The Capacity of Vehicular Ad Hoc Networks

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Abstract—This paper initiates the study of scaling laws for vehicular ad hoc networks (VANET)s. It provides a general framework to study VANETs. The fundamental capacity limits of these networks are investigated and we show how the road geometry affects this capacity. The VANET capacity is calculated for different road structures. The need for new capacity metrics is discussed. These results are obtained by combining geometrical analysis, network flow arguments, and probabilistic study of VANETs.

I. INTRODUCTION

Inter-vehicle communication (IVC) is an important component of the intelligent transportation system (ITS) architecture. IVC has attracted research attention from both academia and industry in the US, EU, and Japan. The most important feature of IVC is its ability to extend the horizon of drivers and on-board devices (e.g., radar or sensors) and, thus, to improve road traffic safety and efficiency. IVC will enable a wide range of novel applications such as accident avoidance messaging, congestion sensing, traffic metering, and general information services (e.g., Internet access). The allocation of 75 MHz in the 5.9 GHz band for dedicated short-range communications (DSRC) may also enable future delivery of rich media content to vehicles at short to medium ranges via both inter-vehicle and road-vehicle communication. Recently, ASTM Committee E17.51 endorsed a variant of the IEEE wireless LAN standard, denoted 802.11a Roadside Applications, as the platform for the DSRC link and data link layer [1]. Considerable efforts have been dedicated to road safety applications. One of the earliest studies on IVC was started by JSK (Association of Electronic Technology for Automobile Traffic and Driving) of Japan in the early 1980s. Later, research results have been demonstrated by California PATH [2] and Chauffeur of EU [3]. The cooperative driving systems of Japan in the late 1990s and 2000 (e.g., DEMO 2000 [4]) . The newly initiated European Project CarTALK 2000 [5] covers problems related to safe and comfortable driving based on IVC. CarTALK 2000 also co-operates with other projects like German FleetNet [6] for the development of IVC. The soon-to-be-launched Vehicle Infrastructure Integration (VII) [7], a major initiative at the United States Department of Transportation envisions that a future vehicle will be equipped with an On-Board Equipment (OBE) which consists of an On-Board Unit (OBU) which is essentially a transceiver, a GPS receiver, and a computer. The communication infrastructure will include Roadside Equipments (RSEs) that are deployed at strategic locations at the roadside. An RSE consists of a roadside unit (RSU) which is essentially a transceiver, and a computer. A number of papers have published results related to media access protocols for safety application. PATH project [8, 9] applies Wireless Token Ring Protocol (WTRP) for communicating between all vehicles. In DOLPHIN protocol [10] the vehicles broadcast emergency messages repetitively. Non-persistent and p-persistent CSMA are used on each repetition to increase the probability of reception. R-ALOHA [11] has been proposed to access and reserve slots in a framed medium based on TDMA. In Location-Based Channel Access protocol (LCA) [12], channels are allocated to the nodes involved in the communication based on their current location. The authors have proposed a variant of Enhanced Distributed Channel Access (EDCA) MAC protocol of IEEE802.11e [13, 14]. The protocol is applicable to both infrastructure and ad hoc MANETs and is compatible with the existing 802.11e standard. Multicasting and broadcasting protocols for IVC have been published in [15–17].
A. Motivation

Despite the increasing amount of research on VANETs, there is currently no rigorous mathematical framework to study the fundamental limits and scaling laws of VANETs. There exists a considerable literature on the fundamental limits of wireless networks and some papers address MANETs. However, our results show that VANETs have some unique characteristics and thus their scaling laws differ significantly from other wireless networks.

We observed that the road geometry plays a significant role in the fundamental properties of VANETs. As it will be seen, even a single isolated road (e.g. rural area) can potentially have every possible capacity scaling just based on its path geometry. Such a phenomenon was not observed in ordinary analysis of wireless networks. It is usually mentioned that as long as the deployment region has smooth boundaries the scaling laws for capacity do not change. This is because it is assumed that the deployment region has circular, rectangular or similar geometry. Thus, there is a need to categorize roads based on their geometric properties. An example of this categorization is the road sparseness condition which is discussed in Section III.

Another issue is that a unique mobility paradigm exists in VANETs. There is some interesting literature on the effects of mobility on the capacity of wireless networks [18–24]. In these analyses it is usually assumed that there is high delay tolerance, nodes have huge buffer sizes, and the network topology changes over time-scale of packet delivery. Indeed none of the assumptions hold in VANETs. For example, emergency and safety-related messages are extremely delay sensitive in VANETs. More importantly, a very common assumption in the literature is that the nodes move independently of each other. This assumption by no means holds in VANETs. In fact our results suggests unlike the existing literature, mobility does not improve capacity scaling in VANETs. This is in contrast with the previous conception on mobility and capacity.

Finally, new capacity and throughput metrics should be defined for VANETs. In the study of transport capacity it is usually assumed that each node has a random destination chosen uniformly from the available nodes in the network. In VANETs, different applications have different priorities. For example, in safety applications vehicles need to communicate and cooperate with vehicles that are in their vicinities. This significantly affects the scaling laws for throughput.

In this paper we initiate the study of VANET’s scaling laws focusing on the road geometry factor and the capacity metrics. We assume a VANET without infrastructure support (no roadside units are available) and adopt the protocol model introduced in [25]. The methodology can be extended to other models as well. The transport capacity of a VANET with \( n \) nodes is shown by \( \Lambda(n) \). In this paper we focus on traditional definitions of capacity. We have provided new capacity metrics and their calculations in [26]. It is assumed that each node has a communication radius \( r_L \).

II. Formulation and Preliminaries

To provide a rigorous analysis of VANETs, we provide some definitions. We show a lane on a road by a parameterized smooth continuous curve \( X_n(s) = (x_n(s), y_n(s)), s \in [0, 1] \) on the plane, see Figure 1. The length of each section of the curve is obtained using the Hausdorff one-dimensional measure [27]. The subscript \( n \) shows the number of vehicles on the road and can be dropped for simplicity, if it is clear from the context. The curve shows the trajectory of the road. \( X_n(0) \) is the beginning of the lane and \( X_n(1) \) shows the end. It is assumed a road can intersect itself only a finite number of times. Multi-lane roads are indicated by several parallel curves. A transportation network is usually consists of several roads. The geometry of the roads plays an important role in the performance of the corresponding VANET. It is assumed that the movements of vehicles on the roads follow a stationary stochastic process. In particular, the density, \( k_n(s) \), of a road is defined by the average number of vehicles per unit length at point \( X_n(s) \). At any part of the road, the density of cars is assumed to be a bounded positive number as in reality the density is limited by the physical size of cars. In this paper, for simplicity we assume \( k_n(s) = k \), for all \( s \in [0, 1] \) in all proofs. However, the results are easily extendable to the general case. For transportation networks consisting of several roads, the values of densities are chosen in a way that the flow conservation principle is satisfied at the intersections.

The mobility model for vehicles is an important factor in vehicular ad hoc networks. Note that in
VANETs, the vehicles do not move independently from each other. However, it has been observed that at any time $t$, the positions of vehicles can be modeled based on a Poisson process on the road, thus the spacing between them has exponential distribution [28–30]. In this paper we follow this assumption, however, it can be shown that the results hold for more general mobility models that satisfy some specific conditions. To define our model rigorously, we extend the lane $X_n(s)$ from both ends to infinity. Then, we place a Poisson point process with density $k$ on the extended curve. Any point of the process will correspond to a vehicle. At time $t = 0$, all vehicles on the same lane will choose a common speed $v \in [0, v_{max}]$ uniformly at random. It is assumed that $v_{max}$ is a fixed and bounded real number. It is assumed that the vehicles do not change their speed. Thus, at any time $t$, the positions of vehicles is still a Poisson process. Since we assume Poisson distribution and are interested in scaling laws, we can often combine parallel lanes to obtain one curve whose density is given by the summation of densities, i.e., $k = k_1 + k_2 + ... + k_l$ to simplify the analysis. However, it is important to note that this is possible only when we are providing macroscopic analysis, otherwise we need to consider each lane separately and account for the interactions between lanes. We assume $B(X, r)$ is the closed ball with radius $r$ centered at $X$ in $\mathbb{R}^2$. Also, $C(X, r)$ is the circle with radius $r$ centered at $X$.

We consider transportation networks that consist of $n$ cars equipped with OBEs. We are interested in the fundamental limits of these networks as $n$ grows large. Since the density of cars is a bounded positive number, to have a large number of nodes, the total lengths of the roads are assumed to be large, $L = \frac{N}{k} = \Theta(n)$. In particular, if the transportation system consists of only one road, we have

$$n = k \int_{s=0}^{s=1} \sqrt{(\frac{dx}{ds})^2 + (\frac{dy}{ds})^2} \, ds. \quad (1)$$

We assume $\sqrt{(\frac{dx}{ds})^2 + (\frac{dy}{ds})^2}$ is $\Theta(n)$, except for a countable number of values of $s$. By these assumptions, at any time $t$, the number $n(t)$ of vehicles on $X(s), s \in [0, 1]$ is $n(1 + o(1))$. In particular, for any $\epsilon > 0$, there exists a fixed number $\xi$ such that

$$\text{Prob}\{|n(t) - n| > \epsilon n\} < e^{-\xi n}. \quad (2)$$

We also make the assumption that the roads are not highly dense on the plane. We call this the sparseness condition. To make this rigorous, for any point $Y \in \mathbb{R}^2$, let $l(Y, r)$ be the Hausdorff one-dimensional measure (combined length) of the sections of the roads inside $B(Y, r)$. It is assumed that $l(Y, \alpha r_i) = O(r_i)$ for all $Y \in \mathbb{R}^2$ and any constant $\alpha > 0$. For any point $X$ on a road let $n(X)$ be the number of times that $C(X, r_i)$ intersects with the road curves. It is assumed that $n(X)$ is bounded. Let $A_{r_i}$ be the sections of the roads consisting of points with $n(X) > 2$. We say that the road system is sparse if the combined length of $A_{r_i}$ is $o(L)$. If $n(X) \leq 2$ for all points on the roads, then the system is said to be highly sparse. At any intersection, it is assumed that only a bounded number of roads can intersect with each other.

**III. RESULTS AND DISCUSSIONS**

Here we provide our results and discuss the implications. The detailed proofs are provided in Section IV. Generally, the results are obtained by combining three analytical tools. The first tool is network flow arguments. Since the roads are somewhat similar to the wired networks, it is expected that the results should be somewhat similar to network flow problems. The second important tool is the geometrical tools that capture the road structures. Finally, all the tools developed in the analysis of scaling laws of wireless networks such as the methods developed in [25] are
used. Before stating the results we need a simple lemma. This lemma is used in the proofs of the capacity results.

**Lemma 1.** Consider a transportation network that consists of $u$ single roads with lengths $l_1, l_2, ..., l_u$. Suppose that we divide the roads to sections of lengths $\beta r_t$, where $\beta$ is a constant. We can place these sections into a bounded finite number of non-interfering groups.

This result states that we can schedule parallel transmissions in the network as long as the transmissions belong to different groups. This is a standard method used to obtain lower bound on the capacity of wireless networks. The lemma can be proved using graph coloring.

**A. Transport capacity of Single Roads**

The simplest geometry is a single road, i.e., a road that is not connected to any other roads. This could be the case where we study the section of an interstate road passing through an unpopulated area. To consider the connectivity, define $\lim_{n \to \infty} \left( kr_t(n) \frac{ln n}{ln n} \right) = \alpha$.

It is easy to see that the VANET is connected with high probability if $\alpha > 1$, and it not connected with high probability if $\alpha < 1$. The capacity of single-road VANETs is characterized by the following theorem.

**Theorem 1.** Consider a single road $X(s), s \in [0, 1]$, with density $k$. Assume that the road is highly sparse. Then the transport capacity of the corresponding VANET is $\Theta(\frac{1}{\sqrt{n ln n}})$.

The theorem states that the transport capacity of a single-road VANET scales as $\Theta(\frac{1}{\sqrt{n ln n}})$. Note that this result is somewhat trivial, as one might say this is the capacity of a line network. Nevertheless, it is important to note that the road shape can significantly affect the capacity. Indeed, if the sparseness condition does not hold, the capacity of a single road can be as large as $\Theta(\frac{1}{\sqrt{n ln n}})!$ Figure 2 shows an example of such roads. Another point is that the mobility cannot improve capacity scaling. This is because the topology of the network changes much more slowly than the packet delivery rate and the acceptable delay. Nevertheless, these conclusions are based on the specific definitions and assumptions of transport capacity such as the fact that the destinations of nodes are chosen uniformly at random from the available nodes in the network. For example, the results are significantly different when other definitions of capacity are considered. The $\Theta(\frac{1}{\sqrt{n ln n}})$ capacity is significantly lower than the ordinary wireless networks [25]. The loss of transport capacity comes from the loss of area. Indeed, the cars are limited to be on the road curve.

**B. Transport capacity of Grid Transportation Networks**

We next turn our attention to a more interesting case in which our transportation network consists of several streets. In particular, we consider a case of a grid-like geometry that can appear in downtowns of cities, see Figure 3. Examples of this geometry are Manhattan, Anchorage, etc. To make the argument rigorous, we define a transportation grid of order $m$, as a set of $m$ parallel streets intersected with another set of $m$ parallel streets. We assume the two sets of roads are orthogonal to each other, see Figure 4. We refer to this structure as $Grid(m)$.

**Theorem 2.** Consider the grid $Grid(m)$:

- If the sparseness condition holds, then $m = O(\sqrt{\frac{n}{ln n}})$ and the transport capacity of the VANET is given by $\Theta(\frac{m}{ln n})$.
- If $m = \Omega(\sqrt{\frac{n}{ln n}})$, then the transport capacity of the VANET is $\Theta(\frac{1}{n ln n})$.

Theorem 2 states that the transport capacity increases as $m$ increases until, $m = \Theta(\sqrt{\frac{n}{ln n}})$. After this point the transport capacity is equivalent to that
of random ad hoc networks [25]. Figure 5 shows the transport capacity as the function of $m$.

**IV. ANALYSIS AND PROOFS**

Here we provide the proofs. For brevity, when we mainly focus on the new techniques that are used. If we use an idea that has been used in another theorem or a previous paper, we only highlight the general methodology. We need one definition before providing the proofs. The distance two curves $C_1$ and $C_2$, is defined as

$$D(C_1, C_2) = \inf \{ d(X, Y) : X \in C_1, Y \in C_2 \}. \quad (3)$$

**Proof of Lemma 1**

**Proof:** Construct the interference graph $G$ in the following way. Any section of any road will be a vertex in $G$. Two vertices in $G$ are connected to each other if the distance between the two corresponding sections is less than or equal to $r_l$. The requirements of sparseness condition guarantee that the maximum degree of $G$ is a finite bounded number. Thus the chromatic index of $G$ is a bounded number, too. We conclude that the vertices of $G$ can be colored using a finite number of colors so that no two vertices with the same color are adjacent. Each color group represents the sections of the road belonging to one of the non-interfering groups. The number of groups is bounded because the chromatic index is bounded.

![Fig. 5. Transport capacity of $Grid(m)$. The capacity increases linearly with $m$ until $m = \Theta(\sqrt{\ln n})$. After that the capacity remains constant.](image-url)

**Proof of Theorem 1**

**Proof:** Here, we are concerned with the conventional transport capacity as defined in [25]. By our assumption, any vehicles will have speed $v \in [0, v_{max}]$. Since $v_{max}$ is a fixed and bounded real number, in any time interval of length $\Delta t$, each vehicle can move at most $O(\Delta t)$. Thus, in any time interval $\Delta t = o(n)$, each vehicle will move $o(L)$, where $L$ is the total length of the road. That implies that the topology of the network changes much more slowly than the packet delivery rate. Each vehicle chooses a target vehicle on the $X(s), s \in [0, 1]$ at random and establishes a multi-hop communication as will be discussed. Note that if the target vehicles moves out of $X(s), s \in [0, 1]$, then the relative position will be obtained by the target vehicles.
but the first vehicle is still on the road, it will choose another target vehicle at random. Suppose that our road consists of \( l \) parallel lanes. Consider the lane with the highest speed. There are \( n \) (sender, receiver) pairs in the network. Since the target vehicles are chosen at random, for \( \frac{n}{l} \) pairs, both the sender and the target nodes are in the high speed lane. Moreover, for half of these pairs, the target nodes is located in the direction of the traffic from the source node. This means that any vehicle that becomes neighbor to the source node will never approach the target node. Moreover, the distance between the source and the target vehicles does not change in time. This means that, the capacity of the system cannot grow faster than the capacity of a static system with same parameters. Note that this discussion is based on the fact that the cars in the same lane do not change their speed and follow each other continuously. Nevertheless, this is a reasonable assumption to show that at least we cannot guarantee that mobility helps the capacity scaling. Now we are ready to prove that the capacity scales as \( \Theta \left( \frac{1}{n^2} \right) \).

Note that because we assume that the road is highly sparse, all transmissions have to occur along the road. That is, all transmissions consume at least \( O(r_t) \) length on the road. Also, two randomly chosen cars are \( O(L) \) away from each other. Thus we can use the method in [25] to obtain an upper bound on the achievable throughput. Specifically, if the throughput \( \lambda(n) \) is achieved, we need at least \( n \lambda(n) \frac{l}{r} \) concurrent transmissions. However, the number of concurrent transmissions is limited by \( O \left( \frac{n}{l} \right) \), thus \( \lambda(n) = O \left( \frac{l}{n} \right) \).

To show that \( \Theta \left( \frac{1}{n^2} \right) \) is achievable we provide a routing strategy. Since the topology of the network changes much more slowly than the packet delivery rate, we can have dynamic routing protocols, in which the routes need to be slowly adjusted as the vehicles move.

Choose \( r_t = \frac{2 \ln n}{k} \). Divide the road into sections of length \( \frac{r}{n} \). In each section, there exist at least one node. Divide the sections into a finite number of non-interfering groups. This is possible based on Lemma 1. Route the messages along the road through the sections. Each section has to support at most \( \Theta(n) \) routes, thus \( \lambda(n) = \Theta \left( \frac{1}{n} \right) \) is achievable.

**Proof of Theorem 2**

Suppose that the total length of the roads is \( L = 2ml \). First note that if \( m = O(1) \), then the result is trivial and can be shown similar to the proof of Theorem 1. Thus we assume \( m(n) = \omega(1) \). We now note that the sparseness condition implies that \( r_t(n) = o(\Delta) = o \left( \frac{1}{m(n)} \right) \) in Figure 4. This is because, if \( r_t(n) = \Theta \left( \frac{1}{m(n)} \right) \), then the combined length \( A_r \) of the sections of road contradicting the sparseness condition is \( O(L) \).

Thus

\[
r_t(n) = o \left( \frac{1}{m(n)} \right). \tag{4}
\]

Also it is easy to see that the connectivity of the network implies that

\[
r_t(n) = \Omega \left( \frac{ml}{n} \ln n \right). \tag{5}
\]

Combining Equations (4) and (5), we conclude

\[
m = o \left( \sqrt{\frac{n}{\ln n}} \right). \tag{6}
\]

We now show that the throughput \( O \left( \frac{m}{n} \right) \) is achievable. We provide a routing strategy and show that it achieves \( O \left( \frac{m}{n} \right) \) throughput. Again, since the topology of the network changes much more slowly than the packet delivery rate, we can have dynamic routing protocols, in which the routes need to be slowly adjusted as the vehicles move.

There are \( m \) horizontal and \( m \) vertical streets, Figure 4. Choose \( r_t(n) = \frac{2ml}{m} \ln n \). Each street is divided into sections of length \( \frac{r}{m} \). In the intersections, the sections consist of four parts of length \( \frac{r}{m} \). The sections are divided to a finite number of non-interfering groups using Lemma 1. The algorithm is as follows. Assume a car located at point \( X_s \) wants to transmit to a car located at \( X_d \). The information is transferred through the closest vertical street to \( X_s \), and the closest horizontal road to \( X_d \). The packets are transferred from each section to the neighboring sections until they reach the destination. To find the achievable capacity, we need to obtain the amount of traffic passed through each section. Indeed, it can be easily seen that the number of information paths traveling through each section is

\[
O \left( n \left( \frac{1}{m} + \frac{1}{m^2} \right) \right) = O \left( \frac{n}{m} \right). \tag{7}
\]

Since each section can communicate using the constant
V. CONCLUSION

In this paper we have calculated VANET’s capacity for both rural and urban areas while considering the road geometry. We have shown that the VANET capacity results differ significantly from the known capacity results obtained for MANETs. In particular, it is observed that the road geometry plays an important role in the capacity of VANETs. To the best of the author’s knowledge this is the first paper that analytically studies VANET’s capacity. This paper opens up an important direction for future research on understanding VANETs. Indeed, several simplifying assumptions regarding the mobility models, geometric properties, communication models, and capacity definitions are adopted in this paper. Future work will consider developing and analyzing more realistic models.

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