A SIMPLIFIED KINEMATIC WAVE MODEL AT A MERGE BOTTLENECK

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Abstract

A merge is a point of a highway where two or more streams of traffic flow into one. It is always easy to solve the demand-supply problem at a merge when the merge is operating under uncongested condition. However, when congestion backs up exceeding the merging point where multiple streams of traffic meet, one is typically faced with splitting downstream supply among the merging branches. Solutions of this situation are multiple and several merge queuing models have been proposed in the past. To address the drawbacks of the past models, this paper proposes a capacity-based weighted fair queuing (CBWFQ) model that is characterized by its fidelity (approximation to real situation), simplicity, and extensibility. Based on the CBWFQ merge queuing model, a simplified kinematic wave model is formulated to model traffic operation at a merge bottleneck.

Keywords: Simulation and Modeling, Traffic flow, Simplified kinematic wave, Queuing, Merge.
INTRODUCTION

A merge is a point of a highway where two or more streams of traffic flow into one. Queuing at the merge is of interest because it is often related to highway congestion that traffic engineers are trying their best to mitigate. As an essential part of Intelligent Transportation Systems (ITS), Advanced Traffic Management System (ATMS) relies heavily on various traffic models to identify causes of congestion, advise control traffic strategies, and suggest diverting routes. Newell’s simplified theory of kinematic wave (Newell, 1993a, b, and c) is particularly efficient to assist these efforts in terms of implementation and application. Unfortunately, the simplified theory deals only with freeway mainline. It would be of great interest to both theory and practice if the simplified theory could be extended to address network traffic. As part of such effort, this paper develops a simplified kinematic wave model to address traffic flow at a merge bottleneck.

Several merge queuing models have been reported in the literature. Among the well-established are Daganzo’s (1995) priority-based model and Lebacque’s (1996) lane-based model. Both models are comprehensive in that their solution spaces contain virtually all possibilities. Considering that not every possibility in the solution space is equally likely and motivated by reducing the cost of calibration, Jin and Zhang (2003) proposed a demand-based distribution scheme under the framework of Daganzo’s model. Though it may yield unrealistic results under certain conditions, this model has been the simplest one proposed so far. Newell implicitly proposed a merge queuing model in his original theory. Unlike what is commonly believed, the model assumes full priority for ramp traffic so that it always bypasses any queue at a merge and experiences no delay. Though unrealistic to a certain extent, this model does make the simplified theory easy and elegant. As part of his attempt to study the impact of ramp-metering on traffic delay, Banks (2000) extended Newell’s model by constraining ramp out flow by capacity and metering rate. The resulting model considers the case where the ramp is dictated by forward waves. However, this model does not address another important case where backward waves reach the ramp probably because this case is unlikely to occur under ramp-metering.

The goal of this paper is two-fold. First, it proposes a merge queuing model to preserve the desirable features of the existing models while eliminating their drawbacks. Second, based on the proposed merge
queuing model, a simplified kinematic wave model is formulated to model traffic flow at a merge bottleneck. This is a necessary step to enable the simplified theory to address network traffic.

**REVIEW OF EXISTING WORK ON MERGE QUEUING MODELS**

To facilitate future discussion, a merge is sketched in Figure 1 which has two upstream links/branches and one downstream link/branch. A merge can be a junction where an on-ramp joins a freeway, two freeways or highways come into one, or even a multi-leg intersection if properly defined. Suppose at time $t$, link 1 wants to send $S_1$ vehicles, link 2 wants to send $S_2$ vehicles, and the downstream link can receive $R$ vehicles. $d_1$ denotes the out flow (i.e. departure count) of link 1, $d_2$ the out flow of link 2, and $d$ the inflow of the downstream link, where $d = d_1 + d_2$. $p_1$ denotes the priority factor or splitting coefficient of link 1 and $p_2$ that of link 2. $p_1$ and $p_2$ are non-negative fractions. $Q$ denotes the capacity of a link.

![Figure 1 Sketch of a merge](image)

**Lebacque's Lane-Based Merge Queuing Model**

This model is based on user optimal strategy, i.e. the model tries to maximize outflows at the merging branches and sum them up to obtain the downstream inflow. The model works as follows:
The splitting coefficients, \( p_1 \) and \( p_2 \), defined as the ratios of maximum densities of the upstream links to that of the downstream link, often translate to the ratios of their respective number of lanes. There is no guarantee that the splitting coefficients sum up to 1.

The solution space of Lebacque's model is illustrated by the shaded area in Figure 2. There are several lines emitting from the origin. The solid line 0-6-5 represents one possible ratio of the splitting coefficients, while the two dotted lines represent other possible ratios. If \( R \) is fixed and \( S_1 + S_2 \leq R \), the solution lies on the bold line \( S_2-1-2-3-4-S_1 \). If \( S_1 + S_2 > R \), the solution lies on bold line \( S_2-1-6-4-S_1 \). The above cases hold when \( p_1 + p_2 = 1 \). If \( p_1 + p_2 > 1 \), a solution found may not be realistic because the sum of upstream out flows can be greater than downstream supply. In this case, something has to be done to make the solution feasible. Nevertheless, this model is comprehensive and yields the largest solution space.

\[
\begin{align*}
    d_1 &= \min \{ S_1, p_1 R \} \\
    d_2 &= \min \{ S_2, p_2 R \} \\
    d &= d_1 + d_2
\end{align*}
\]

Note: \( p_1 + p_2 \) may not equal to 1 \\
\( d_1 + d_2 \) may not equal to \( R \)

FIGURE 2 Lebacque's lane-based merge queuing model
Daganzo’s Priority-Based Merge Queuing Model

Unlike Lebacque’s model, this model is based on system optimal strategy, i.e. the model tries to maximize the downstream inflow while keeping upstream outflows feasible. The model works as follows:

\[
\begin{align*}
    d_1 &= S_1 & \text{if } R \geq S_1 + S_2 \\
    d_2 &= S_2
\end{align*}
\]

\[
\begin{align*}
    d_1 &= \text{mid}\{S_1, R - S_2, p_1R\} & \text{if } R < S_1 + S_2 \\
    d_2 &= \text{mid}\{S_2, R - S_1, p_2R\}
\end{align*}
\]

where operator mid means the middle point of the three points intersected by line \( R = d_1 + d_2 \).

Again, solution space of this model is shown by the shaded area in Figure 3. The lines emitting from the origin bear the same meaning as in Figure 2. The solution under certain supply and demand conditions can be multiple and all the points in the shaded region are feasible, though they might not be equally likely. To find a unique solution, some additional constraints must be provided. For example, the priority constraint assumes that flow on link 1, \( d_1 \), has higher priority than flow on link 2, \( d_2 \), i.e. \( p_1 > p_2 \). This eliminates virtually half of the solution space. If \( p_1 \) and \( p_2 \) are fixed, the solution space reduces to the bold line 1-2-3-4. Depending on the values of sending flows and receiving flow, unique solution can be found at point 1 if both merging branches are constrained by backward waves, or at point 3 if link 1 is constrained by forward wave and link 2 is constrained by backward wave, or at point 4 if both branches are dictated by forward waves.
Jin and Zhang's Simplest Distribution Scheme

In an attempt to simplify Daganzo's priority constraint, Jin and Zhang proposed a distribution scheme based on contributions of upstream demands. The model works as follows:

\[
\begin{align*}
  d_1 &= S_1 & \text{if } S_1 + S_2 < R \\
  d_2 &= S_2 & \\
  d_1 &= R \times \frac{S_1}{S_1 + S_2} & \text{if } S_1 + S_2 \geq R \\
  d_2 &= R \times \frac{S_2}{S_1 + S_2}
\end{align*}
\]

Based on this assumption the solution space of Daganzo's reduces to the bold line in Figure 4.
Unfortunately, this model may yield unrealistic results under certain conditions. For example, suppose that upstream mainline demand is $S_1 = 2000$, on-ramp demand is $S_2 = 200$, and downstream supply is $R = 2000$. This distribution scheme suggests a solution of $d_1 = 1818$ and $d_2 = 182$. This implies that the on-ramp can depart only 1 vehicle at the departure of every 10 vehicles at the upstream mainline, a scenario that is very rare in real life.

**Newell's Simplified Merge Queuing Model**

As one of his underlying assumptions of the simplified theory of kinematic wave, Newell assumed that ramp-entering vehicles can always bypass a queue, if any, and experience no delay. Unlike Daganzo's model the full priority is given to on-ramp traffic, i.e. $p_2 = 1$ and $p_1 = 0$. The model works as follows:

\[
\begin{align*}
    d_2 &= S_2 & \text{if } & S_1 + S_2 \leq R \\
    d_1 &= S_1 \\
    d_2 &= S_2 & \text{if } & S_1 + S_2 > R \\
    d_1 &= R - S_2
\end{align*}
\]

Note: $p_1 = S_1 / (S_1 + S_2)$; $p_2 = S_2 / (S_1 + S_2)$; $p_1 + p_2 = 1$
With this, the solution space is reduced to the bold line indicated in Figure 5. The solution is at point 1 if link 1 is dictated by backward waves or at point 2 if link 1 is dictated by forward waves. In either case, link 2 is always dictated by forward waves.

**FIGURE 5 Newell’s simplified merge queuing model**

**Banks’ Ramp-Metering Merge Queuing Model**

As part of his effort to analyze the effect of ramp-metering in reducing traffic delay, Banks extended Newell’s assumption on a merge such that ramp out flow is constrained by its demand $S_2$, capacity $Q_2$, and metering-rate $M_2$, i.e. $d_2 = \min\{ S_2, Q_2, M_2 \}$. This model is basically the same as Newell’s, i.e. full priority is still given to link 2 and backward waves never reach this branch. If we combine constraints $S_2, Q_2, and M_2$ into a new demand $S_2'$, Bank’s model yields exactly the same solution space as Newell’s.

Generally, Lebacque’s model yields larger solution space than Daganzo’s due to the former’s relaxation of constraint $p_1 + p_2 = 1$. Newell’s model, Banks’ model, and Jim’s model can be considered as
special cases of the first two models. The above relationship is illustrated in Figure 6. As can be seen later, the model we are proposing can be viewed as another special case.

Lebacque's and Daganzo's models are comprehensive. It is informative to know the entire solution space. However, not all solutions are feasible (as in Lebacque's model), nor are feasible solutions equally likely (as in Lebacque's and Daganzo's models). The physically meaningful and thus highly likely solutions are only a small subset of the entire solution space. To obtain the subset, additional effort of calibration may be necessary and such an effort can be costly. The other three models are very simple to implement and require no calibration. However, these models are subject to over-simplification and may yield unrealistic results under certain conditions. This paper proposes a CBWFQ merge queuing model which preserves the advantages of the above models while eliminating their drawbacks.

THE PROPOSED CBWFQ MERGE QUEUING MODEL

This section proposes a CBWFQ merge queuing model as well as its generalized form to facilitate the effort of applying the simplified theory to network traffic. Figure 7a shows a general merge where there are $\alpha$ merging branches.
We define the following notations:

- \( a_i \) denotes upstream arrival of branch \( i \), i.e. the number of vehicles that are expected to arrive at branch \( i \) based on further upstream conditions. In this and the definitions below, \( i = 1, 2, \ldots, \alpha \).

- \( Q_i \) denotes the capacity of the branch \( i \) and \( Q \) denotes the capacity of the downstream common link.

- \( \Delta_i \) denotes the fair share of downstream supply by branches \( i \). \( \Delta_i = R \times \frac{Q_i}{\sum_{j=1}^{\alpha} Q_j} \).

- \( S_i \) denotes the number of vehicles that branch \( i \) is able to send. \( S_i \) takes the minimum of upstream arrival and the amount allowed by local capacity, i.e. \( S_i = \min \{ a_i, Q_i \tau \} \) where \( \tau \) is time increment.

- \( d_i \) denotes the outflow of branch \( i \), i.e. the number of vehicles that branch \( i \) can actually send based on local capacity and upstream and downstream conditions.

- \( R \) denotes the downstream supply, i.e. number of vehicles that can be received by the downstream link.

- \( B \) denotes the set of branches \( \{1, 2, \ldots, \alpha\} \).

In two-dimensional (2D) case, the merge has two upstream branches. The proposed CBWFQ model determines the outflows of the two upstream branches as illustrated in the left part of Figure 7b.
The two axes represent the outflows of the two upstream branches 1 and 2. After determining the fair shares of the two branches, we construct a fair share rectangle O-?_1P-?_2 by running two lines based on ?_1 and ?_2. We also construct a supply line P_1P_2 with the coordinates of every point on the line summing up to R.

If the demands on the two branches (i.e. S_1 and S_2) are greater than their corresponding fair shares (?_1 and ?_2 respectively), the demand point must fall in the half space ?_1-P-?_2, e.g. point 5. The solution is point P, i.e. d_1 = ?_1 and d_2 = ?_2. Otherwise, there must be at least one branch (e.g., branch 2) whose demand (e.g. S_2) is less than its fair share (e.g. ?_2). We satisfy this demand and allocate the remaining supply to the other branch. For example, if the demand is point 6, the solution is point 6’, i.e. the intersection of the horizontal line through point 6 and the supply line. If the demand is point 3, the solution is the point itself. Similar treatment applies if branch 1 has more demand than its fair share, e.g., points 4 and 2. If both branches have lower demands than their corresponding fair shares, we satisfy these demands immediately, e.g. point 1.

The above CBWFQ model is readily extensible at no extra cost. For example, the solution to the three-dimensional (3D) case (i.e. the merge has three upstream branches) is shown in the right part of

FIGURE 7b Generalized CBWFQ merge queuing model
Figure 7b where the three axes represent the outflows of the three upstream branches, respectively. We construct a fair share box (a rectangular prism to be precise) O-Δ₂-P₁₂-Δ₁-Δ₃-P₂₃-P₁₃. We also construct a supply plane P₁-P₂-P₃ with the coordinates of every point on it summing up to R. Obviously, the fair share box and the supply plane contact at point P. If all three branches have higher demands than their corresponding fair shares, the demand point must fall in the half space P₁-P₁₂-P₁₃-P₂₃ (which points outward) and the solution is point P. Otherwise, there must be at least one branch (e.g. branch 3) whose demand is less than its fair share. We satisfy this demand, subtract this amount from the downstream supply, and remove this branch from further consideration. This is equivalent to cutting the picture with a horizontal plane at height S₃, shown as the dotted triangle T₁-T₂-T₃. This essentially reduces the problem to a 2D case whose solution has been discussed ready.

Summing up, the algorithm for the generalized CBWFQ model works as follows:

Step 1: compute the fair share of the downstream supply for each of the merging branches proportional to its capacity.

Step 2: for each merging branch, if its demand is less than or equal to its fair share, set its outflow to its demand S, subtract this amount from the downstream supply, and remove this branch from the set. Repeat this step until all merging branches have been processed. i.e.:

\[
\begin{align*}
\text{IF } a_i & \leq \Delta_i \text{ THEN } \left\{ \\
& d_i = S_i \\
& R = R - d_i \\
& B = B - \{i\}
\end{align*}
\]

Step 3: for the remainder of the downstream supply and the remainder of the branches, repeat steps 1 and 2 until no new branch's demand is satisfied or \( B \) is empty.

Step 4: based on the remainder of the downstream supply and the remainder of the branches, recalculate the fair share of the remaining supply for each of the remaining branches and set their outflows to their fair shares.

The CBWFQ model, as well as its generalized form, merits the following advantages: (1) it deals with both forward and backward waves, i.e. it considers both upstream and downstream conditions when determining outflows; (2) it yields unique solution, so it is well-formulated; (3) its solution is physically
meaningful and highly likely, so the model eliminates many unnecessary possibilities that are both unlikely and costly; (4) it is simple to understand and easy to implement; (5) it is able to account for many factors related to traffic operation such as demand, supply, road geometry, capacity, ramp-metering strategies, etc.; (6) it is readily extensible to merges with multiple upstream branches at no extra cost.

SIMPLIFIED THEORY OF KINEMATIC WAVE AT A MERGE BOTTLENECK

The goal of this section is to extend the simplified theory of kinematic wave to incorporate queuing at a merge bottleneck. Figure 8 shows a sketch of the merge bottleneck. There are two upstream links (i.e. \((x_i, x_f)\) and \((x_j, x_f)\)) and one downstream link (i.e. \((x_f, x_n)\)). \(x_r\) denotes any further destination of \(x_f\) via \(x_n\). \(A_{r,n}(x_f^-, t)\) denotes the cumulative number of vehicles arriving at somewhere slightly upstream of \(x_f\) at time \(t\) originated from node \(x_i\) and destined for node \(x_n\) and beyond at time \(t\). In this expression, capital letter “A” means “cumulative arrival curve” and minus sign “-” means “to the left of” or “slightly upstream of.” \(D_{i,n}(x_f^+, t)\) denotes the cumulative number of vehicles past somewhere slightly downstream of \(x_f\) originated from node \(x_i\) and destined for node \(x_n\) and beyond at time \(t\). In this expression, capital letter “D” means “cumulative departure curve” and plus sign “+” means “to the right of” or “slightly downstream of.” \(Q_{ij}, k_i, v_i, n_i, l_i, u_i\) denote the capacity, jam density, forward wave speed (the same as free flow speed in the simplified theory), number of lanes, length, and backward wave speed of link \((x_i, x_f)\), respectively. Capacity incorporates factors such as number of lanes, per lane capacity, and traffic control strategies. The meaning of the above notations apply to other similar symbols.
Starting from boundary conditions, the algorithm evaluates all nodes one by one from the upstream end to the downstream end. Then time tick advances and the above process repeats. Suppose at time $t$ we know $D_{l,z}(x^{+}_{l},t)$ and $D_{j,z}(x^{+}_{j},t)$ (where $z = l, n, r$) from boundary conditions and previous time steps. Suppose also that geometry data and traffic characteristics data are well defined for each link. Our goal is to determine $D_{l,z}(x^{+}_{l},t)$ and $D_{j,z}(x^{+}_{j},t)$. This can be done by a 5-step procedure as follows:

A. Departure to the right of node $x_{i}$

As proposed by Newell (1993b), the cumulative departure curve to the right of $x_{i}$ originated from $x_{i}$ destined for $x_{n}$ and beyond, $D_{l,z}(x^{+}_{l},t)$, is the minimum of the following:

a. Upstream arrival $A_{l,z}(x^{+}_{l},t)$

According to the forward wave propagation rule (Newell, 1993b), the cumulative arrival curve to the left of $x_{i}$ originated from $x_{i}$ destined for $x_{n}$ and beyond, $A_{l,n}(x^{-}_{l},t)$, can be obtained by translating $D_{l,n}(x^{+}_{l},t)$ to the right by a link travel time at free flow speed, i.e. $A_{l,n}(x^{-}_{l},t) = D_{l,n}(x^{+}_{l},t-l_{il} \frac{v_{il}}{v_{fl}})$. 

FIGURE 8 Data of simplified kinematic wave at a merge bottleneck
Similarly, \( A_{j,i,n}(x_i, t) \) can be obtained from \( D_{j,i,n}(x_i^+, t) \), i.e. \( A_{j,i,n}(x_i^+, t) = D_{j,i,n}(x_i^+, t - l_{ji} / v_{ji}) \). The arrival to the right of \( x_i \), \( A_{i,n}(x_i^+, t) \), is the sum of \( A_{i,n}(x_i^-, t) \) and \( A_{j,i,n}(x_i^+, t) \), i.e.

\[
A_{i,n}(x_i^+, t) = A_{i,n}(x_i^-, t) + A_{j,i,n}(x_i^+, t) = D_{i,n}(x_i^-, t - l_{ij} / v_{ij}) + D_{j,i,n}(x_i^+, t - l_{ji} / v_{ji})
\]

b. The constraint by the capacity to the right of node \( x_i \)

\[
D_{i,n}(x_i^+, t - \tau) + \tau Q_{in}
\]

where \( \tau \) is time increment and \( Q_{in} \) is the capacity of link \( (x_i, x_n) \).

c. The constraint if link \( (x_j, x_n) \) is congested

\[
D_{l,n}(x_n^-, t - l_{in} / u_{in}) + l_{in} k_{in}
\]

where \( l_{in} \) is the length of link \( (x_j, x_n) \), \( u_{in} \) is the backward wave speed of the link, and \( k_{in} \) is the jam density of the link. This is the place where the backward wave propagation rule (Newell, 1993b) applies.

d. The constraint by the capacity to the left of node \( x_i \)

The maximum number of vehicles that are allowed to depart from link \( (x_j, x_i) \) at current time step is \( \tau \times Q_{il} \) and the maximum number that are allowed to depart from link \( (x_j, x_i) \) is \( \tau \times Q_{jl} \). Therefore, the departure constrained by the capacity to the left of node \( x_i \) is:

\[
D_{l,n}(x_i^+, t - \tau) + \tau Q_{il} + \tau Q_{jl}
\]

Summing up, \( D_{l,n}(x_i^+, t) \) is the minimum of the above four, i.e.

\[
D_{l,n}(x_i^+, t) = \min \left\{ A_{l,n}(x_i^+, t), D_{l,n}(x_i^+, t - \tau) + \tau Q_{il}, D_{l,n}(x_n^-, t - l_{in} / u_{in}) + l_{in} k_{in}, D_{l,n}(x_i^+, t - \tau) + \tau Q_{jl} + \tau Q_{jl} \right\}
\]

B. Departure to the left of node \( x_i \)
Now, we want to know how much of $D_{i,n}(x^+_i,t)$ is contributed by link $(x_i,x_j)$ and how much by $(x_j,x_i)$. This is the place where our proposed CBWFQ merge queuing model plugs in. Let:

\[ a_{i,n}(x^-_i,t) \] be the upstream arrival at current step to the left of $x_i$ originated from $x_j$ destined for $x_n$ and beyond. $a_{i,n}(x^-_i,t) = A_{i,n}(x^-_i,t) - D_{i,n}(x^-_i,t - \tau)$. \[ a_{j,n}(x^-_j,t) \] be the upstream arrival at current step to the left of $x_j$ originated from $x_i$ destined for $x_n$ and beyond. \[ a_{j,n}(x^-_j,t) = A_{j,n}(x^-_j,t) - D_{j,n}(x^-_j,t - \tau) \].

\[ d_{i,n}(x^-_i,t) \] be the amount at current step that can actually depart to the left of $x_i$ originated from $x_j$ destined for $x_n$ and beyond. This is an unknown variable.

\[ d_{j,n}(x^-_j,t) \] be the amount at current step that can actually depart to the left of $x_j$ originated from $x_i$ destined for $x_n$ and beyond. This is another unknown variable.

$S^n_{il}$ be the amount at current step that link $(x_i,x_j)$ can send. $S^n_{il} = \min \{ a_{i,n}(x^-_i,t), \tau Q_{il} \}$.

$S^n_{jl}$ be the amount at current step that link $(x_j,x_i)$ can send. $S^n_{jl} = \min \{ a_{j,n}(x^-_j,t), \tau Q_{jl} \}$.

$R^n_{in}$ be the supply of link $(x_i,x_n)$ at current step. $R^n_{in} = D_{i,n}(x^+_i,t) - D_{i,n}(x^-_i,t - \tau)$.

$\Delta^n_{in}$ be link $(x_i,x_i)$'s fair share of downstream supply. $\Delta^n_{in} = R^n_{in} \times \frac{Q^n_{il}}{Q^n_{il} + Q^n_{jl}}$.

$\Delta^n_{jn}$ be link $(x_j,x_i)$'s fair share downstream supply. $\Delta^n_{jn} = R^n_{jn} \times \frac{Q^n_{jl}}{Q^n_{il} + Q^n_{jl}}$.

We can determine $d_{i,n}(x^-_i,t)$ and $d_{j,n}(x^-_j,t)$ by applying the proposed CBWFQ merge queuing model which is not repeated here. Therefore, we can compute $D_{i,n}(x^-_i,t)$ and $D_{j,n}(x^-_j,t)$ as follows:

\[ D_{i,n}(x^-_i,t) = D_{i,n}(x^-_i,t - \tau) + d_{i,n}(x^-_i,t) \]

\[ D_{j,n}(x^-_j,t) = D_{j,n}(x^-_j,t - \tau) + d_{j,n}(x^-_j,t) \]
Since this is a merge, no traffic exits. The cumulative departure curve to the left of \( x_i \) originated from \( x_i \) destined for \( x_i \) and beyond, \( D_{i,j} (x_i^- , t) \), is the same as \( D_{i,n} (x_i^- , t) \). Similarly, \( D_{j,j} (x_i^- , t) \) is the same as \( D_{j,n} (x_i^- , t) \), i.e.

\[
D_{i,j} (x_i^- , t) = D_{i,n} (x_i^- , t) \\
D_{j,j} (x_i^- , t) = D_{j,n} (x_i^- , t)
\]

C. Link travel time

According to Newell (1993b), link travel time is obtained by comparing the cumulative departure curves at both ends of a link. Therefore, the travel time at link \( (x_i, x_j), T_{il} (t) \), can be found by comparing the following pair of cumulative departure curves:

\[
D_{i,j} (x_i^+ , t) \text{ vs. } D_{i,j} (x_j^-, t)
\]

If we trace \( D_{i,j} (x_i^+ , t) \) backwards until some prior time \( t' \) such that \( D_{i,j} (x_i^+ , t') \) is equal to \( D_{i,j} (x_i^- , t) \), then \( T_{il} (t) = t - t' \) is the travel time of the vehicle bearing the “number” \( D_{i,j} (x_i^+ , t) \) at this link. Travel times of other vehicles at the same link are assumed to be the same regardless of their destinations. Similarly, travel time on link \( (x_j, x_l), T_{jl} (t) \), can be found by comparing curve pair

\[
D_{j,j} (x_j^+ , t) \text{ vs. } D_{j,j} (x_j^- , t)
\]

D. Departure to the left of node \( x_i \) (multi-destinations)

According to Newell (1993b), \( D_{i,j} (x_i^- , t) \) can be obtained by simply translating \( D_{i,r} (x_i^+ , t) \) to the right by a link travel time \( T_{il} (t) \). The same applies to other multi-destination cumulative departure curves \( D_{j,i} (x_j^- , t) \), i.e.

\[
D_{i,r} (x_i^- , t) = D_{i,r} (x_i^+ , t - T_{il} (t))
\]
\[ D_{j,s}(x^-_i, t) = D_{j,s}(x^+_j, t - T_{jl}(t)) \]

E. Departure to the right of node \( x_i \) (multi-destinations)

The cumulative departure curve past the right of \( x_i \) originated from \( x_i \) destined for other destination \( x_r \), \( D_{i,r}(x^+_i, t) \), is simply:

\[
D_{i,r}(x^+_i, t) = D_{i,s}(x^-_i, t) + D_{j,r}(x^-_i, t)
\]

So far we have formulated the simplified kinematic wave model at a merge bottleneck.

EMPIRICAL TESTS

Empirical tests of the proposed model are based on data collected from GA-400 by Georgia NAVIGATOR system, an automatic traffic surveillance system covering the greater Atlanta metropolitan area.

The Test Site

The test site is a merge on the northbound of GA-400, as illustrated in Figure 9:
This site consists of two upstream mainline links (4000051-4000053 and 4000053-5008), one downstream mainline link (5008-4000055), and an on-ramp link (4005008-5008). All mainline links have 3 lanes with approximately the same capacity. The merge point, node 5008, might be a bottleneck because the capacity of its downstream link (5008-4000055) is less than the sum of its upstream links (4000053-5008 and 4005008-5008). Another potential bottleneck is the downstream of node 4000055 because queues might pile up from further downstream and back onto our test site.

The geometry data and traffic characteristic data of the test site are summarized in Table 1.

<table>
<thead>
<tr>
<th>Link</th>
<th>Up Node</th>
<th>Down Node</th>
<th>Length (mi)</th>
<th>Lanes</th>
<th>Type</th>
<th>FFS (mi/h)</th>
<th>Capacity (veh/h/ln)</th>
<th>Jam Density (veh/mi/ln)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4000051</td>
<td>4000053</td>
<td>0.65</td>
<td>3</td>
<td>Mainline</td>
<td>65</td>
<td>2200</td>
<td>180</td>
</tr>
<tr>
<td>3</td>
<td>4000053</td>
<td>5008</td>
<td>0.16</td>
<td>3</td>
<td>Mainline</td>
<td>65</td>
<td>2200</td>
<td>180</td>
</tr>
<tr>
<td>4</td>
<td>5008</td>
<td>4000055</td>
<td>0.45</td>
<td>3</td>
<td>Mainline</td>
<td>60</td>
<td>2200</td>
<td>180</td>
</tr>
<tr>
<td>7</td>
<td>4005008</td>
<td>5008</td>
<td>0.50</td>
<td>1</td>
<td>On-ramp</td>
<td>20</td>
<td>1800</td>
<td>180</td>
</tr>
</tbody>
</table>

**Empirical Test 1**

Data for test 1 was collected on Thursday, October 14, 2002 from 5:52 in the morning to 19:57 in the evening, roughly 14 hours. Comparison of model prediction and field observation is based on qualitative as well as quantitative measures.

Figure 10 shows the density vs. time curves of each link in test 1. The solid line is observed density and the dashed line is predicted density. The X axis is time of day and the Y axis is density in veh/mi/ln. There are two peaks on this day, one in the morning and the other in the afternoon. Both are originated from the downstream of node 4000055, probably due to insufficient capacity there. Since the real bottleneck is located outside of our test site, a dummy link is added at the downstream of node 4000055 and a time varying capacity is applied at the dummy link to simulate the effect of the real bottleneck which is located somewhere further downstream. If one views the plots in a reverse order from Part C to Part A, the impact of the peaks is decreasing and eventually disappears. As what the merge queuing model
predicts, there is almost no peak at the on-ramp link 4005008-5008 because its fair share of downstream supply is enough to meet its demand.
Figure 11 shows density contour for the traffic condition of the day based on density level of 45 veh/mi/ln. This density level is used in this study to delineate the boundary of congestion in time-space domain. The X axis represents location (nodes along the freeway mainline) and the Y axis represents time of day. There are two congested regions backing up from downstream node 4000055 towards upstream node 4000051. The solid line is the observed contour and the dashed line is the predicted contour. The figure qualitatively shows a good fit between the prediction and the observation. Quantitative comparison result will be presented later.
Figure 11 shows the frequency of modeling error, i.e. the residuals resulted after subtracting the observed values from the predicted values. As expected, residuals are densely concentrated around zero with the rest balanced at both sides – a bell-shaped distribution. There are 676 samples in total.
A simultaneous statistical test based on batch means technique proposed by Goldsman and Tokol (2000) is performed. There are two hypotheses in the test: (1) the mean of prediction error is not statistically different than 0 (i.e. the model is unbiased) and (2) the variance of prediction error is sufficiently small. The results show a lack of evidence that the mean of prediction error is statistically different than 0 at 95% level of confidence. The second hypothesis translates to a 95% confidence interval for percentage error of (-0.08, 0.04) × 100% if the sample variance is taken as small enough.

Empirical Test 2

The data for test 2 was collected on Friday, September 6, 2002 from 00:01 to 23:51, almost a whole day. Description of test 2 generally follows the format of test 1. To be concise, we only highlight the results and skip much of the discussion.

Figure 13 shows two major peaks, one in the morning and the other in the afternoon. Both are originated from the downstream of node 4000055.
Figure 14 indicates that the prediction matches the observation well in queue formation and dissipation, but less satisfactory in transition area between queued and unqueued traffic.
Figure 15 also confirms a good fit between the prediction and the observation.

Simultaneous statistical test shows that there is also lack of evidence that the mean of prediction error is statistically different than 0 at 95% level of confidence. The test suggests a 95% confidence interval of (-0.04, 0.06) × 100% for percentage prediction error.

CONCLUSION

In this paper, we proposed a CBWFQ model as well as its generalized form to model traffic merging behavior. We allocated the supply at the downstream branch of a merge among the upstream branches proportionally to their capacities. Capacity seems to be a good indicator to split the downstream supply because capacity incorporates factors such as number of lanes, per lane capacity, and traffic control strategies. Based on the proposed CBWFQ merge queuing model, we formulated a simplified kinematic wave model to deal with traffic operation at a merge bottleneck. This is a necessary step to enable network
applications of the simplified theory which was originally proposed to address freeway mainline only. Empirical test results support the validity of the CBWFQ model and show that the kinematic wave model at a merge bottleneck is accurate with narrow confidence interval.

REFERENCE


