Electroweak Phase Transition, Scalar Dark Matter, & the LHC

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http://www.physics.wisc.edu/groups/particle-theory/

ANL Seminar, February 2012
Recent Work

- M. Gonderinger, Y. Li, H. Patel, & MRM, arXiv:1202.1316
- M. Buckley & MRM JHEP 1109 (2011) 094
- C. Wainwright, S. Profumo, MRM Phys Rev. D84 (2011) 023521
- M. Gonderinger, H. Lim, & MRM, arXiv:1202.1316
What role do scalar fields play (if any) in the physics of the early universe?
Scalar Fields in Particle Physics

Scalar fields are a simple

Scalar fields are theoretically problematic

$\Delta m^2 \sim \lambda \Lambda^2$

Fundamental scalars have yet to be observed
# Scalar Fields in Cosmology

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<tr>
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Scalar Fields & Cosmic History

EW Symmetry
Breaking: Higgs?

New Scalars?

Standard Model Universe

QCD:
$n+p \rightarrow n,p...$

QCD:
$q+g \rightarrow n,p...$

Astro: stars, galaxies,..
Scalar Fields & Inflation

EW Symmetry Breaking: Higgs?

New Scalars?

Standard Model Universe

• Flatness
• Isotropy
• Homogeneity

Scalar Field: Inflaton

QCD: \( n+p \rightarrow \text{nuclei} \)

Astro: stars, galaxies,..
Scalar Fields & Dark Energy

- $\Lambda$CDM
- Supernovae
- BAO

Cosmological constant $\Lambda$?

\[ \dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left( \rho + \frac{3p}{c^2} \right) + \frac{\Lambda c^2}{3} > 0 \]

Scalar Field: Quintessence?
Scalar Fields & the Origin of Matter


- BBN ($t \sim 100 \text{ s}$)
- CMB ($t \sim 10^5 \text{ y}$)

\[ Y_B = \frac{n_B}{s_\gamma} = (9.29 \pm 0.34) \times 10^{-11} \]
Scalar Fields & the Origin of Matter

Electroweak Phase Transition? New Scalars

Baryogenesis: When?
CPV? SUSY? Neutrinos?
Non-equilibrium dynamics or CPT violation?

- BBN (t ≈ 100 s)
- CMB (t ≈ 10^5 y)

\[ Y_B = \frac{n_B}{S_{\gamma}} = (9.29 \pm 0.34) \times 10^{-11} \]
Scalar Fields & the Origin of Matter

Electroweak Phase Transition ? New Scalars


• Rotation curves
• Lensing
• Bullet clusters


\[
Y_B = \frac{n_B}{s_\gamma} = (9.29 \pm 0.34) \times 10^{-11}
\]
Scalar Fields & the Origin of Matter


Electroweak Phase Transition? New Scalars: CDM?


• Rotation curves
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$$Y_B = \frac{n_B}{s_\gamma} = (9.29 \pm 0.34) \times 10^{-11}$$
## Scalar Fields in Cosmology

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Could experimental discovery of a fundamental scalar point to early universe scalar field dynamics?
Scalar Fields in Cosmology

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Focus of this talk, but perhaps part of larger role of scalar fields in early universe
Questions for this Talk:

• Was there a cosmic phase transition at $T \sim 100$ GeV?
• Did it have the characteristics needed for successful electroweak baryogenesis and/or production of gravity waves?
• Are its dynamics coupled to dark matter?
• What are the experimental signatures?
• How robust is the underlying theory?
Outline

I. EWPT: General Features

II. Three minimal extensions of the Standard Model scalar sector

III. How will we know it’s a scalar?

IV. Theoretical issues
I. General Features
Effective Potential

Tree level

\[ \mathcal{L} = (D_{\mu} \varphi)^\dagger (D^{\mu} \varphi) - V(\varphi) \]

Quantum corrections

\[ V(\varphi) \Rightarrow V_{\text{EFF}}(\varphi, T) \]

- \( T=0 \): Coleman-Weinberg (RG improved)
- \( T>0 \): Finite-\( T \) effective potential

* Many applications: Effective action
EW Phase Transition: New Scalars

Increasing $m_h$ → New scalars
**EW Phase Transition: New Scalars**

1st order  
2nd order

Increasing $m_h$  
New scalars

- Baryogenesis
- Gravity Waves
- Scalar DM
- LHC Searches
EW Phase Transition: New Scalars

"Strong" 1st order EWPT

Increasing $m_h$ → New scalars

Baryogenesis
Gravity Waves
Scalar DM
LHC Searches
**EW Phase Transition: New Scalars**

- **1st order**
- **2nd order**

Increasing $m_h$ leads to new scalars.

- **Baryogenesis**
- **Gravity Waves**
- **Scalar DM**
- **LHC Searches**

“Strong” 1st order EWPT

Bubble nucleation

EWSB
EW Phase Transition: New Scalars

1st order

2nd order

Increasing $m_h$

New scalars

Baryogenesis
Gravity Waves
Scalar DM
LHC Searches

“Strong” 1st order EWPT

Bubble nucleation

$Y_B$: CPV & EW sphalerons

EWSB
**EW Phase Transition: New Scalars**

- 1st order
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Increasing $m_h$

New scalars

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“Strong” 1st order EWPT

Bubble nucleation

$Y_B$ : diffuses into interiors

EWSB
**EW Phase Transition: New Scalars**

- **1st order**
- **2nd order**

- **Increasing** $m_h$

- **New scalars**

- **Baryogenesis**
- Gravity Waves
- Scalar DM
- LHC Searches

- **“Strong” 1st order EWPT**
  - Preserve $Y_B^{\text{initial}}$
  - Bubble nucleation

- **1st order EWPT**
  - Quench EW sph
  - $Y_B$ : diffuses into interiors

- **EWSB**
**EW Phase Transition: New Scalars**

- **Increasing** $m_h$
- **New scalars**

- **Baryogenesis**
- **Gravity Waves**
- **Scalar DM**
- **LHC Searches**

**1st order EWPT**

- **“Strong”**
- **1st order EWPT**
- **Bubble nucleation**

- **Preserve** $Y_B^{\text{initial}}$

**Quench**

- **$Y_B$ : diffuses into interiors**

**EWSB**

- **$T_C$**, $E_{\text{sph}}$, $S_{\text{tunnel}}$

- **$F(\phi)$**
**EW Phase Transition: New Scalars**

- **1st order EWPT**
  - Detonation & turbulence
- **2nd order**
  - Bubble nucleation

Increasing $m_h$ → New scalars

- Baryogenesis
- Gravity Waves
- Scalar DM
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**“Strong” 1st order EWPT**
**EW Phase Transition: New Scalars**

Increasing $m_h$  

New scalars

- Baryogenesis
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“Strong”  

1st order EWPT

Detonation & turbulence  

Bubble nucleation

GW Spectra: $\Delta Q$ & $\Delta t_{EW}$

$F(\phi)$
**EW Phase Transition: New Scalars**

- **1st order**
- **2nd order**

**“Strong” 1st order EWPT**
- Detonation & turbulence
- Bubble nucleation

**GW Spectra:**
- $\Delta Q$ & $\Delta t_{EW}$

**EWSB**

**nMSSM**

**Caprini et al.**

- $h_s^2 \Omega$
- $f / \text{Hz}$

- LISA
- BBO

- $\Delta Q$
- $\Delta t_{EW}$
EW Phase Transition: New Scalars

1st order

2nd order

Increasing $m_h$

New scalars

Baryogenesis
Gravity Waves
Scalar DM?
LHC Searches?

“Strong” 1st order EWPT

Y_B preservation, detonation & turbulence
Bubble nucleation

EWSB

$T_C, E_{sph}, S_{tunnel}$
$\Delta Q & \Delta t_{EW}$

$F(\phi)$
Higgs Boson Searches

SM Higgs?
SM Higgs?

- LHC Indications
- LHC Exclusion
- Tevatron Excl
- LEP Exclusion
- Non-SM Higgs(es)?
- Precision Tests
- SM Higgs properties?
Dark Matter: $\Omega_\chi$ & $\sigma^{SI}$

Thermal DM: WIMP

Direct detection: Spin-indep DM-nucleus scattering
## II. Simplest Scalar Extensions

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May be low-energy remnants of UV complete theory & illustrative of generic features
The Simplest Extension

Simplest extension of the SM scalar sector: add one real scalar S (SM singlet)

\[ V_{HS} = \frac{a_1}{2} \left( H^\dagger H \right) S + \frac{a_2}{2} \left( H^\dagger H \right) S^2 \]

- EWPT: \( a_{1,2} \neq 0 \) & \( \langle S \rangle \neq 0 \)
- DM: \( a_1 = 0 \) & \( \langle S \rangle = 0 \)

O’Connel, R-M, Wise; Profumo, R-M, Shaugnessy; Barger, Langacker, McCaskey, R-M Shaugnessy; He, Li, Li, Tandean, Tsai; Petraki & Kusenko; Gonderinger, Li, Patel, R-M; Cline, Laporte, Yamashita; Ham, Jeong, Oh; Espinosa, Quiros; Konstandin & Ashoorioon…
The Simplest Extension

Independent Parameters:
\( v_0, x_0, \lambda_0, a_1, a_2, b_3, b_4 \)

H-S Mixing:
\[ H_1 \leftrightarrow H_2 \]

Mass matrix:
\[ M_2^2 = \mu_h^2 + \mu_h s^2 + \mu_s^2 \]

Stable S (dark matter?)

Tree-level \( Z_2 \) symmetry: \( a_1 = b_3 = 0 \) to prevent \( s-h \) mixing and one-loop \( s \rightarrow h h \)

\( x_0 = 0 \) to prevent \( h-s \) mixing

EWPT Scenario

\[ V_{HS} = \frac{a_1}{2} (H^\dagger H) S + \frac{a_2}{2} (H^\dagger H) S^2 \]

- Raise barrier
- Lower \( T_C \): \( a_2 < 0 \)

Signal Reduction Factor

Production

Decay

EWPT Scenario

Raise barrier

Lower \( T_C \): \( a_2 < 0 \)
Finite Temperature Potential

Multiple fields & new interactions: novel patterns of symmetry breaking, lower $T_C$, greater super-cooling, “stronger 1st order EWPT”
The Simplest Extension

Low energy phenomenology

\[ V_{HS} = \frac{a_1}{2} \left( H^\dagger H \right) S + \frac{a_2}{2} \left( H^\dagger H \right) S^2 \]

- Raise barrier
- Lower \( T_c \)

- Mixing
- Modified BRs

Two mixed (singlet-doublet) states with reduced SM branching ratios
EWPT & LHC Phenomenology

Signatures

Light: all models
Black: LEP allowed

Signal Reduction Factor

Production

Decay

Profumo, R-M, Shaugnessy
EWPT & LHC Phenomenology

Signatures

- LHC exotic final states: 4b-jets, $\gamma + 2$ b-jets…
- LHC: reduced $BR(h \rightarrow SM)$

$m_2 > 2 m_1$

$m_1 > 2 m_2$

Signal Reduction Factor

$\xi^2_i = V_{ij}^2 \frac{BF(H_j \rightarrow X_{SM})}{BF(h_{SM} \rightarrow X_{SM})}$

Light: all models
Black: LEP allowed

Profumo, R-M, Shaugnessy
**EWPT & LHC Phenomenology**

**Signatures**

- LHC exotic final states: 4b-jets, $\gamma + 2$ b-jets...
- LHC: reduced $\text{BR}(h \rightarrow \text{SM})$

**Scan: EWPT-viable model parameters**

- $m_2 > 2m_1$
- $m_1 > 2m_2$

**Signal Reduction Factor**

$$\xi_i^2 = V_{ij}^2 \frac{\text{BF}(H_j \rightarrow X_{SM})}{\text{BF}(h_{SM} \rightarrow X_{SM})}$$

**Profumo, R-M, Shaugnessy**
**EWPT & LHC Phenomenology**

**Signatures**

- **LHC exotic final states:** $4b$-jets, $\gamma + 2\ b$-jets...
- **LHC: reduced BR($h \rightarrow SM$)**

**EWPO compatible**

- $m_2 > 2m_1$
- $m_1 > 2m_2$

**Signal Reduction Factor**

$$\xi_i^2 = \frac{V_{ij}^2 \text{BF}(H_j \rightarrow X_{SM})}{\text{BF}(h_{SM} \rightarrow X_{SM})}$$

**Production**

**Decay**

Profumo, R-M, Shaugnessy
EWPT & LHC Phenomenology

Signatures

LHC exotic final states: $4b$-jets, $\gamma$ + 2 $b$-jets...

LHC: reduced $BR(h \rightarrow SM)$

EWPO compatible

$m_2 > 2m_1$

$m_1 > 2m_2$

SM-like
Singlet-like

Barger, Langacker, McCaskey, R-M, Shaugnessy
The Simplest Extension

DM Scenario

\[ V_{HS} = + \frac{a_2}{2} \left( H^\dagger H \right) S^2 \]

\[ \Omega_{DM} & \sigma_{SI} \]

\[ + \ldots \]
DM Phenomenology

Relic Density

He, Li, Li, Tandean, Tsai

Direct Detection

Barger, Langacker, McCaskey, R-M, Shaugnessy
Vacuum Stability & Perturbativity

Preserving EW Min

“Funnel plot”

\[ V_{\text{EFF}} \]

EW vacuum

top loops

naïve stability scale \( \Lambda \)

\[ \beta_{\lambda} = \frac{1}{16\pi^2} \left( 4\lambda^2 + 12a_2^2 - 36y_t^4 + 12\lambda y_t^2 - 9\lambda g^2 - 3\lambda g'^2 + \frac{9}{4}g'^4 + \frac{9}{2}g^2g'^2 + \frac{27}{4}g^4 \right) \]

DM-H coupling

top loops

Gonderinger, Li, Patel, R-M
Vacuum Stability & Perturbativity

Preserving EW Min

- $V_{\text{EFF}}$
- EW vacuum
- top loops
- naive stability scale $\Lambda$

$m_{S^2} = b_2 + a_2 v^2$

Gonderinger, Li, Patel, R-M

“Funnel plot”

- perturbativity
- $m_H$?

No DM: $Z_2$-breaking vac
LHC & Higgs Phenomenology

Invisible decays

LHC discovery potential

Signal Reduction Factor

\[ \xi_i^2 = \frac{V_{ij}^2 \text{BF}(H_j \rightarrow X_{SM})}{\text{BF}(h_{SM} \rightarrow X_{SM})} \]

Production

Decay

Dijet azimuthal distribution

Look for azimuthal shape change of primary jets (Eboli & Zeppenfeld ‘00)
LHC & Higgs Phenomenology

Invisible decays

He, Li, Li, Tandean, Tsai

LHC discovery potential

Signal Reduction Factor

Production

Decay

\[ \xi^2 = \frac{V_{ij}^2 \text{BF}(H_j \rightarrow X_{SM})}{\text{BF}(h_{SM} \rightarrow X_{SM})} \]

Invis search

CMS 30 fb⁻¹

ATLAS, CMS @ 30 fb⁻¹

\( \Gamma (h \rightarrow SS) = 0 \)
Complex Singlet: EWB & DM?

Barger, Langacker, McCaskey, R-M Shaugnessy

Spontaneously & softly broken global $U(1)$

\[
V_{HS} = \delta_2 \frac{1}{2} H^\dagger H |\tilde{S}|^2 = \delta_2 \frac{1}{2} H^\dagger H (S^2 + A^2)
\]

Controls $\Omega_{CDM}$, $T_C$, & H-S mixing

\[
V_{\tilde{S}} = \frac{b_2}{2} |\tilde{S}|^2 + \frac{b_1}{2} \tilde{S}^2 + \text{c.c.} + \cdots
\]

Gives non-zero $M_A$
Complex Singlet: EWB & DM

Barger, Langacker, McCaskey, R-M, Shaugnessy

$\delta_2$ controls $\Omega_{CDM}$ & EWPT

decreasing $T_C$

$M_{H1} = 120$ GeV, $M_{H2}=250$ GeV, $x_0=100$ GeV
Complex Singlet: Direct Detection

Two component case ($x_0=0$)

Little sensitivity of scaled $\sigma_{SI}$ to $\delta_2$
SM Higgs?

SM Branching Ratios?

Three scalars: $h_{1,2}$ (Higgs-like)
$D$ (dark matter)

LHC: WBF “Invisible decay” search
SM Higgs?

Three scalars: $h_{1,2}$ (Higgs-like)
$D$ (dark matter)

Viable Dark Matter?

SM Branching Ratios?

LHC: WBF “Invisible decay” search

WIMP-Nucleus Scattering

Gonderinger, Lim, R-M

2nd light state

- $\Omega_{TH} < \Omega_{WMAP}$
- $\sigma < \sigma_{Xenon}$
- $Br(inv) > 0.6$
Real Triplet

\[ \Sigma^0, \Sigma^+, \Sigma^- \sim (1, 3, 0) \]

\[ V_{H \Sigma} = \frac{a_1}{2} H^\dagger \Sigma H + \frac{a_2}{2} H^\dagger H \text{ Tr } \Sigma^2 \]

**EWPT:** \( a_{1,2} \neq 0 \) \& \( \langle \Sigma^0 \rangle \neq 0 \)

**DM \& EWPT:** \( a_1 = 0 \) \& \( \langle \Sigma^0 \rangle = 0 \)

**Small:** \( \rho \)-param

Multi-step EWSB transition:

- Step 1: quench sphalerons
- Step 2: move to EW/DM vac
Real Triplet: DM Search

**Basic signature:**

\[ x_0 = 0 : H^\pm \to H_2 \pi^\pm \]

**Charged track disappearing after ~ 5 cm**

\[ q\bar{q} \to W^{\pm*} \to H^\pm H_2 \quad q\bar{q} \to Z^*, \gamma^* \to H^+ H^- \]

**Trigger:** Monojet (ISR) + large \( \not{E}_T \)

**SM Background:**

QCD jZ and jW w/ Z \( \to \nu\nu \) & W \( \to l\nu \)
Real Triplet : DM Search

**Basic signature:**

\[ x_0 = 0 : H^\pm \rightarrow H_2 \pi^\pm \]

**Charged track disappearing after \( \sim 5 \) cm**

\[ q\bar{q} \rightarrow W^{\pm*} \rightarrow H^\pm H_2 \quad q\bar{q} \rightarrow Z^*\gamma^* \rightarrow H^+H^- \]

**Trigger:** Monojet (ISR) + large \( \not{E}_T \)

**SM Background:**

QCD \( jZ \) and \( jW \) w/ 
\( Z \rightarrow \nu\nu \) & \( W \rightarrow l\nu \)

**Cuts:**

large \( \not{E}_T \)

hard jet

One 5cm track
Real Triplet : DM Search

Basic signature: \[ x_0 = 0 : H^\pm \rightarrow H_2 \pi^\pm \]

Charged track disappearing after \(~5\) cm

\[ q\bar{q} \rightarrow W^\pm \rightarrow H^\pm H \quad q\bar{q} \rightarrow Z^* , \gamma^* \rightarrow H^+ H^- \]

Cirelli et al:

\[ M_\Sigma = 500 \text{ GeV:} \]

\[ \Omega_\Sigma / \Omega_{CDM} \sim 0.1 \]
SM Higgs?

SM Branching Ratios?

Four scalars: $h_1$ (Higgs-like), $\Sigma^0$ (dark matter), $\Sigma^+, \Sigma^-$ (new states)

Fileviez-Perez, Patel, R-M, Wang
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Mixed Higgs-like states and/or modified BRs: signal reduction $\xi$, invisible search…
**III. Is it a Scalar?**

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Could be fermionic triplet [e.g., Buckley, Randall, Shuve, JHEP 1105 (2011) 97]. How to distinguish?
Di-Jet Correlations

Buckley, R-M JHEP 1109 (2011) 094

R-hadrons from gg fusion

Fermions

Scalars
Di-Jet Correlations

\[ \mathcal{M}_1 \rightarrow J_1 \]
\[ \mathcal{M}_{\text{pair}} \rightarrow V \rightarrow p_1 \]
\[ \mathcal{M}_2 \leftarrow J_2 \]

**Scalars:**
\[ A_3 \propto |\mathcal{M}_{\text{pair}}|^2 > 0 \]

**Fermions:**
\[ A_3 < 0 \]
- Abelian
- Non-abelian
  
  for large \( \Delta \eta \)

\[ \frac{d\sigma}{d\Delta \phi} = A_0 + A_1 \cos \Delta \phi + A_3 \cos 2\Delta \phi \]

\[ = (PS)\mathcal{M}_1(+1)\mathcal{M}_1(-1)^*\mathcal{M}_2(-1)\mathcal{M}_2(+1)^* \times \mathcal{M}_{\text{pair}}(+1,-1)\mathcal{M}_{\text{pair}}(-1,+1)^* + (1 \leftrightarrow -1) \]
IV. Theoretical Issues

Gauge-dependence in $V_{\text{EFF}}(\phi, T)$

$V_{\text{EFF}}(\phi, T) \rightarrow V_{\text{EFF}}(\phi, T; \xi)$

Ongoing research: approaches for carrying out tractable, GI computations

- C. Wainwright, S. Profumo, MRM Phys Rev. D84 (2011) 023521
- H. Gonderinger, H. Lim, & MRM, arXiv:1202.1316
Origin of Gauge Dependence

Effective Action

\[ \Gamma[\phi_{c1}(x)] = W[j] - \int d^4x \, j(x)\phi_{c1}(x) \]

\[ W[j] = -i \ln Z[j] \]

\[ Z[j] = \int \mathcal{D}\phi \, \mathcal{D}A \, \mathcal{D}\eta \, \mathcal{D}\eta^\dagger \, e^{i \int d^d x [\mathcal{L}(x;\eta,\xi)]} \]

Effective Potential

\[ \phi_{c1}(x) \rightarrow \phi_{c1} \quad \Gamma(\phi_{c1}) = -(\text{vol}) \, V_{\text{eff}}(\phi_{c1}) \]
**Nielsen Identities**

*Identity:*

\[
\frac{\partial \Gamma}{\partial \xi} = \int d^d x \, d^d y \left[ C(\phi, A; \, x, y) \frac{\delta \Gamma}{\delta \phi(x)} + E^a_\mu(\phi, A; \, x, y) \frac{\delta \Gamma}{\delta A^a_\mu(x)} \right]
\]

*Extremal configurations:*

\[
\frac{\delta \Gamma}{\delta \phi(x)} = \frac{\delta \Gamma}{\delta A^a_\mu(x)} = 0 \quad \rightarrow \quad \frac{\partial \Gamma}{\partial \xi} = 0
\]

*Effective potential:*

\[
\phi \rightarrow \phi_{\text{min}}(\xi) \quad \rightarrow \quad \frac{\partial V_{\text{eff}}}{\partial \xi} = -\tilde{C}(\phi, \xi) \frac{\partial V_{\text{eff}}}{\partial \phi} = 0
\]
Baryon Number Preservation

\[ S \equiv \frac{\rho_B(\Delta t_{EW})}{\rho_B(0)} > e^{-N} \]

Washout factor 

\[ \zeta = F(\varphi) \] \[ \zeta \equiv \frac{\hat{E}_{\text{sph}}}{T} \bigg|_{T=T_C} \]

Two qtns of interest:

- \( T_C \) from \( V_{\text{eff}} \)
- \( E_{\text{sph}} \) from \( \Gamma_{\text{eff}} \)
Baryon Number Preservation: Pert Theory

\[ S \equiv \frac{\rho_B(\Delta t_{EW})}{\rho_B(0)} > e^{-N} \]

\[ \zeta = F(\varphi) \]

Conventional treatments

\[ \frac{\varphi(T_C)}{T_C} \geq 1 \]

Gauge Dep

- GI \( T_C \) from hbar exp, \( V_{\text{eff}}(\phi^+\phi) \), or Hamiltonian formulation
- Use GI scale in \( E_{\text{sph}} \) computation

“Baryon number preservation criterion” (BNPC)

H. Patel & MRM, JHEP 1107 (2011) 029
Nielsen Identities: Application to $T_C$

Critical Temperature

\[ V_{\text{eff}}(\phi_{\text{min}}, T_C) = V_{\text{eff}}(0, T_C) \]

Apply consistently order-by-order in $\hbar$

\[ V_{\text{eff}}(\phi, T) = V_0(\phi) + \hbar V_1(\phi, T) + \hbar^2 V_2(\phi, T) + \ldots \]

\[ \phi_{\text{min}} = \phi_0 + \hbar \phi_1(T, \xi) + \hbar^2 \phi_2(T, \xi) + \ldots \]

Implement minimization order-by-order (defines $\phi_n$)

\[ V_{\text{eff}}[\phi_{\text{min}}(T), T] = V_0(\phi_0) + \hbar V_1(\phi_0, T) \]

\[ + \hbar^2 \left[ V_2(\phi_0, T, \xi) - \frac{1}{2} \phi_1(T, \xi) \frac{\partial^2 V_0}{\partial \phi^2} |_{\phi_0} \right] + O(\hbar^3) \]
Obtaining a GI $T_C$

Track evolution of minima with $T$ using $\hat{h}_v$ expansion

Track evolution of different minima with $T$ using

$$V_{\text{eff}}[\phi_{\text{min}}(T), T] = V_0[\phi_{\text{min}}(T)] + \hat{h}V_1[\phi_{\text{min}}(T), T]$$

Illustrative results in SM:

$$V_{\text{eff}}(\phi_{\text{min}}(T), T) = V_0(\phi) + \hat{h}V_1(\phi, T)$$

Full $\phi$

$$V_{\text{eff}}[\phi_{\text{min}}(T), T] = V_0(\phi_0) + \hat{h}V_1(\phi_0, T)$$
Gravity Waves from EWPT: Pert Theory

Abelian Higgs Model

C. Wainwright, S. Profumo, R-M
Phys Rev. D84 (2011) 023521
Vacuum Stability & Gauge Dependence

Complex Singlet Model

Possible runaway direction for $\delta_2 < 0$ (EWPT)

GI Stability Condition

\[
\delta_2^2 (\mu) < \lambda (\mu) d_2 (\mu) \\
\lambda (\mu) > 0 \\
d_2 (\mu) > 0
\]

$\forall \mu < \Lambda$

Use $\beta$-fns

$M. \text{Gonderinger, H. Lim, M. R-M}$

Conclusions

• Cosmology loves scalar fields (inflation): although no fundamental scalar yet observed, perhaps scalar fields can address a number of questions about the early universe

• Simple extensions of the SM scalar sector and lead to observable ($\sigma_{SI}$, LHC) particle dark matter and/or EWPT with interesting implications: baryogenesis, GW, new Higgs states/modified Higgs properties….

• Perhaps observation of these scalars will point to a richer structure for scalar fields in the early universe involving both visible and dark physics