Traffic viscosity due to speed variation: modeling and implications

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Abstract
The analogy of traffic flow to water flow has been well known. But some essential differences exist, e.g. anisotropy, viscosity, and extent to which conservation law holds. Not surprisingly, this is due to different manners in which water particles and vehicles move and interact. In this study, we investigate and model the viscous behavior exhibited by traffic flow. We attribute the viscous effect of traffic flow to drivers’ heterogeneity in terms of their preferred driving speeds. A model incorporating the diffusion effect is developed based on the idea of characteristic curve. In particular, the governing equation of traveling platoon is explicitly derived from this model. In accordance with our postulation, the derived equation includes one viscosity term. The proposed model is best suited for the analysis of local fine structure of traffic flow which is conventionally represented as a shock.

Keywords
Viscosity, traffic flow, platoon dispersion, method of characteristics
1. Introduction

For purposes of planning and traffic control, it is essential to understand various traffic behaviors. The kinematic wave (KW) theory, initiated in (1)(2), provides an indispensable insight into freeway traffic at the macroscopic level. In the perspective of KW theory, traffic flow is treated as a fluid-like continuum. Its associated density waves are of primary concern. The mathematical form of the simplest kinematic wave model, known as LWR (namely, Lighthill-Whitham-Richards) model, is as follows,

\[ k_t + Q(k)_x = 0 \]  

(1)

where \( k = k(x,t) \) denotes the traffic flow density at location \( x \) and time \( t \), and \( Q(k) \) the traffic flow corresponding to density \( k \). The subindices \( t, x \) stand for the partial derivative of corresponding variables with respect to time and space, respectively. Equation (1) is a scalar conservation law. It reflects the fact that total number of vehicles on a long enough stretch of arterial road keeps unchanged. The propagation of kinematic wave is roughly revealed by following quasi-linear representation of (1),

\[ k_t + Q'(k)k_x = 0 \]  

(2)

where \( Q' \) is the derivative of traffic flow \( Q \). Equation (2) implies that the total derivative of traffic density keeps constant along curve \( x'(t) = Q'(k(x,t)) \). This motivates the so-called “method of characteristics” to solve LWR model. More accurately, it is the starting point to construct weak solution of equation (1). For traffic flow, it is conventionally assumed that \( Q \) is concave. Examples include quadratic and triangle traffic stream models employed (1)(2). As such, regardless of the specific form of \( Q \), solutions to the LWR model are qualitatively similar: The density profile of traffic is a result of interactions of various shock and rarefaction waves. When time approaches infinity, the density profile tends to be stable regions separated by shocks.

The flaws of LWR model have been noticed for a long time (1)(3), some of which are relevant to this study: The LWR model does not reproduce the dispersion of a platoon and the shock predicted by LWR is of zero width. These problems are not surprising. The LWR model is initially proposed to model the crowded traffic on long arterial roads (1). Speed variation of vehicles within a platoon is neglected. However, this is not common in light traffic, and intuitively is a significant contributing factor to platoon dispersion. Moreover, in the LWR model the adaption of speed to traffic density is instantaneous, because the
relation \( v(x,t) = Q(k(x,t))/k(x,t) \) holds if functions \( Q \) and \( k \) are appropriately defined. This implies that it takes zero time for a driver to change driving speed. As such, it is normal that some fine structure of traffic is missed in LWR model.

High-order models are proposed to address the above problems. Payne-Whitham (PW) model (4)(8) and subsequent variants and generalization (5)(6)(7) model the dynamics of velocity by explicitly considering the relaxation and anticipation time. This rectifies the problems of abrupt speed change in LWR model. More complicated wave patterns arise in these non-equilibrium models. For example, (5)(6)(7) preserve the anisotropic property of traffic flow overlooked in (4).

Another type of correction (8) to the LWR model, however, receives little attention in literature. This correction reads,

\[
k_x + \tilde{Q}(k)_x = \nu k_{xx}
\]  

where \( \nu \) is diffusion coefficient. Such examples can be found in (15)(17). In particular, Nelson showed in (15) that many physical phenomenon can be reproduced after including the diffusion correction into the LWR model.

There is controversy if \( \tilde{Q} \) should be interpreted as traffic flow at corresponding density. If so, the conservation property is violated. In fact, model (3) is postulated by assuming that the flow of traffic is a function of both traffic density and its gradient, i.e.,

\[
Q(k)_x = \tilde{Q}(k)_x - \nu k_{xx} = (\tilde{Q}(k) - \nu k_x)_x
\]

Therefore, \( Q = \tilde{Q} - \nu k_x + C \), where \( C \) is an integration constant. In another word, drivers in this model are aware of the magnitude and gradient of local traffic density, and change the driving speed in the following fashion (for brevity, rename the \( \tilde{Q} + C \) as \( \tilde{Q} \)),

\[
v = v(k, k_x) = Q/k = \tilde{Q}/k - \nu k_x/k
\]

Model (3) was seriously criticized in (3). Through qualitative analysis, it is shown that Eq. (3) has the wrong-way-traveling issue. That is, vehicles at the end of a stopping queue will move backward. This is counter-intuitive, and is an indication of the violation of anisotropic property. Nonetheless, (3) seems still attractive for its motivation. Moreover, it has some extra nice properties. It replaces shock by a transition with finer details, thus approximating the reality better than LWR model in many cases. Mathematically, it reduces to the LWR model as \( \nu \to 0 \), always adopts a unique smooth solution, and
algorithm to solve this equation is abundant. These features make it favorable for the purposes of both modeling and analysis. Thus, some questions naturally arise: what is the real implication of $\nu_k\nu$, how to correctly model the effect associated with this term, and what constraints can be imposed to resolve the wrong-way-traveling issue.

The remaining part of this paper attempts to answer these questions. Section 2 is devoted to the derivation of a traffic flow model incorporating the viscous effect. It starts with a brief sketch of documented studies of diffusion effect. Our analysis indicates that the viscous effect can be attributed to the distribution of actual driving speeds under identical driving conditions. Section 3 presents an analysis of the model proposed in Section 2. Linkage of the proposed model with other models is shown. In particular, with the proposed model, a fine characterization of traveling platoon front is obtained. This paper concludes with Section 4, where our findings are summarized and some remarks are given.

2. A new model of traffic viscosity

A sketch of previous studies

- Diffusion vs. Viscosity: While Eq. (3) describes a convection-diffusion process, it is at the same time the governing equation of viscous flow. This is an interesting perspective to understand this model, although what ‘diffusion’ and ‘viscosity’ really indicate in traffic flow is still arguable. The appearance of diffusion term in Eq. (3) seems purely phenomenological. Admittedly, it can be regarded as a result of driver’s perception of ‘pseudo-density’, as in (9), but this point of view is hardly tested against real life data since ‘pseudo-density’ is not a measurable quantity. It is not the diffusion in conventional sense, which is due to the local random motion of particles. In (10), the traffic flow was examined as viscous fluid, and a viscous traffic flow model was developed. The motive comes from the following analogy: a driver has the tendency to resist sudden changes of speed (which induces so-called speed-constant fluctuation on phase plane), the same as particles, so speed dynamics with viscosity presumably exists. It is found that a non-equilibrium model with viscosity can be derived from a car-following model when the latter assumes the driver memory, i.e. speed of a later time depends on speed before.

- Platoon Dispersion: Another very relevant and insightful study is (11), in which platoon dispersion is modeled. The formation and propagation of platoon dispersion is a feature of traffic overlooked by the LWR model. The proposed platoon dispersion model explicitly considers the distribution of individual speeds across drivers within a platoon. In a short time, it can be reasonably assumed that there is no interaction between drivers and each driver maintains a constant speed. When further assuming that the distribution of
speed follows the Gaussian law, the authors find that the derived density profile \( k(x,t) \) happens to be the solution of the following equation,

\[
k_t + mk_x = m^2 \alpha^2 tk_{xx}
\]

where \( m \) is the average vehicle speed, and \( \alpha = \sigma / m \), with \( \sigma \) being the standard deviation of vehicle speed. The dimensionless parameter \( \alpha \) is the coefficient of the variation of the speed distribution. The meaning of the right hand side of Eq. (6) is evident when compared with Eq. (3): approximately, the platoon propagates at speed \( m \). Of particular interests is the right hand side. Information disclosed from that term is rich. For example, the structure and magnitude of diffusion coefficient can be estimated. It is natural to ask whether the cause-effect relation revealed in (6), which pertains to a very special traffic flow pattern (the short time behavior of downstream propagating platoon), is possibly general. We provide an analysis in the coming subsection.

**Modeling traffic diffusion**

Interpretation and derivation of traffic flow models like (3) can be carried out in multiple ways. This depends on the problem to address and can be based on quite different modeling philosophies. In (8), the term \( vk_{xx} \) is introduced, indicating that the equilibrium relation of \( v - k \) no longer exists. This relation is extended as \( v - (k, k_x) \), i.e. speed is related to more local information of density. In a mathematical point of view, such a revision indicates that, the driver is aware of the traffic density in the neighborhood of the driver (denoted as \( x_0 \)) since \( k(x,t) \) is usually well approximated by \( k(x_0,t) + k_x(x_0,t)(x-x_0) \) in such situation.

Extra caution should be taken when developing high order models of traffic flow, because erroneous results can usually be obtained unconsciously. For example, the Payne-Whitham model (4)(8) has one characteristic speed greater than traffic speed, which imposes a serious difficulty for interpretation and triggers lots of theoretical controversy (3)(5)(6)(7). Such discrepancy is obviously not an inherent character of high-order model (5)(6). Rather, as mentioned in (3), the flaw of PW and models alike is possibly due to the inappropriate approximation of PDE to its corresponding prototype car-following model.

Realizing the intricate structure of high order model, we are not directly attaching a diffusion term into the LWR model. Instead, we start from a microscopic description of driver speed heterogeneity, and use the method of characteristics to define the model. Note that the heterogeneity of the driver population has been extensively explored by other researchers (12)(13)(14). The main difference is that they start from a Boltzmann-like equation, and represent the driver heterogeneity by a distribution of desired speeds which
in turn leads to a modification to the basic Prigogine-Boltzmann model. Our approach to construct the model is unique, and interesting in its own right, in that information of characteristic curve is explicitly incorporated.

A more relevant work is (16), in which the authors actually obtained a 3rd order diffusive PDE as the limit of a microscopic stochastic description of traffic. Our approach is similar to theirs at large, in the sense that both researches assume certain microscopic dynamics which more easily characterize the fine details of traffic. The difference is that we do not resort to Taylor expansion to obtain the PDE approximation, out of the concerns that we mentioned above. Moreover, (16) intends to explain a spectrum of elusive traffic phenomena, such as stop-and-go traffic, spontaneous congestion, etc. Therefore realism issue is stressed. In contrast, our model concerns more with the usefulness of a new formalism, i.e. one modeling possibility directly related to the characteristic method. As such, the modeling flexibility and interpretation is more emphasized at current stage. It would be interesting to compare these two models and other models alike once the explicit form of our model becomes available.

-METHOD OF CHARACTERISTICS: Before elaborating on the new model, it is beneficial to review how the solution of LWR model is constructed. As is known, the solution to Eq. (1) or equivalently Eq. (2) is given by,

\[
\begin{align*}
  k(x, t) &= k_0(\xi) \\
  x &= \xi + c(k_0(\xi))t \\
\end{align*}
\]

in short time \( t \leq t_B = -1/F'(\xi_B) \), where \( k_0(\xi) \) describes the density profile at location \( \xi \) and time zero, \( c(\cdot) = Q'(\cdot) \) [check visual effect in PDF, the derivative sign] is the characteristic speed associated with specific types of traffic flow, and \( F(\cdot) = c \circ f(\cdot), \) \( \xi_B = \arg \max_{F(\xi) < 0} |F'(\xi)|, \) and \( t_B \) is called breaking time. Solution of this form implies the following properties of traffic flow: in reasonably short time, the traffic density at a given location and time is fully determined by a single point of the initial traffic density profile. Moreover, along the curve characterized by the second equation of (7), the value of traffic density virtually stays constant. The quantity \( c(k_0(\xi)) \) gives the instantaneous speed at which the information of density carried by a kinematic wave propagates. A graphical description of (7) is given in Figure 1.

Simple and enlightening as they are, the above properties of traffic flow implied by the LWR model seem to be questionable. In real traffic, vehicles within the same small region are supposed to exhibit certain variations in their behavior, even if they travel at approximately the same speed (a relation implied by the fundamental diagram). The variations fall in two categories, as mentioned in (3): (i) the temporal variability of
preferred speed of each individual vehicle, and (ii) the spatial variability of preferred speeds across vehicles. The former of the two is regarded less important in terms of its effect, while the latter is generally overlooked in the LWR model. It is noted that, in a PW-like model, with the speed dynamics governed by a ‘momentum’ equation, the resulted traffic density and speed \((\hat{k}, \hat{v})\) deviates systematically from an equilibrium curve \((k, v(k))\) in the phase plane. In this sense, the speeds constitute a spectrum with respect to any given density. This, however, is not exactly the second variation that we plan to discuss. The distributional property of speed in a PW-like model is totally determined by a law like,

\[
\begin{cases}
  dv/dt = f(k, v) \\
  x'(t) = v
\end{cases}
\] (8)

where \(f(\cdot)\) is a function representing the interaction of drivers with local traffic environment (for example, behaviors of anticipation and relaxation), whose specific form is not important here. Essentially, the equations (7) specify a relation that applies uniformly to all drivers. No driver is assumed to be different from the others. In comparison, variation (ii) is due to the driver inhomogeneity, i.e. drivers have various driving preferences, and the resulted effect seems more local. The difference is best illustrated with the following example. Imagine a platoon of traffic of sufficient length is propagating downstream freely. The LWR model and model like (8) predict that two drivers within the platoon drive identically as long as its interior still keeps intact. In reality and by (ii), one normally expects that drivers within this platoon prefer to keep different headway, so the relative locations of vehicles gradually change even in the interior of this platoon.

- A new model: Now it should be manifested what we meant by claiming that the variation (ii) is local. In one small region where traffic condition (in terms of traffic density and aggregate speed) is literally regarded as uniform, the speeds could still vary, in a manner beyond the description of the LWR or the PW-like model. It is such speed variation (or called speed distribution) that is modeled here and leads to a new explanation to the diffusion or viscosity in traffic flow. Our work is based on the following assumptions:

a1) The traffic flow in one lane (i.e. bypassing is not allowed) is a continuum;
a2) At space and time \((x, t)\), the traffic flow is described by the following three aggregate variables, i.e. the speed \(v(x, t)\), density \(k(x, t)\) and flow \(q(x, t)\);
a3) Density \(k(x, t)\) is conserved;
a4) There exists a monotone decreasing function \(V(\cdot)\) such that \(v = V(k)\) holds;
a5) The flow \(q = kv\);
a6) At each instant, the aggregate speed \(v(x, t)\) is the average of a speed distribution \(v_1, v_2, \ldots\) of local vehicles. The distribution is dependent on density \(k(x, t)\), with
variance $\sigma^2(k)$ or alternatively $\sigma^2(v)$ when abuse of notation does not cause confusion;

Compared with previous models, the main difference is assumption a6), which is a mathematical representation of variation (ii) mentioned above. Excluding assumption a6), with only assumptions a1-5), one obtains exactly the classical kinematic wave model, i.e. the LWR model (recall that the speed $v$ defined in (1) is the same as aggregate speed in our formulation). It will be interesting to investigate what arises from assumption a6).

The first and foremost observation is that the single point dependence (Figure 1) we mentioned at the very beginning of this subsection does not hold anymore. The rationale is: due to the distribution of speeds, some vehicles travel slower than the aggregated (average) speed while the others travel faster than this speed. This implies that the characteristics associated with the LWR model and depicted by (7) can be understood as an aggregation to the corresponding rarefaction fan. As such, the density $k(x,t)$ at any space-time point is dependent on the density values at a previous time in a region. This region is presumably getting larger when evolution within a longer time is concerned. Moreover, points along the characteristic given in (7) still play the dominant role in determining the value of $k(x,t)$. We illustrate this point in Figure 2. To accommodate this observation, a weighting function $w = w(\xi,t;\xi_0)$ is defined, with the following desirable properties:

$$ w \geq 0, \int w(\xi,t;\xi_0)d\xi = 1, \int \xi w(\xi,t;\xi_0)d\xi = \xi_0 $$

where $\xi_0$ is a location parameter. Moreover, it is required that $w$ is continuous in space and time $(\xi,t)$, $\lim_{t \to 0} w(\xi,t;\xi_0) = \delta(\xi_0)$ and $\int \xi^2 w(\xi,t;\xi_0)d\xi$ is an increasing function of $t$. The weighting function is, mathematically, a distribution function, but it does not indicate any random behavior of traffic flow in our current formulation. Rather, it is meant to capture the fine detail, namely the existence and interplay of disaggregate speeds, of traffic flow. Corresponding to the disaggregation of speeds, each characteristic curve is decomposed into a family of sub-characteristics (refer to Figure 2). This argument indicates that, a simple characterization of traffic flow like (7) no longer holds even for short time. The dependence domain, when assumption a6) is added, should in general be an interval rather than a single point. Our analysis finally leads to a new model in the resemble form of Eq. (7),

$$\begin{align*}
  k(x,t) &= \int k_0(\xi)w(\xi,t;\xi_0)d\xi \\
  x &= \xi_0 + c(k_0(\xi_0))t
\end{align*}$$

where $k_0(\cdot)$ represents the density profile at time zero. Other notations have been previously explained. It is noted that, besides the defining properties of $w$, it should also ensure that the density is a conserved quantity. This seems not naturally true in the generic formulation (10). Existence of such function is obvious, e.g. the Delta function. But it is
difficult to answer how many such $w$ actually exist at the moment. This, however, does not affect our analysis of two specific examples in the next section.

One analogy motivates us to call (10) a viscous model of traffic flow. Suppose a driver is traveling along a traffic flow with the local aggregate speed $v$. According to the LWR model, this driver observes a net flow $q - vk = 0$ relative to her/him. Whereas due to the distribution of speeds, i.e. assumption a6), the driver finds local vehicles spread at very small relative speeds. Roughly, this motion is in correspondence to the fluctuations of particles while they transport in a systematic way (macroscopically observable) with the fluid. It is already known that the systematic movement of traffic flow is modeled by the LWR model or its variants, e.g. PW-like models. Yet such small fluctuation is generally overlooked. It is known that viscosity is related to the microscopic fluctuations. Therefore model (10) is supposed to have certain linkage with Partial Differential Equation (PDE) of the form,

$$k_t + c(k)k_x = \nu(k)k_{xx} \tag{11}$$

where the right hand side is the diffusion term and $\nu(k)$ is the viscosity (diffusion) coefficient. We shall derive (11) from (10) in the coming section, revolving the specification of $w$ and $\nu$.

3. Analysis of the new model

Linkage with other models

Depending on the selection of function $w$, model (10) can be reduced to some familiar models, which usually are in the form of PDEs. Before showing the examples, it is necessary to elaborate a little more on the function $w$. A reasonably defined $w$ must rely on the local density $k$, since the speed distribution, which potentially determines $w$, cannot be independent of the aggregate speed (confirmed by numerous field observations). Therefore, a complete characterization of $w$ over one space-time domain $dx \times dt$ is impossible when density $k$ over this domain is also unknown. Fortunately, comprehensive knowledge of $w$ is not necessary in certain special cases, as shown below.

Case 1: LWR model. This case is straightforward since model (10) inherits a mathematical structure similar to that of Eq. (7), which is essentially the LWR model restricted to short time. The Eq. (10) reduces to (7) by letting $w(\xi; t; \xi_0) = \delta(\xi_0)$. The first equation of (10) then reads,

$$k(x, t) = \int k_0(\xi)w(\xi, t; \xi_0)d\xi = \int k_0(\xi)d\xi = k_0(\xi_0) \tag{12}$$
Rename $\xi_0$ as $\xi$, then (10) and (7) are exactly the same. Since $w(\xi, t; \xi_0) = \delta(\xi_0)$ is actually a degenerated weighting function, LWR may be regarded as a degenerated case of the proposed viscous model in this sense.

**Case 2: Platoon diffusion model.** Consider a platoon of congested vehicles discharged to downstream when a signal turns green (see Figure 3, location of signal is defined as $x = 0$). Of interest is the evolution of the platoon front, namely, the density $k(x, t)$ for $t$ not large and $x \sim v_f t$, where $v_f$ denotes the free flow speed. Since in short time the density profile is nearly constant, it can be assumed that the speed distribution is identical across the platoon. Moreover, we already restrict the focus to short time behavior. Therefore, it is assumed that $w$ is location independent, symmetric with respect to $\xi_0$. A choice of $w$ is the Gaussian kernel,

$$ w(\xi, t; \xi_0) = \frac{1}{\sqrt{2\pi \eta t}} \exp\left(-\frac{(\xi - \xi_0)^2}{2\eta^2 t^2}\right) $$

where $\eta > 0$ is an adjusting factor of the spread of the function. Practically, this particular choice of $w$ approximates many continuous symmetric functions. Mathematical analysis involving this function usually keeps tractable.

With the weighting function specified, one can obtain the solution of (10). This is by no means easy, since solution of (7) is already non-trivial and (10) is much more complex. Therefore a series of approximations are introduced in the following analysis. By the first equation of (10),

$$ k(x, t) = \int k_0(\xi) \frac{1}{\sqrt{2\pi \eta t}} \exp\left(-\frac{(\xi - \xi_0)^2}{2\eta^2 t^2}\right) d\xi $$

Notice that the scenario discussed here is similar to that of a Riemann problem

$$ k_i + c(k)k_x = 0 $$

with initial data $(k_j, 0)$, it is reasonable to approximate $k_0(\xi)$ with the solution to (15), which is known to be a rarefaction wave. Moreover, it is the intermediate part of the rarefaction solution that matters in our analysis. This part of the solution assumes the self-similarity form, which can be written as (Note: To illustrate the key point, $u$ is assumed to be identity function. The form of $u$ is relevant but does not influence the derived solution in terms of its general structure.),

$$ k_0(\xi) \sim u(x / t) = a \frac{x}{t} + b $$

for $x \in [c(k_j)t, c(0)t]$, where $a, b$ can be solved from the equations,
\[
\begin{align*}
    \begin{cases}
        ac(0) + b = 0 \\
        ac(k_j) + b = k_j
    \end{cases}
\end{align*}
\]

Qualitatively, \(a < 0, b > 0\). Combine the above results, we obtain density profile after a short time \(\Delta t\) from density at time \(\tau\),

\[
k(x, \tau + \Delta t) \sim \int \frac{1}{\sqrt{2\pi \eta \Delta t}} \exp\left(-\frac{(\xi - \xi_0)^2}{2\eta^2 \Delta t^2}\right) d\xi
\]

(17)

where \(\Delta t \to 0\), so that \(k(x, \tau) \sim k(x, \tau + \Delta t) + O(\Delta t)\). Moreover, let

\[
g(s) = \frac{1}{\eta \Delta t} \exp\left(-\frac{s^2}{2\eta^2 \Delta t^2}\right)
\]

(18)

\[
f(s) = a \frac{s^2}{\tau} + b
\]

The right hand integral of (17) is indeed a convolution,

\[
\theta(x, t) = \frac{1}{\sqrt{2\pi}} \int f(\xi) g(\xi_0(x, t) - \xi) d\xi
\]

(19)

This is the solution of

\[
\theta_x = \kappa \theta_{xx}
\]

(20)

if we specify \(\tau = \Delta t^2, \bar{x} = \xi_0\) and \(\kappa = \eta^2 / 2\). This essentially shows that our model leads to a traffic flow model with a diffusion term, which can be explicitly calculated by transforming \((\bar{x}, \bar{t})\) back to \((x, t)\). Formula (20) indicates that, qualitatively parameter \(\eta\) reflects the strength of viscosity/diffusion in traffic. When \(\eta \to 0\), the Gaussian kernel essentially reduces to Kronecker kernel and the viscosity term disappears. This corresponds to the scenario that local individual speeds are strictly uniform and a non-viscous description of traffic may be as good as one can expect.

4. Concluding remarks

The LWR model is perhaps the most important prototype of all kinematic traffic flow models. That partly explains why its deficiencies have provoked so much attention and discussion. This paper focuses on an intriguing and arguable extension of the LWR model, i.e. including a viscosity (diffusion) term in the conservation equation. Multiple reasons could be found for such extension, e.g. to smooth the shocks which are inconsistent with empirical observations, to reproduce the scattering on the phase plane of flow-density relationship, to reflect drivers’ awareness of density gradient, etc. However, it is recognized that a model of this type may lead to erroneous result, e.g. backward travelling at the end of a stopping queue.
This paper proposes another perspective to understand such extensions. We postulate that, associated with each aggregate speed, there is a speed distribution due to the inhomogeneous nature of drivers and their varied preference of traveling speeds in the essentially identical environment (in terms of local aggregate density and flow). This postulation leads to a model of traffic flow that resembles the characteristics solution of a scalar conservation law in short time but generalizes the single point dependence. We find that, through the example of discharging queue, a diffusion PDE can be derived from the proposed model.

It is remarked that the proposed model, which is similar to the characteristic method, is best suited for the analysis of local fine structure of traffic flow, because new construction is needed when characteristic curves start to cross each other. Moreover, the assumption a6), which claims the existence of a spectrum of disaggregates speeds, underlies the local behavior rather than system dynamics. In another perspective, the viscosity effect is mostly dominated by transport at the system level, especially when traffic density does not change abruptly. Therefore, it is expected that the proposed model is most useful when shock structure is concerned. An example in this case is the boundary of two stable regions, in particular, the front and end of a traveling queue.
References

FIGURE 1 Illustration of the method of characteristics
FIGURE 2 The proposed diffusion model of traffic flow
FIGURE 3 The problem of discharged platoon (Up: front of platoon predicted by different models; Down: characteristics)