v_i \) driver \( i \)'s desired speed, \( s_{ij} \) is the actual spacing between vehicle \( i \) and its leading vehicle \( j \), and \( s^*_{ij} \) is the desired value of \( s_{ij} \).

No further motivation for this special case is provided other than the following claims: (1) it takes a simple functional form that involves physically meaningful parameters but not arbitrary coefficients (see this and the next section), (2) it makes physical and empirical sense (see this and Section 4), (3) it provides a sound microscopic basis to aggregated behavior, i.e. traffic stream modeling (see the remainder of this section and Section 4), and (4) it is simple and easy to apply (see Section 5).
The determination of desired spacing $s^*_ij(t)$ admits safety rules. Basically, any safety rule that relates spacing to driver’s speed choice can be inserted here. Of particular interest is an algorithm for desired spacing that allows vehicle $i$ to stop behind its leading vehicle $j$ after a perception-reaction time $\tau_i$ and a deceleration process (at rate $b_i > 0$ which driver $i$ believes that he or she is capable of applying in an emergency) should the leading vehicle $j$ apply an emergency brake (at rate $B_j > 0$). After some math, the desired spacing can be determined as:

$$s^*_ij(t) = \frac{\dot{x}_i^2(t)}{2b_i} - \frac{\dot{x}_j^2(t)}{2B_j} + \dot{x}_i \tau_i + l_j$$ (3)

where $s^*_ij \geq l_j$ and $l_j$ is vehicle $j$’s effective length (i.e., actual vehicle length plus some buffer spaces at both ends). Note that the term $\frac{\dot{x}_i^2(t)}{2b_i} - \frac{\dot{x}_j^2(t)}{2B_j}$ represents degree of aggressiveness that driver $i$ chooses. For example, when the two vehicles travel at the same speed, this term becomes $\gamma_i \dot{x}_i^2$ with:

$$\gamma_i = \frac{1}{2} \left( \frac{1}{b_i} - \frac{1}{B_j} \right)$$ (4)

where $B_j$ represents driver $i$’s estimate of the emergency deceleration which is most likely to be applied by driver $j$, while $b_i$ can be interpreted as the deceleration which driver $i$ believes that he or she is capable of applying in an emergency. Attention should be drawn to the possibility that $b_i$ might be greater than $B_j$ in magnitude, which translates to the willingness (or aggressive characteristic) of driver $i$ to take the risk of tailgating.
It is necessary to point out that, though both parameters $B_j$ and $b_i$ carry a sense of “emergency”, the model itself (i.e., Equations and ) is meant to describe all situations including both “emergency” and “normal” operations. Or put it in another way, LCM models a driver’s operational control $x_i$ over a wide range based on the interaction of a set of parameters, some of which concern the driver’s emergency responses, e.g., $B_j$ and $b_i$. This modeling philosophy echoes the “complete” car-following model described in Treiber and Kesting (2013) (page 158).

### 2.2 Macroscopic model

Under steady-state conditions, vehicles in the traffic behave uniformly and, thus, their identities can be dropped. Therefore, the microscopic LCM (Equations 2.1 and 2.1) can be aggregated to its macroscopic equivalent (traffic stream model):

$$v = v_f[1 - e^{1 - k^*}]$$

where $v$ is traffic space-mean speed, $v_f$ free-flow speed, $k$ traffic density, and $k^*$ takes the following form:

$$k^* = \frac{1}{s^*} = \frac{1}{\gamma v^2 + \tau v + l}$$

where $\gamma$ and $\tau$ denote the aggressiveness and average response time, respectively, that characterize driver population, and $l$ denotes average effective
vehicle length. Equivalently, the macroscopic LCM can be expressed as:

\[ k = \frac{1}{s} = \frac{1}{(\gamma v^2 + \tau v + l)[1 - \ln(1 - \frac{v}{v_f})]} \]  \hspace{1cm} (7)

Note that LCM refers to the model (both microscopic and macroscopic forms) formulated herein by default. If a distinction has to be made to differentiate this model from its earlier version which was proposed in Ni (2011c), Ni (2011a) without explicitly considering the effect of drivers’ aggressiveness, the model in this paper may be referred to as LCMx4 (x means longitudinal and 4 means four parameters), while the earlier version LCMx3 (without aggressiveness).

3 Model Properties

LCM features a set of appealing properties that makes the model unique. First of all, it is a one-equation model that applies to a wide range of situations. More specifically, the microscopic LCM not only captures car-following regime, but also other regimes such as starting up, free-flow, approaching, cutting-off, stopping, etc., see Ni et al. (2011) for more details. The macroscopic LCM applies to the entire range of density and speed without the need to identify break points.

Secondly, LCM makes physical sense since it is rooted in basic principles (such as field theory and Newton’s second law of motion). In addition, LCM employs a set of model parameters that are not only physically meaningful
but also easy to calibrate. For example, the microscopic LCM involves desired speed $v_i$, perception-reaction time $\tau_i$, desired maximum acceleration when starting from standing still $A_i$, the deceleration which driver $i$ believes that he or she is capable of applying in an emergency $b_i$, emergency deceleration $B_j$ by driver $j$ in front, and effective vehicle length $l_j$. The macroscopic LCM includes aggregated parameters including free flow speed $v_f$, aggressiveness $\gamma$, average response time $\tau$, and effective vehicle length $l$. Data to calibrate the above parameters are either readily available in publications (such as Motor Trend and human factors study reports) or can be sampled in the field with reasonable efforts.

Lastly, LCM represents a consistent modeling approach, i.e., the macroscopic LCM is derived from its microscopic counter-part when aggregated over vehicles and time. Such micro-macro consistency not only supplies macroscopic modeling with a microscopic basis but also ensures that microscopic modeling aggregates to a predictable macroscopic behavior.

More properties are discussed in the following subsections.

### 3.1 Boundary conditions

The macroscopic LCM has two clearly defined boundary conditions. When density approaches zero ($k \rightarrow 0$), traffic speed approaches free-flow speed ($v \rightarrow v_f$); when density approaches jam density ($k \rightarrow k_j = 1/l$), traffic speed approaches zero ($v \rightarrow 0$), see Figure 9 for example.

Kinematic wave speed at jam density $\omega_j$ can be determined by finding the
first derivative of flow \( q \) with respect to density \( k \) and evaluating the result at \( k = k_j \). Hence,

\[
q = kv = \frac{v}{(\gamma v^2 + \tau v + l)[1 - \ln(1 - \frac{v}{v_f})]}
\]

Take derivative of flow \( q \) with respective to density \( k \),

\[
\frac{dq}{dk} = \frac{v - s}{s'} = \frac{(\gamma v^2 + \tau v + l)[1 - \ln(1 - \frac{v}{v_f})]}{(2\gamma v + \tau)[1 - \ln(1 - \frac{v}{v_f})] + (\gamma v^2 + \tau v + l)[\frac{1}{v_f - v}]}
\]

Therefore, \( \omega_j \) can be evaluated as:

\[
\omega_j = \frac{dq}{dk} \bigg|_{k=k_j,v=0} = -\frac{l}{\tau + \frac{l}{v_f}}
\]

Meanwhile, capacity \( q_m \) can be found by first setting Equation 9 to zero to solve for optimal speed \( v_m \) or optimal density \( k_m \) and then plugging \( v_m \) or \( k_m \) into Equation 8 to calculate \( q_m \). However, it appears that an analytical solution of \((q_m, k_m, v_m)\) is not easy to find and this is a limitation of LCM. Fortunately, the problem can be easily addressed numerically.

On another note, the spacing-speed relationship is:

\[
s = (\gamma v^2 + \tau v + l)[1 - \ln(1 - \frac{v}{v_f})]
\]

The slope of the speed-spacing relationship when traffic is jammed can be determined by finding the first derivative of \( v = f(s) \) with respect to spacing
s and evaluate the result at \( s = l \) and \( v = 0 \):

\[
\frac{dv}{ds}\bigg|_{s=l,v=0} = \frac{1}{(2\gamma v + \tau)[1 - \ln(1 - \frac{v}{v_f})] + (\gamma v^2 + \tau v + l)[\frac{1}{v_f - v}]}|_{s=l,v=0} = \frac{1}{\tau + \frac{l}{v_f}}
\]

(12)

### 3.2 Model flexibility

The macroscopic LCM employs four parameters that allow sufficient flexibility to fit data from a wide range of facilities, see details in the next section. As originally noted by Whitham (1974) and later by Del Castillo and Benitez (1995) and Rakha (2009) that concavity is a desirable property of flow-density relationship. This property is empirically evident in field observations from most highway facilities, especially in outer lanes, and the shape of flow-density relationship looks like a skewed parabola. In addition, some researchers such as Newell (1993); Daganzo (1994); Del Castillo and Benitez (1995); Rakha (2009); Del Castillo (2012) recognize the attractiveness of having a triangular flow-density relationship. Moreover, an inverse-lambda shape was reported by Koshi et al. (1983); Banks (1989), most likely in the inner lane of freeway facilities. Therefore, a desirable property of a traffic stream model is its flexibility to represent a variety of flow-density shapes ranging from skewed parabola to triangle to inverse lambda.

The shape of LCM is related to the second derivative of flow with respect to density:
where

\[
\frac{d^2 q}{dk^2} = -\frac{s^3 s''}{s'^3}
\]  

(13)

\[
s' = \frac{ds}{dv} = (2\gamma v + \tau)[1 - \ln(1 - \frac{v}{v_f})] + (\gamma v^2 + \tau v + l)[\frac{1}{v_f - v}]
\]

(14)

and

\[
s'' = \frac{d^2 s}{dv^2} = 2\gamma[1 - \ln(1 - \frac{v}{v_f})] + \frac{4\gamma v + 2\tau}{v_f - v} + \frac{\gamma v^2 + \tau v + l}{(v_f - v)^2}
\]

(15)

Note that \(s\) is always positive, so the shape of flow-density relationship is determined by the signs of \(s'\) and \(s''\). If \(s'\) and \(s''\) are both positive, \(d^2 q/dk^2\) is negative and the shape of flow-density relationship is concave. Otherwise, the flow-density relationship may consist of a combination of concave, straight, and convex sections. In particular, it is possible to result in an almost triangular shape and even an inverse-lambda shape under certain combinations of parameters \(v_f\), \(\tau\), \(\gamma\), and \(l\), among which \(\gamma\) plays a critical role in controlling the shape of flow-density relationship. For example, when the driver population is not aggressive, i.e., \(\gamma \geq 0\), a concave flow-density relationship is resulted; moderately aggressive driver population may give rise to an almost triangular shape, and aggressive driver population could lead to an inverse-lambda flow-density relationship.

The above discussion is further illustrated in Figure 2 where a family of fundamental diagrams are generated from the macroscopic LCM with the
following parameters: $v_f = 30 \text{ m/s}$, $k_j = 1/5 \text{ veh/m}$, $\tau = 1 \text{ s}$, and aggressiveness $\gamma$ ranging from 0 to $-0.03 \text{ s}^2/\text{m}$. In the flow-density subplot, the lowest curve exhibiting a skewed parabolic shape is generated using $\gamma = 0$, the second highest curve showing nearly a triangular shape is generated using $\gamma = -0.027$, and the highest curve, which takes an inverse-lambda shape, is generated using $\gamma = -0.030$. From the definition of aggressiveness in Equation 4, one recognizes that smaller values of $\gamma$ correspond to more aggressive drivers who are willing to accept shorter car-following distances. Therefore, the values of $\gamma$, the shape of $q - k$ curves, and field observations are consistent. Further quantitative analysis of the effect of aggressiveness and its interaction with other model parameters warrants further research and is not discussed here.

4 Empirical Results

Initial test results of LCM at both microscopic and macroscopic levels without the consideration of driver aggressiveness were reported in Ni et al. (2011). Hence, this paper focuses on testing LCM with consideration of aggressiveness by fitting the model to traffic flow data collected from a variety of facilities at different locations including Atlanta (US), Orlando (US), Germany, California (US), Toronto (Canada), and Amsterdam (Netherlands).

Figures 3 through 8 illustrate field data observed at these facilities with data “clouds” in the background labeled as “Empirical”. The fitted result
of LCM is illustrated as solid lines labeled as “LCM”. Also shown are the fitted results of other traffic stream models including Underwood model (Underwood, 1961) (which employs two parameters) and Newell model (Newell, 1961) (three parameters). As such, the reader is able to visually compare goodness-of-fit of two-, three-, and four-parameter models and examine how fit quality varies with number of parameters. Consisting of four subplots (namely, speed-density, speed-flow, flow-density, and speed-spacing), each figure illustrates the fundamental diagrams represented by empirical data and these models.

The empirical data in Figure 3 were collected on GA400, a toll road in Atlanta, GA, at station 4001116. Consisting of 4787 observation points, the abundant field data reveal the relationships among flow, density, and speed by means of cloud density, i.e. the intensity of data points. Meanwhile, the wide scatter of data points seems to suggest that any deterministic, functional fit is merely a rough approximation and a stochastic approach such as Wang et al. (2013) might be more statistically sound. By examining the cloud density, one is able to identify the trend of the these relationships. For example, the flow-density relationship appears to be an inverse-lambda shape. Meanwhile, the speed-flow relationship features a \( \supset \) shape with its “nose” leaning upward.

In order to fit LCM to the empirical data, a bi-level optimization procedure similar to Rakha and Arafah (2010) is adopted. First, each set of raw data is aggregated in order to reduce its size to a manageable level. When aggregating the data set, its distribution with respect to density is obtained
and the entire density range is divided into intervals delimited by equally spaced quantiles. Then the data are aggregated by computing an empirical mean (i.e., Emp mean) for each group consisting of the same amount of consecutive observations. Next, the bi-level optimization procedure is carried out. The inner loop searches for the minimum distance from each dot of “Emp mean” \((v_i, k_i, \text{and } q_i)\) to LCM curve \((\hat{v}_i, \hat{k}_i, \text{and } \hat{q}_i)\) normalized by \((v_f, k_j, \text{and } q_m)\) given a set of model parameters \((v_f, \tau, \gamma, \text{and } l)\):

\[
\min d_i = \sqrt{\left(\frac{v_i - \hat{v}_i}{v_f}\right)^2 + \left(\frac{k_i - \hat{k}_i}{k_j}\right)^2 + \left(\frac{q_i - \hat{q}_i}{q_m}\right)^2} \tag{16}
\]

Then, the outer loop searches for a set of optimized parameters that minimizes the total of minimized distances \(D(v_f, \tau, \gamma, l)\):

\[
\min D = \sum d_i \text{ subject to } v_f, \tau, \gamma, \text{and } l \tag{17}
\]

Normally, this would end the fitting process. However, the optimized model does not always match the empirical capacity condition \((q_m, k_m, \text{and } v_m)\) since it consists only of a limited number of observations. If the capacity condition is also part of the fitting objective, one may need to tweak the optimized model and this is typically done manually by visual inspection.

The fitting results are indicated in Tables 1 and 2. Table 1 compares fitted capacity condition with empirical capacity condition. The relative error of capacity is less than 5 percent and those of optimal density and speed are generally under 10 percent. Table 2 lists fitted parameters of
LCM. For example, the GA400 data set suggests a free-flow speed $v_f$ of 29 m/s (104.4 km/h), effective vehicle length $l = 6$ m (or jam density $k_j = 167$ veh/km), average response time $\tau = 1.3$ s, and aggressiveness $\gamma = -0.041$ s$^2$/m. In addition, kinematic wave speed at jam condition $\omega_j$ is calculated using Equation 10. Though $\omega_j$ typically lies in a relatively narrow range between 15 to 25 km/h, outliers are observed in field data. For example, the California Performance Measurement System (PeMs) data set does not provide a clear clue to estimate $\omega_j$, while the Autobahn data set does suggest an $\omega_j$ of 31.4 km/h or even higher.

Two additional models are fitted to the data sets by matching empirical free-flow speed and capacity and the results are presented in Table 2. It is apparent that the more parameters a model employs, the more flexible the model becomes and hence the more likely to result in a good fit. In the speed-flow subplot of Figure 3, Underwood and Newell models are comparable in the congested regime (i.e., the lower portion of the graph), while in the free-flow regime (i.e., the upper portion of the graph) Newell model outperforms Underwood model since Newell model is closer to the dense cloud. In contrast, LCM (which employs four parameters) yields the best fit among the three, as indicated by the close approximation of LCM curve to the empirical data. More specifically, LCM runs through the dense cloud in the free-flow regime and follows the trend nicely in the rest of the graph. In the flow-density subplot, both Underwood model and Newell model peak later than do empirical data. In the congested regime (i.e., the portion after the peak),
both models exhibit a lack of fit with Newell model slightly better in terms of concavity and closeness to data points. In contrast, LCM is superior on all accounts. Not only does it exhibit an inverse-lambda shape but its proximity to empirical observations is much closer. In addition, the curve peaks at the same location where the empirical data peaks ($k_m = 25 \text{ veh/km}$). In the speed-density subplot, LCM appears to over-fit when density is very small. Except for this, the three models have their own relative merits since each appears to fit the empirical data reasonably well. The speed-spacing subplot emphasizes the free-flow regime which is the flat portion in the top of the graph. It appears that Underwood model takes a long way to approach free-flow speed, while Newell model and LCM adapt to free-flow speed sooner. Unfortunately, the congested regime (the slope at the beginning portion of this graph) does not reveal much difference among the three models since they all cluster tightly together.

As shown in Figure 4 and Table 1, I-4 data in Orlando, FL feature a capacity $q_m$ of 1953 veh/hr which is achieved at an optimal density $k_m$ of 24.9 veh/km and optimal speed $v_m$ of 78.4 km/hr. What’s striking in this set of data is that the free-flow regime in the speed-flow subplot is almost flat and this condition sustains almost up to capacity. This graph clearly differentiates fitting quality of models with different number of parameters. More specifically, the two-parameter Underwood model exhibits the least fit since its upper branch (i.e. free-flow regime), nose (i.e. capacity), and lower branch (i.e. congested regime) are far from empirical observations. The
three-parameter Newell model is better as indicated by the closer fit of its upper branch, nose, and lower branch. The four-parameter LCM is superior in all aspects. For example, its upper branch is almost a flat line running through empirical data points, its nose leans upward and roughly coincides with empirically observed capacity, and its lower branch cuts evenly through empirical observations. Though there are discrepancies between the empirical data and the fitted curve, no systematic over- or under-fit is observed in this graph. In the remaining three subplots, the differences among the three models and their fit quality are consistent with those observed in the speed-flow subplot.

In Figure 5, the Autobahn data collected from Germany exhibit an unusually high free-flow speed $v_f$ of 42.4 m/s (or 152.6 km/hr). Unlike the I-4 data which feature an almost constant free-flow speed $v_f$ up to capacity, traffic speed in the Autobahn data gradually decreases in free-flow regime, resulting in an optimal speed $v_m$ of only about 60 percent of $v_f$ as shown in the speed-flow subplot. Unfortunately, the particular nature of this set of data poses a great challenge to any effort that attempts to fit the data. In the speed-flow subplot, one has difficulty to fit a model that meets the observed free-flow regime, the congested regime, and the capacity simultaneously, so a trade-off has to be made among the three portions. The LCM curve has been tweaked between free-flow and congested regimes while emphasizing the capacity. Though better than Underwood and Newell models, LCM still exhibits some discrepancies compared with the empirical data.
The PeMS data collected from California is plotted in Figure 6. This set of data heavily emphasizes the free-flow regime (which is virtually a flat band in the speed-flow subplot) with observations elsewhere sparsely scattered. In addition, a remarkable feature in the flow-density subplot is the spike at capacity which clearly indicates an inverse-lambda flow-density relationship. As expected, LCM is able to be fitted to such a shape and, thus, approximate free-flow and capacity condition very well. Since observations in the congested regime are few, model fit in this area appears to be quite arbitrary. In comparison, LCM approximates the free-flow regime and capacity condition the best, while Underwood and Newell models are slanted and significantly underestimate optimal speed $v_m$.

Though field observation on Highway 401 in Toronto do not have abundant data points, a trend is still clearly established in each subplot of Figure 7. Much like the results in the I-4 data, there are clearly differences in capabilities among the models, with two-parameter Underwood model being the least and the four-parameter model the best. Notice that no systematic under- or over-fit is observed in LCM curves. The same comments as above apply to Ring Road data in Amsterdam, see Figure 8.

5 Applications

Since LCM takes a simple mathematical form that involves physically meaningful parameters, the model can be easily applied to help investigate traffic
phenomena at both microscopic and macroscopic levels. For illustrative purpose, a concrete example is provided below, in which a moving bottleneck is created by a sluggish truck. Microscopic modeling allows LCM to generate profiles of vehicle motion so that the cause and effect of vehicles slowing down or speeding up can be analyzed in exhaustive detail; macroscopic modeling may employ LCM to generate fundamental diagrams that help determine shock paths and develop graphical solutions; Since LCM is consistent at the microscopic and macroscopic levels, the two sets of solutions not only agree with but also complement each other.

In addition, LCM can be adopted by existing commercial simulation packages to improve their internal logic of car following, or it can serve as the basis of a new simulation package. Moreover, LCM can be adopted in highway capacity and level of service (LOS) analysis. For example, conventional LOS analysis procedure involves the use of speed-flow curves to determine traffic speed, see HCM (2000) for the family of curves in EXHIBIT 23-3 and the set of approximating equations underneath. The macroscopic LCM can help make the analysis more effectively by providing more realistic speed-density curves to facilitate analytical, numerical, and graphical solutions. Furthermore, the LCM can be adopted by transportation planners to be used as the basis of a highway performance function which realistically estimates travel time (via traffic speed) as a function of traffic flow assigned to a route. The resultant travel time is the basis of driver route choice behavior, which in turn alters dynamic traffic assignment.
5.1 An illustrative example

A freeway segment contains an on-ramp (which is located at 2000 m away from an arbitrary reference point denoting the upstream end of the freeway) followed by an off-ramp 2000 m apart. The freeway was initially operating under condition A (flow 0.3333 veh/s or 1200 veh/hr, density 0.0111 veh/m or 11.1 veh/km, and speed 30 m/s or 108.0 km/hr). At 2:30pm, a slow truck enters the freeway traveling at a speed of 5.56 m/s (20 km/hr) which forces the traffic to operate under condition B (flow 0.3782 veh/s or 1361 veh/hr, density 0.0681 veh/m or 68.1 veh/km, and speed 5.56 m/s or 20 km/hr). After a while, the truck turns off the freeway at the next exit. The impact on the traffic due to the slow truck is illustrated macroscopically in subsection 5.2 and microscopically in subsections 5.3 and 5.4.

A fundamental diagram, which is illustrated in Figure 9, is generated using the macroscopic LCM to characterize the freeway with the following parameters: free-flow speed $v_f = 30$ m/s, aggressiveness $\gamma = -0.028$ s$^2$/m, average response time $\tau = 1$ second, and effective vehicle length $l = 7.5$ m. In addition, the above-mentioned traffic flow conditions, free-flow condition O, and capacity condition C are tabulated in Table 3.

To illustrate the application of LCM, the above problem is addressed in two approaches: macroscopic graphical solution and microscopic simulation solution. The microscopic simulation is conducted in deterministic and random fashions.
5.2 Macroscopic approach - graphical solution

The graphical solution to the problem involves finding shock paths that delineate time-space (t-x) regions of different traffic conditions. Figure 10 illustrates the time-space plane overlaid with the freeway on the right and a mini-version of the flow-density plot in the top-left corner. The point when the slow truck enters the freeway (2:30pm) roughly corresponds to $P_1(t_1 = 65, x_1 = 2000)$ on the time-space plane, while the point when the truck turns off the freeway is roughly $P_3(t_3 = 425, x_3 = 4000)$. Therefore, constrained by the truck, the t-x region under $P_1P_3$ should contain traffic condition B. On the other hand, the t-x regions before the truck enters and before congestion (i.e. condition B) should have condition A. As such, there must be a shock path that delineates the two regions, and such a path should start at $P_1$ with a slope equal to shock wave speed $U_{AB}$ which can be determined according to Rankine-Hugonoit jump condition (Rankine, 1870) (Hugoniot, 1887):

$$U_{AB} = \frac{q_B - q_A}{k_B - k_A} = \frac{0.3782 - 0.3333}{0.0681 - 0.0111} = 0.7877 \text{m/s} \quad (18)$$

Meanwhile, at downstream of the off-ramp, congested traffic departs at capacity condition C, which corresponds to a t-x region that starts at $P_3$ and extends forward in time and space. Hence, a shock path forms between the region with condition C and the region with condition B. Such a shock path starts at $P_3$ and runs at a slope equal to shock wave speed $U_{BC}$:
If the flow-density plot is properly scaled, one should be able to construct the above shock paths on the t-x plane. The two shock paths should eventually meet at point \( P_2(t_2, x_2) \). Its location can be found by solving the following set of equations:

\[
\begin{align*}
  x_2 - x_1 &= U_{AB} \times (t_2 - t_1) \\
  x_2 - x_3 &= U_{BC} \times (t_2 - t_3) \\
  (x_2 - x_1) + (x_3 - x_2) &= 2000
\end{align*}
\]  (20)

After some calculations, \( P_2 \) is determined roughly at \((716.8, 2513.4)\). After the two shock paths \( P_1P_2 \) and \( P_3P_2 \) meet, they both terminate and a new shock path forms which delineates regions with conditions C and A. The slope of the shock path should be equal to shock speed \( U_{AC} \):

\[
U_{AC} = \frac{q_C - q_A}{k_C - k_A} = \frac{0.5983 - 0.3333}{0.0249 - 0.0111} = 19.2029 \text{m/s} \]  (21)

As such, the shock path can be constructed as \( P_2P_4 \). Lastly, the blank area in the t-x plane denotes a region with no traffic, i.e. condition O.
5.3 Microscopic approach - deterministic simulation

In order to double check on LCM and to verify if its macroscopic and microscopic solutions agree with each other reasonably, the microscopic LCM is implemented in Matlab, a computational software package. As a manageable starting point, the microscopic simulation is made deterministic with the following parameters: desired speed $v_i = 30$ m/s, maximum acceleration $A_i = 4$ m/s$^2$, emergency deceleration $B_i = 6$ m/s$^2$, the deceleration which driver $i$ believes that he or she is capable of applying in an emergency $b_i = 9$ m/s$^2$, perception-reaction time $\tau_i = 1$ second, and effective vehicle length $l_i = 7.5$ m, where $i \in \{1, 2, 3, ..., n\}$ are unique vehicle identifiers. Vehicles arrive at the upstream end of the freeway at a rate of one vehicle every three seconds, which corresponds to a flow of $q = 1200$ veh/hr. Simulation time increment is one second and simulation duration is 1000 seconds.

Figure 10 illustrates the simulation result in which vehicle trajectories are plotted on the t-x plane. The varying density of trajectories outlines a few regions with clearly visible boundaries. The motion or trajectory of the first vehicle is pre-determined, while those of the remaining vehicles are determined by LCM. The first vehicle enters the freeway at time $t = 65$ seconds (2:30pm) after the simulation starts. This moment is calculated so that the second vehicle is about to arrive at the on-ramp at this particular moment. Hence, the second vehicle and vehicles thereafter have to adopt the speed of the truck, forming a congested region where traffic operates at condition B.
Upstream of this congested region B is a region where traffic arrives according to condition A. The interface of regions B and A, $P_1P_2$, denotes a shock path in which vehicles in fast platoon A catch up with and join slow platoon B ahead. The situation continues and the queue keeps growing till the truck turns off the freeway at $t = 425$ seconds into the simulation (2:36pm). After that, vehicles at the head of the queue begin to accelerate according to LCM, i.e. traffic begins to discharge at capacity condition C. Therefore, the front of the queue shrinks, leaving a shock path $P_3P_2$ that separates region C from region B. Since the queue front shrinks faster than the growth of queue tail, the former eventually catches up with the later at $P_2$, at which point both shock paths terminate denoting end of congestion. After the congestion disappears, the impact of the slow truck still remains because it leaves a capacity flow C in front followed by a lighter and faster flow with condition A. Hence the trace where faster vehicles in platoon A join platoon C denotes a new shock path $P_2P_4$.

Comparison of the macroscopic graphical solution and the microscopic deterministic simulation reveals that they agree with each other very well, though the microscopic simulation contains much more information about the motion of each individual vehicle and the temporal-spatial formation and dissipation of congestion.
5.4 Microscopic approach - random simulation

Since the microscopic approach allows the luxury to account for randomness in drivers and traffic flow, the following simulation may replicate the originally posed problem more realistically. The randomness of the above example is set up as follows with the choice of distribution forms being rather arbitrary provided that they are convenient and reasonable:

- Traffic arrival follows Poisson distribution, in which the headway between the arrival of consecutive vehicles is exponentially distributed with mean 3 seconds, i.e. $h_i \sim \text{Exponential}(3)$ s, which corresponds to a flow of 1200 veh/hr;

- Desired speed follows a normal distribution: $v_i \sim N(30, 2)$ m/s;

- Maximum acceleration follows a triangular distribution: $A_i \sim \text{Triangular}(3, 5, 4)$ m/s$^2$;

- Emergency deceleration: $B_i \sim \text{Triangular}(5, 7, 6)$ m/s$^2$;

- The deceleration which driver $i$ believes that he or she is capable of applying in an emergency: $b_i \sim \text{Triangular}(8, 10, 9)$ m/s$^2$;

- Effective vehicle length: $l_i \sim \text{Triangular}(5.5, 9.5, 7.5)$ m;

The result of one random simulation run is illustrated in Figure 11 where the effect of randomness is clearly observable. Trajectories in region B seem to exhibit the least randomness because vehicles tend to behave uniformly
under congestion. Trajectories in region C are somewhat random since the metering effect due to the congestion still remains. In contrast, region A appears to have the most randomness not only because of the Poisson arrival pattern but also the random characteristics of drivers. Consequently, the shock path between regions B and C, $P_3P_2$, remains almost unaltered, while there are some noticeable changes in shock path $P_1P_2$. The first is the roughness of the shock path and this is because vehicles in platoon A now join the tail of the queue in a random fashion. The second is that the path might not be a straight line. As a matter of fact, the beginning part of the shock path has a slope roughly equal to $U_{AB}$, while the rest part has a slightly steeper slope (due to less intense arrival from upstream during this period) resulting in the termination of congestion earlier than the deterministic case (which is somewhere near $P_2$). This, in turn, causes the slope of the shock path between regions C and A to shift left. Note that the slope of this shock path remains nearly the same since this scenario features a fast platoon that is caught up with by an even faster platoon.

6 Related Work

The microscopic LCM is a dynamic model which stipulates the desired motion (or acceleration) of a vehicle as the result of the overall field perceived by the driver. Other examples of dynamic model are General Motors (GM) models (Chandler et al., 1958; Gazis et al., 1961) and the Intelligent Driver Model
A dynamic model may reduce to a steady-state model when vehicle acceleration becomes zero. A steady-state model essentially represents a safety rule, i.e., the driver’s choice of speed as a result of car-following distance or vice versa. Examples of steady-state models include Pipes model (Pipes, 1953), Forbes model (Forbes et al., 1958; Forbes, 1963; Forbes and Simpson, 1968), Newell nonlinear car-following model (Newell, 1961), Gipps car-following model (Gipps, 1981), and Van Aerde car-following model (Van Aerde, 1995; Van Aerde and Rakha, 1995). Interested readers are referred to (Ni, 2011b) for a detailed discussion on the relation among LCM and other car-following models including a unified diagram that summarizes such relation.

The microscopic LCM incorporates a term called desired spacing $s_{ij}^*$ (Equation 2.1) which generally admits any safety rule and consequently any steady-state model. However, Equation 2.1 instantiates $s_{ij}^*$ in a quadratic form as a simplified version of Gipps car-following model (Gipps, 1981). The result coincides with the speed-spacing relation documented in Highway Capacity Manual (HCM, 1950) and Chapter 4 of Revised Monograph of Traffic Flow Theory (Gartner et al., 2001) as a result of 23 observational studies. The speed-spacing relation incorporates three terms: a constant term representing effective vehicle length; a first order term which is the distance traveled during perception-reaction time $\tau$; a second order term, which is the difference of the breaking distances by the following and leading vehicles, is interpreted in this paper as the degree of aggressiveness that the follow-
ing driver desires to be. If one ignores the second order term, Pipes model (Pipes, 1953) and equivalently Forbes model (Forbes et al., 1958; Forbes, 1963; Forbes and Simpson, 1968) are resulted.

The macroscopic model is a single-regime traffic stream (or equilibrium) model with four parameters. Also in the single-regime category, Van Aerde model (Van Aerde, 1995; Van Aerde and Rakha, 1995) and IDM (Treiber et al., 2000; Helbing et al., 2002) employ four parameters, Newell model (Newell, 1961) and Del Castillo models (Del Castillo and Benitez, 1995; Del Castillo, 2012) have three parameters, and early traffic stream models such as Greenshields (1934), Greenberg (1959), Underwood (1961), and Drake et al. (1967) models necessitate only two parameters, though their flexibility and quality of fitting vary as illustrated in Section 4.

7 Conclusions

This paper proposed a simple yet efficient traffic flow model, the longitudinal control model (LCM), which is a result of modeling from a combined perspective of Physics and Human Factors. LCM model is formulated in two consistent forms: the microscopic model describes vehicle longitudinal operational control and the macroscopic model characterizes steady-state traffic flow behavior and further the fundamental diagram.

LCM model is tested by fitting to empirical data collected at a variety of facility types in different locations including GA400 in Atlanta, I-4 in Orlando
(US), Autobahn in Germany, PeMs in California, Highway 401 in Toronto, and Ring Road in Amsterdam. The wide scatter of these data sets suggest that any deterministic, functional fit is merely a rough approximation and a stochastic approach might be more statistically sound. Test results support the claim that LCM has sufficient flexibility to yield quality fits to these data sets, some of which even exhibit inverse-lambda flow-density relationships. Meanwhile, two more models are fitted to the same data sets in order to compare with LCM. These models include the two-parameter Underwood model and the three-parameter Newell model. Fitting results reveal that the more parameters a model employs, the more flexible the model becomes and hence the more potential to a good fit. Consistently, Underwood model yields the least goodness-of-fit, while Newell model represents an upgrade and LCM maintains the best fit to empirical data.

The unique set of properties possessed by LCM lend itself to various transportation applications. For example, LCM can be easily applied to help investigate traffic phenomena. An illustrative example is provided showing how to apply the LCM to the impact of a sluggish truck at both microscopic and macroscopic levels. Noticeably, the two sets of solutions agree with and complement each other due to the consistency of LCM. In addition, LCM can be adopted by existing commercial simulation packages to improve their internal logic of car following, or perhaps serves as the basis of a new simulation package. Moreover, LCM may help make highway capacity and level of service (LOS) analysis more effectively by providing more realistic speed-
density curves to facilitate analytical, numerical, and graphical solutions. Furthermore, LCM can assist effective transportation planning by providing a better highway performance function that helps determine driver route choice behavior.

**Acknowledgement**

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Figure 1: Forces acting on a vehicle
Figure 2: Family of curves generated from LCM with varying aggressiveness

Figure 3: LCM fitted to GA400 Data
Figure 4: LCM fitted to I-4 Data

Figure 5: LCM fitted to Autobahn Data
Figure 6: LCM fitted to PeMS Data

Figure 7: LCM fitted to Highway 401 Data
Figure 8: LCM fitted to Amsterdam Data

Figure 9: Fundamental diagram of the freeway generated from LCM
Figure 10: A moving bottleneck due to a slow truck, deterministic simulation

Figure 11: A moving bottleneck due to a slow truck, random simulation
Table 1: Result of fitting LCM to various facility types

<table>
<thead>
<tr>
<th>Data source</th>
<th>Location</th>
<th>Facility</th>
<th>No. obs.</th>
<th>Fitted capacity</th>
<th>Empirical capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$q_m$ v/h</td>
<td>$k_m$ v/km</td>
</tr>
<tr>
<td>Atlanta</td>
<td>GA400</td>
<td></td>
<td>4787</td>
<td>2372</td>
<td>25.0</td>
</tr>
<tr>
<td>Orlando</td>
<td>I-4</td>
<td></td>
<td>288</td>
<td>1954</td>
<td>26.0</td>
</tr>
<tr>
<td>Germany</td>
<td>Autobahn</td>
<td></td>
<td>3405</td>
<td>2266</td>
<td>17.6</td>
</tr>
<tr>
<td>CA/PeMs</td>
<td>Freeway</td>
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<td>2576</td>
<td>1382</td>
<td>13.4</td>
</tr>
<tr>
<td>Toronto</td>
<td>Hwy 401</td>
<td></td>
<td>286</td>
<td>2177</td>
<td>22.7</td>
</tr>
<tr>
<td>Amsterdam</td>
<td>Ring Rd</td>
<td></td>
<td>1199</td>
<td>2745</td>
<td>30.4</td>
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Table 2: Comparison of traffic stream models fitted to various facility types

<table>
<thead>
<tr>
<th>Location</th>
<th>Model</th>
<th>Estimated parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atlanta</td>
<td>Underwood $v_f = 29.0 \text{ m/s}, k_m = 0.051 \text{ v/m}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Newell $v_f = 29.0 \text{ m/s}, l = 6.0 \text{ m}, \lambda = 0.95 \text{ 1/s}$</td>
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</tr>
<tr>
<td></td>
<td>LCM $v_f = 29.0 \text{ m/s}, l = 6.0 \text{ m}, \tau = 1.3 \text{ s}, \gamma = -0.041 \text{ s}^2/\text{m}; w_j = 14.3 \text{ km/hr}$</td>
<td></td>
</tr>
<tr>
<td>Orlando</td>
<td>Underwood $v_f = 24.2 \text{ m/s}, k_m = 0.055 \text{ v/m}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Newell $v_f = 24.2 \text{ m/s}, l = 8.6 \text{ m}, \lambda = 1.09 \text{ 1/s}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>LCM $v_f = 24.2 \text{ m/s}, l = 8.6 \text{ m}, \tau = 1.1 \text{ s}, \gamma = -0.043 \text{ s}^2/\text{m}; w_j = 21.3 \text{ km/hr}$</td>
<td></td>
</tr>
<tr>
<td>Germany</td>
<td>Underwood $v_f = 42.4 \text{ m/s}, k_m = 0.037 \text{ v/m}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Newell $v_f = 42.4 \text{ m/s}, l = 11.0 \text{ m}, \lambda = 1.12 \text{ 1/s}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>LCM $v_f = 42.4 \text{ m/s}, l = 11.0 \text{ m}, \tau = 1.0 \text{ s}, \gamma = -0.021 \text{ s}^2/\text{m}; w_j = 31.4 \text{ km/hr}$</td>
<td></td>
</tr>
<tr>
<td>CA/PeMs</td>
<td>Underwood $v_f = 31.0 \text{ m/s}, k_m = 0.029 \text{ v/m}$</td>
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</tr>
<tr>
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<td>Newell $v_f = 31.0 \text{ m/s}, l = 6.3 \text{ m}, \lambda = 0.50 \text{ 1/s}$</td>
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<tr>
<td></td>
<td>LCM $v_f = 31.0 \text{ m/s}, l = 6.3 \text{ m}, \tau = 2.8 \text{ s}, \gamma = -0.080 \text{ s}^2/\text{m}; w_j = 7.6 \text{ km/hr}$</td>
<td></td>
</tr>
<tr>
<td>Toronto</td>
<td>Underwood $v_f = 29.5 \text{ m/s}, k_m = 0.050 \text{ v/m}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Newell $v_f = 29.5 \text{ m/s}, l = 12.0 \text{ m}, \lambda = 1.3 \text{ 1/s}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>LCM $v_f = 29.5 \text{ m/s}, l = 12.0 \text{ m}, \tau = 1.0 \text{ s}, \gamma = -0.036 \text{ s}^2/\text{m}; w_j = 30.7 \text{ km/hr}$</td>
<td></td>
</tr>
<tr>
<td>Amsterdam</td>
<td>Underwood $v_f = 28.4 \text{ m/s}, k_m = 0.064 \text{ v/m}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Newell $v_f = 28.4 \text{ m/s}, l = 7.5 \text{ m}, \lambda = 1.5 \text{ 1/s}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>LCM $v_f = 28.4 \text{ m/s}, l = 7.5 \text{ m}, \tau = 0.8 \text{ s}, \gamma = -0.028 \text{ s}^2/\text{m}; w_j = 24.9 \text{ km/hr}$</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Traffic flow conditions

<table>
<thead>
<tr>
<th>Condition</th>
<th>Flow, $q$ veh/s (veh/hr)</th>
<th>Density $k$ veh/m (veh/km)</th>
<th>Speed $v$ m/s (km/hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.3333 (1200.0)</td>
<td>0.0111 (11.1)</td>
<td>30 (108.0)</td>
</tr>
<tr>
<td>B</td>
<td>0.3782 (1361.6)</td>
<td>0.0681 (68.1)</td>
<td>5.56 (20.0)</td>
</tr>
<tr>
<td>C</td>
<td>0.5983 (2154.0)</td>
<td>0.0249 (24.9)</td>
<td>24.03 (86.5)</td>
</tr>
<tr>
<td>O</td>
<td>0 (0)</td>
<td>0 (0)</td>
<td>30 (108.0)</td>
</tr>
</tbody>
</table>
References


