CAN WE RULE OUT THE POSSIBILITY THAT THERE IS MORE THAN ONE DIRECTION IN TIME?

AN EXPLORATION OF ALTERNATE MODELS

by
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ABSTRACT

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The literature was examined to see if a proposal for the existence of additional time-like axes could be falsified. Some naïve large-time models are visualized and examined from the standpoint of simplicity and adequacy. Although the conclusion here is that the existence of additional time-like dimensions has not been ruled out, the idea is probably not falsifiable, as was first assumed. The possibility of multiple times has been dropped from active consideration by theorists less because of negative evidence than because it can lead to cumbersome models that introduce causality problems without providing any advantages over other models. Even though modern philosophers have made intriguing suggestions concerning the causality issue, usability and usefulness issues remain. Symmetry considerations from special relativity and from the conservation laws are used to identify restrictions on large-time models. In view of recent experimental findings, further research into Cramer's transactional model is indicated.

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TABLE OF CONTENTS

I. Introduction p. 6

II. History of the Question

- A. Previous Mental Models and Cultural Interpretations p. 8
- B. Previous Findings of Modern Physical Theorists p. 8
- C. Some Findings of Cognitive Science: How We Perceive p. 17

III. New Ideas and Questions

- A. Symmetry Considerations p. 18
- B. Concerning Our Perceptions of Time p. 33
- C. Reinterpreting the Meaning of the Ensemble p. 41
- D. Reinterpreting the Meaning of the Quantum *i* p. 51
- E. Philosophical Considerations: Implications for Free Will p. 63

IV. Conclusions

- A. Summary p. 67
- B. Suggestions for Future Research p. 69

Appendices

- A. Difficulty in Defining a Conservation Law Through Space p. 72
- B. Inversions Equal Rotations p. 74
- C. Einstein Tensor p. 76
- D. The Ensemble and Interference: A Visual Representation p. 78
- E. 1 + 1 Feynman Checkerboard p. 86

Endnotes p. 88

References p. 91

TABLE OF FIGURES

Figure 1. Complex plane p. 9

Figure 2. Space invariant (animated) p. 22

Figure 3. Minkowski spacetime invariant p. 25

Figure 4. Translations (4a animated) p. 26

Figure 5. Rotations (5a animated) p. 27

Figures 6-7. Increase of universe with time p. 35

Figure 8a-c. Expansion of the universe p. 37

Figure 9. Single time direction p. 39

Figure 10. Electron cloud p. 42

Figures 11a-b. One time, two times p. 45

Figure 12. Two times, multiple 3-d spaces p. 46

Figures 13-15. Three-dimensional time p. 47

Figure 16. Density of the electron cloud (animated) p. 48

Figure 17. Cross section of Fig. 14 (electron cloud in three-D time) p. 50

Figures 18-19. Pattern of light through slits p. 52

Figures 20-21. $e^{-i\omega t}$ part of the wave function (animated) p. 55

Figures 22-23. Advanced wave $e^{+i\omega t}$ (animated) p. 60

Figures 24-25. Adding $e^{+i\omega t}$ and $e^{-i\omega t}$ (animated) p. 61

Figures B1-B4. Inversions equal rotations p. 74

Figures D1-D3. Wave interference (D-3 animated) p. 78

Figures D4-D8. Particle wave interference p. 80

Figure D9. Probability wave p. 83

I. INTRODUCTION

Why do we perceive time moving forward? Why can we not jump ahead a few minutes just by willing it, or step back and undo a mistake just made? If Einstein is correct that space and time are part of the single indivisible entity known as spacetime, why is it that we can affect at will (within certain limits) our change of position in space but not our change of position in time? These questions have motivated speculation among mystics, philosophers, and those men and women of science who delve into the fundamental quantities of our physical universe, into the very foundations of space and time. The questions have also provided the motivation for the present work.

The provocative suggestion has been made in a number of contexts that there are additional directions in time parallel to our own. It has been asked (Tegmark, 2003) whether there could be another "you" closer than your own skin. To be an armchair philosopher or a science fiction writer and ponder these matters is one thing, but to do so from the perspective of the rigors of science is quite another. Those who have written most persuasively (but not at all uniformly) on the subject by and large are people who possess great intellects and great education. Their minds have been honed for years on General Relativity, Quantum Theory, particle physics, and unified theories such as strings and loop quantum gravity. They are often conversant in wide swaths of mathematics including tensors, cohomology, and higher-order symmetries. Such knowledge takes far more than the work of a lone Master's degree to acquire; typically, the knowledge base of such thinkers spans disciplines and has been informed through the work of multiple degrees. Therefore, an organizing question for the present study has been to ask what chunk of all of this could be broken off and digested within the time allowed. The answer is that it has proven possible to review many of the results and conclusions of the philosopher-scientists of the 20^{th} century. It has also been possible to apply these findings to the initial set of questions, to clarify and focus the questions, to develop suggestions for future inquiry, and to obtain some idea of the kinds of answers it might (and might not) be possible to find.

We begin by noting cultural influences on our current model of time. Then we take a quick tour through the foundational physics of the 20^{th} century with some mathematics and cognitive science thrown in. Part II closes with a summary of all findings that could have the power to rule in or out any aspect of our central question. In Part III, we take the most provocative results and engage in some further speculation. We ask what is required to define a new direction as time-like and examine this question from the standpoint of symmetry considerations. We use mathematics and visualization to evaluate the usefulness of a few simple large-time models. We take a look at implications for the existence of freewill and note some room for expanded possibilities for its exercise. We conclude in Part IV with a discussion of the validity—and the importance—of posing questions that may not be falsifiable, citing reasoning from Feynman to support our stance, and make some proposals for future inquiry.

A question underlying the entire paper is what ideas about time, if any, we can rule out.

II. HISTORY OF THE QUESTION

A. Previous Mental Models and Cultural Influences

There is plenty of anecdotal evidence in the literature suggesting that human individuals and human cultures have had, and in some cases continue to have, a wide variety of ways in which they have experienced time. The concept of cyclical time has been noted in the ancient Greek and Hindu cultures, in the Babylonian, Chinese, Aztec, Mayan, and Norse cyclical calendars, and in the cosmologies of aboriginal Africans, Australians, and Americans {endnote 1}.

In what is generally known as the Western worldview, we hold strongly to the concept of time as an arrow. We see ourselves evolving, growing "toward," whether toward heaven, toward evolutionary change, or simply toward a death. This viewpoint is strongly supported by our sciences. It might be worthwhile to ask ourselves if it is objective research that has shaped our viewpoint or if the viewpoint has shaped—and perhaps restricted--our scientific thought.

B. Previous Findings of Modern Physical Theorists

1. From the founders of quantum theory and relativity

In the early part of the 20th century, as a result of the work done by Einstein, Bohr, Heisenberg, Schrödinger and others, it became clear that mathematics could describe the results of experiments on light quanta and other elementary particles only by employing complex quantities {endnote 2}. These quantities are normally graphed by using a plane that extends at right angles to our usual 3 + 1 spacetime, the *complex plane* (Fig. 1).

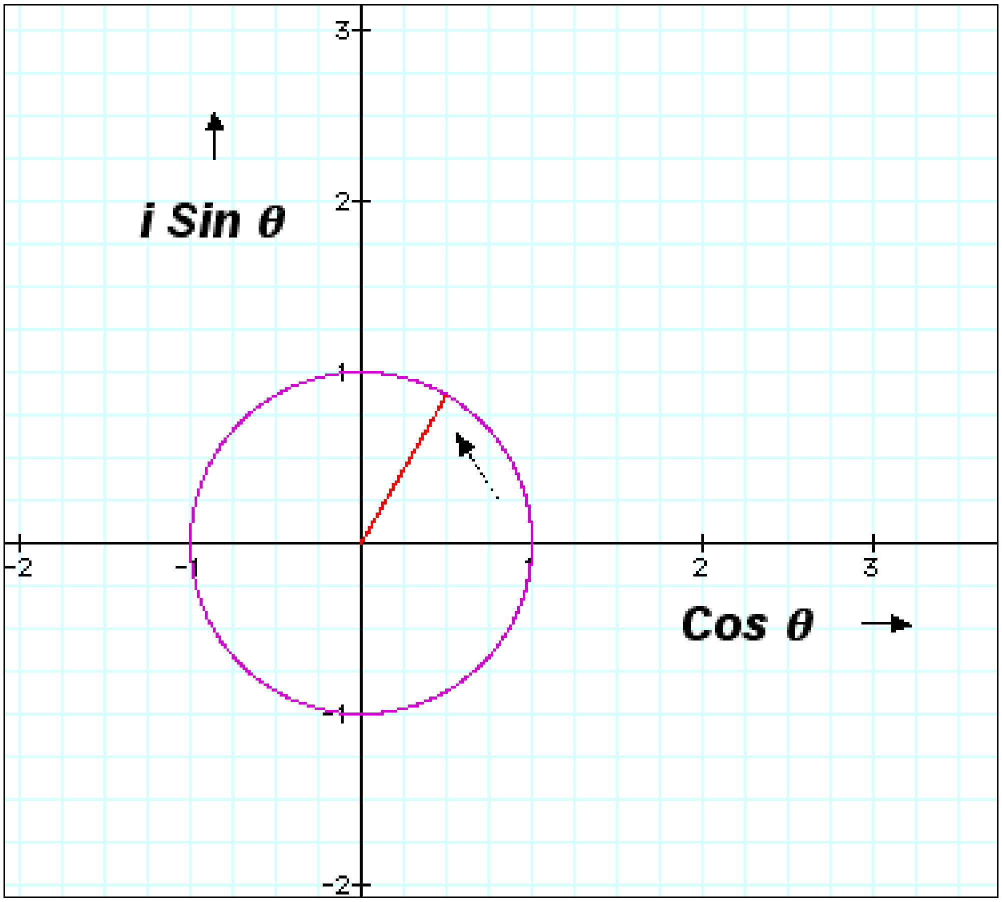


Figure 1. Complex plane. Our usual 3-D space is represented by the horizontal line. The vertical line represents the imaginary numbers.

An assumption made by Bohr (1928) was that when equations depend on imaginary quantities it is an indication that the relation of these equations to reality is a symbolic one. Pauli (1928) equated many-dimensional space with fictitious space, and the common assumption among physicists is that the complex plane cannot exist in physical terms; that is, that it cannot contain energy. But what Bohr actually said was that when the number of dimensions employed in a theory is greater than the number of dimensions of ordinary 3 + 1 spacetime, there can be no immediate connection of that theoretical construction with our ordinary perceptions. It may at first appear that he was assuming that a three-dimensional space is what is actually "out there," but his meaning was more subtle. He used the term *ordinary perceptions*, not *ordinary reality,* and remained consistent with his long-established practice of not trying to speak about what is "really" out there. In fact, he left the question of the number of spacetime dimensions open. He and Rosenfeld (1933), writing only five years after Bohr had written the opinion above, made it clear that they believed the determination of the number of spacetime dimensions would be obtained only by the inclusion into Quantum Theory of the electric charge and the rest masses of the elementary particles. Their passage presages the development of string theories in the 1980's, which do include charge and mass but require many space dimensions in order to do so. (See Section 3a, p. 15.)

Almost from the very beginning of their work, these scientists realized that there was a problem with the treatment of time in their theories. In relativity, there is no sharp notion concerning the simultaneity of events; different observers can disagree on which of two events occurred first. But the ability to order all events is crucial to the formulation of quantum mechanics. Schrödinger (1935) put it this way:

> That 'sharp time' is an anomaly in Q. M. and that besides, . . . the special treatment of time forms a serious hindrance to adapting Q. M. to the relativity principle, is something that in recent years I have brought up again and again, unfortunately without being able to make the shadow of a useful counterproposal. (p. 166)

In the years since these writings by the founders of quantum mechanics, the needs for extra dimensions and for mutually exclusive notions of time to account for all the phenomena that we have observed have been dealt with in a variety of ways. Following is a summary of those attempts that have implications for our current inquiry.

2. Interpretations of the wave function

a. Many worlds. Everett (1957) showed that quantum theory is mathematically consistent with the assumption that every possibility takes place, with multiple copies of our universe branching off at every quantum event—*if* the assumption is also made that those branches cannot interact in the future. It was felt that this assumption must be made because if a particle is recorded at point A, then its effects from point B will never be observed (Bohr, 1949). This interpretation of the formalism obviously places heavy emphasis on time as an arrow. More recently, other theorists have proposed multiple pasts and shown that their existence is also consistent with the mathematics (Laudisa, 2000; Gell-Mann and Hartle, 1990, 1994; Hartle, 2001). This approach says that the effects from B *are* observed if we go far enough into the past or the future, but that they are, at that point in spacetime, consistent with the effects from all of the other histories or futures that intersect there. The different mathematical formulations lead to identical results.

b. Sum over histories. Feynman (1963, 1964, 1965) addressed a slightly different problem with his sum over histories approach. Instead of looking at all the possible points where a particle could be detected, he focused on the probability that a particle would be detected at one particular point. This probability can be computed by multiplying the complex quantities of the wave equation by their complex conjugates {endnote 3}. This produces a real number, so we do not have to deal with the complex plane at this point. However, it is hard to visualize how these probabilities come about unless we imagine the phenomena as waves instead of as particles. Feynman, however, showed that if we look at every possible path a particle can take to get from A to B and sum up the mathematical expressions for these paths, when we square the result, we get the same number that we get when we square the wave function. (These sums, of course, are over infinite series.) Feynman did not say that the particle actually takes every path, but that the mathematics is *as if* the particle takes every path. We will return to this in Part III.

c. Hidden variables. Bohm's interpretation (1952, 1980) posits a causality that does not depend on time or space. His is a non-local theory with a non-linear time. This theory was rejected by the mainstream physics community in the 1950's but was taken up again by a few theorists in the 1990's (Peat, 1997).

d. Backward and forward in time models. Von Neumann (1932) observed that, because of relativistic considerations, two times would be preferable to one in the quantum mechanical wave function for two particles, even though multiple times are not what we observe. Today, the physics student is normally just told that the way one squares a complex quantity is to multiply it by its complex conjugate; for instance, $(e^{-i\omega t})^2 = (e^{-i\omega t})(e^{+i\omega t}) = e^{-i\omega t + i\omega t} = e^0 = 1$. No reason for this procedure is given except that it works in our equations. But we can look at it the way Wheeler and Feynman (1945, 1949) did: the quantities $e^{+i\omega t}$ and $e^{-i\omega t}$ can be viewed as two separate waves, traveling forward and backward in time, respectively. If we superimpose these waves, the imaginary portions cancel, and we obtain the real wave function that shows the probability for detecting our particle at each point in space.

Feynman (1964, 1965) asked why we do not observe the advanced wave $e^{+i\omega t}$. He put it in his typically eloquent and concrete fashion by asking why it is that when we shake an electron, we observe other electrons to move in a distant antenna a few moments later and not a few moments earlier. In his Nobel lecture (1965), he mentioned several instances where Wheeler had described positrons as the time reversal of electrons. In one instance, Wheeler had speculated that all electrons are actually the same electron, traveling back and forth in time with its world line tangling in one tremendous knot!

In addressing Feynman's question, it has been pointed out {endnote 4} that the time reverse of the wave $e^{-i\omega t}$ being emitted by the electron is simply a wave that is being absorbed by the electron. Cramer (1986) carries this quite a bit further in his transactional approach. (See Part III, Section D.)

Quite a bit has been done to try to introduce a sense of irreversibility into the time-reversible equations of quantum mechanics. It is usually postulated that the wave function describes the probability of locating the quantum at each point in space at all later points in time. Once a measurement is made and we have determined where, in fact, the quantum is, the old wave function is said to collapse and a new wave function that is consistent with our updated knowledge to form. Much has been written about this process, including what role the consciousness of the observer might play in precipitating the collapse (Wheeler, 1980; Wigner, 1961). The interesting thing here is that the wave function itself is time symmetric. Theorists rely solely on the *collapse* to introduce asymmetry to their theory so that it can correspond with the asymmetrical time that we experience. But more recent theorists (Aharonov, Bergmann, and Lebowitz, 1964; Dowe, 1997) have posited that a quantum state can be specified from initial knowledge *or* from final knowledge of the event—that is, that a measurement can collapse either the wave function that precedes it or the wave function that follows it.

Therefore, we cannot rely on the collapse to introduce time asymmetry. They then propose the following alternative way to account for the time asymmetry that we observe.

It is given that light can be emitted from only a select number of positions because, as is well established, it requires a source. Aharonov et al. maintain that light does not require a sink; therefore its wavefunction must start at a source and then spread out. However, Finkelstein (1996) and others have suggested that light may, in fact, require a sink. (The only light we have ever detected is that which has had a sink, whether that sink was a photomultiplier or our own retinas. We have no way of detecting light or its effects except by providing a sink for it.) Even if light does require a sink, we can still introduce temporal asymmetry by considering the expansion of the universe. Sinks for light are always, on the average, going to be more spread out than the sources, which, by definition, precede them in time.

Another argument has been made in this regard. Gold (1958) proposed that the expansion of the universe and the increase in entropy of the universe are really the same thing. Shulman (1999), using Gold's result, has suggested that, in regions of the universe where there is contraction, time could flow backwards. But Hawking (1988) has already refuted this idea, saying that entropy would continue to increase even if the universe contracted, so that even in that case, there would be no backwards flow of time.

An interesting note: Devito (1996) has said that some fairly recent astronomical observations, as well as a number of experiments in particle physics, call into question the standard linear model of time.

e. The meaning of i. Various theorists have proposed alternative interpretations for the appearance of the imaginary *i* in the wave equations. We will go into this more in Part III.

3. Attempts at unified theories

a. String Theories. These theories all postulate the existence of extra space dimensions (Horava and Witten, 1996; Witten, 1996, 1997; Randall and Sundrum, 1999; Lukas, Ovrut, Stelle, and Waldram, 1999) and one proposes an extra time dimension (Vafa and Morrison, 1996). Until very recently, it was assumed that if any of these extra dimensions exist, they must be compactified (Kane, Perry, and Zytkow, 2002); that is, rolled up into a space too small for us to detect; in order to explain the fact that we do not encounter them in our 3 + 1-dimensional experience. But in 1999, Randall and Sundrum suggested that some of the extra space dimensions could, in fact, be large ones. (See also Greene, Schalm and Shiu, 2000; Dine, 2001.) Although the existence of large extra time dimensions has not been a part of these proposals, impetus for further thought has been provided by Greene, et al., who have suggested that the infinities in the self-energies of elementary particles reflect the existence of a minute black hole within each particle. If this were to prove true, it would mean that there are many rips in the fabric of our spacetime, and that, on a fundamental level, neither time nor space has the continuous nature we commonly associate with them.

b. Mathematical theories. Kauffman and Smolin (1997) deal with what they call "one of the key problems in conceptual physics at the present time," the fact that time seems to disappear from the theory of quantum gravity. Elsewhere, Smolin (1995) has reiterated Leibniz' position that space is a pattern of relationships {endnote 5}, which would seem to imply that time could also be such a pattern, but the point he and Kauffman are making here is that they believe the idea of time can be rescued. They accomplish the rescue by restricting the theory to finite quantities, saying that the infinities of the original theory are idealizations that may not be physically realizable in a finite time. They build time and causality into the very foundations of their version of the theory.

Finkelstein (1996) has proposed something akin to Leibniz' idea of space as a pattern of relationships. He believes that the primary entities of nature are not elementary particles but *actions*. He has pointed out that quantum theory involves a new kind of relativity. Where special relativity allows an observer to have contact with a remote event in the past or in the future but not in the present, quantum theory denies the very existence of an instantaneous present. Where SR says there is an infinity of possible ways that the concept of simultaneity can be defined, quantum theory says there is *no* reasonable theoretical definition of that concept (von Neumann, 1932). Therefore, rather than building his theory down from dimensions in space and time or up from fundamental particles, Finkelstein is building his theory up from the fundamental actions of which we can be aware. These are the actions that the experimenter + environment can take on a system under study, and can be as small as the absorption of a single quantum.

Rowlands, Cullerne, and Koberlein (2001) have made a very different argument based on symmetry. They posit that a continuous, irreversible, one-dimensional time is actually symmetric to a discrete, reversible, three-dimensional space, and that the existence of one implies the existence of the other.

C. Some Findings of Cognitive Science—How We Perceive

It is important to ask how our cognitive apparatus constrains our perceptions. That it does constrain them has been well documented. According to Grossberg (1982), for instance, we have evolved to sort through a flood of stimuli that are, for the most part, irrelevant to our survival. Not only must we screen out most stimuli that we receive from our environment, but also, for the new information that our perceptual apparatus does bring to our conscious attention, the normal, *evolutionarily necessary* process is for us to fit it into our existing mental models of reality (Redish, 1994). This will be discussed more in Section III-B.

Stuckey (1996) has suggested that our macroscopically-developed spatial instinct is not adequate for developing a theory of quantum gravity. We have evolved to process and respond to stimuli on our size scale, where objects feel solid, space appears safely continuous and three-dimensional, and time, although it may seem to speed up and slow down, almost never has gaps or circles.

III. NEW IDEAS AND QUESTIONS

We will take a few ideas and speculate on their implications for the existence of extra time-like dimensions and on their implications for the answerability of the question of whether such dimensions exist.

A. Symmetry Considerations

One way to define additional time-like dimensions is to define their symmetry characteristics and to specify the differences between those and the characteristics of space-like dimensions. We examine symmetry characteristics implicit in conservation laws of physics (that is, in laws of nature), which predict the change of physical quantities through time as opposed to the change of those same quantities through space. The physics student will recognize, however, that special relativity and quantum theory each place constraints upon the conclusions we can draw from this investigation.

We will also examine symmetry characteristics by looking at what it means to rotate an object through a space-like direction versus a time-like direction, defining these rotations in the sense of special relativity.

1. Conservation laws

As alluded to above, these laws, such as the law of conservation of mass/energy and the law of conservation of momentum, can be viewed as expressions of the symmetry properties of the system (Rosen, 1995). For example, the law of conservation of mass/energy could be restated, “The amount of mass/energy we will measure, if we measure across all space in our universe and for a small length of time (but not too small {endnote 6}), will be the same no matter at which point in time we choose to begin our measurement." Or, more briefly, "The amount of mass/energy in the universe is symmetric with respect to time." But if we measure across all time in our universe, in slices of space that are thin (but not too thin {endnote 7}), would we measure the same amount of mass/energy as we moved from slice to slice? It is not so easy to answer this question, or even to determine if it is meaningful. (See Appendix A.)

So let us put this more concretely. If we imagine moving through our 3+1 universe in a space-like direction, we know that we can encounter a particular object at many points in spacetime. We have to measure that spacetime relative to something, so let us measure it relative to my bedroom, and to the passage of time recorded by my clock radio. For an object, we can consider my house keys (which I have a lot of practice trying to locate in space and time). Now, I can safely assume that I will never encounter my keys both in my hand and also across the room at the same moment in time. More generally, we know that for each point in time, I have the possibility of encountering my keys at only one location in space. But for each point in space, I have the possibility of encountering them at multiple points in time, and in fact there are points at which I prefer to do so. (I would prefer to locate them on my dresser top again each morning at about eight o'clock, and at other times, as well.) This is a conservation property of my keys.

This property holds true for our ability to detect quanta. As mentioned above, Bohr (1949) said that if a particle is recorded at point A, then its effects from point B will never be observed. (Bohr meant something more subtle than just that we will never encounter a single particle in more than one place at once, but the latter is implicit in what he was saying). Certainly, if we encounter it once in the present moment, we will not encounter it twice or four times in the next moment. The amount of its mass/energy is conserved through time.

We also believe that momentum is conserved through time.

A quantity that is not conserved through time is entropy, or the measure of disorder in the universe. There is more order in the past than there is in the future. Our universe is moving toward equilibrium, with a random distribution of mass/energy everywhere in space. There is a lot of speculation about what caused all that order in the first place, but the fact is, everywhere we look, we see order coming undone.

Although theorists do not usually speak in these terms, and in spite of the difficulties mentioned above, we could also formulate some conservation laws that would apply to movement through space. One such law would be the conservation of the laws of physics. We believe that these laws hold true everywhere in space, although there is speculation that they may have changed in some respects through time. Quantities that we believe to remain constant throughout space are the gravitational constant, the speed of light, the strength of the fundamental forces, and the size of the universe that we can see.

If we use these conservation principles to define new time-like dimensions, all of those dimensions will have to have the same asymmetry with respect to entropy as has our current time. Therefore, one way (but not the only way) to imagine additional time-like dimensions is to specify that they, too, are arrows in a direction of increasing entropy. (See more on this in Section III-B.)

2. Special relativity: rotations in spacetime

In the previous section, we used the idea of an observer to investigate the symmetries of the *distribution of physical quantities* through spacetime. In this section, we investigate the symmetries of the *observations* of different observers. Connected with this is an investigation into all the transformations that an object can undergo when it exists in a space of a given dimensionality. (Some of these transformations are equivalent to shifting the coordinate system to that of a different observer.) We are particularly interested in the kinds of transformations that will not destroy a physical object, such as translations and rotations. Inversions can destroy a rigid object, but they are a part of this analysis, too. Comparing the possible transformations can help us see whether a particular space has more or less capacity to correspond to our physical world.

Our normal 3-dimensional space is, at least approximately, a Euclidean space. The invariant quantity in this space is

$$\mathrm{d}r^2 = \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2$$

(see Fig. 2). This reflects the fact that if we translate or rotate an object in Euclidean space, its x, y, and z coordinates will change, but what we compute for physically meaningful quantities (such as length and shape) will not. Rather than rotating the object, we can rotate the coordinate system. (That is, we can rotate the orientation of the observer.) If we do this, the coordinates of points within the space will change, but relative distances between points in the space will remain the same. Therefore, the

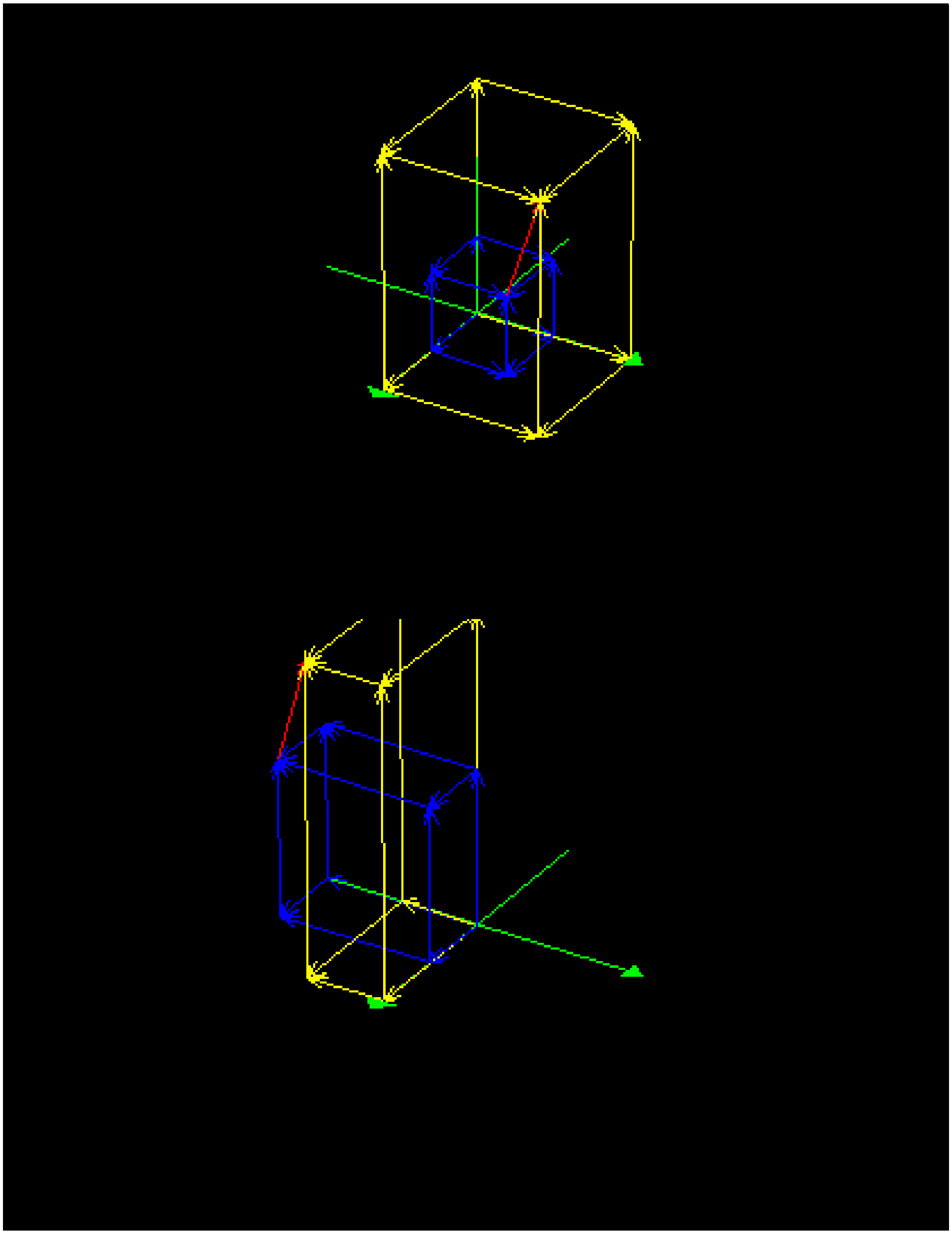


Figure 2. The space invariant. A red line connects two points. This is shown from within two different coordinate systems. The coordinates (blue and yellow) of the points differ in the two systems, but the distance between the points is the same. The distance is said to be *invariant*. (In the black and white version, the invariant is the bright diagonal line. In the QuickTime version, you can double-click either image to animate.)

relative distance dr measured by any observer is invariant under translations and rotations in Euclidean space; this is a symmetry property of the space. (Put another way, different observers may assign different values of *x, y*, and *z* to the same event, but they will all agree about the relative distances between different parts of the event.) We represent this space with the Euclidean metric

$$\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

This simply means that we compute distances by adding the squares of all components.

In spacetime we add another dimension, that of time. But special relativity tells us that the invariant quantity in spacetime is

$$ds^2 = c^2\,d\tau^2 = c^2\,dt^2 - dx^2 - dy^2 - dz^2$$

where ds is the interval between two points in spacetime and $d\tau$ is the proper time, the time that would be measured by a clock traveling between the two points. Since c^2 functions here as a conversion factor, by choosing our units appropriately, we can write this

$$d\tau^2 = dt^2 - dx^2 - dy^2 - dz^2\ .$$

This is represented by the metric

$$\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{matrix}$$

where the minus signs show that we subtract the squares of the three space components. This is the Minkowski metric, and it describes a hyperbolic space. (We show the choice of sign used by particle physics. Theorists in some areas of physics flip all the signs, but the quantities computed are equivalent. To see what it might mean if any of the off-diagonal terms are not zero, see Section 4, p. 30.) Essentially, when computing a distance in this spacetime, we still take the sum of the squares of the space components, but the physically meaningful quantity is the *difference* in the squares between the space components and the time components,

$$d\tau^2 = dt^2 - dr^2 \ .$$

(If the observer is moving at a constant speed, we can use $\Delta\tau$ instead of $d\tau$. See Fig. 3, a diagram designed by the author; Stephens, 2004).

In the coordinate system represented by the above metric, we have the possibility of several kinds of translations and rotations. Translations can be along the individual *x*, *y*, *z*, or *t* axes or they can involve more than one axis. But all translations involving one or more space axes are equivalent. So the only physically significant

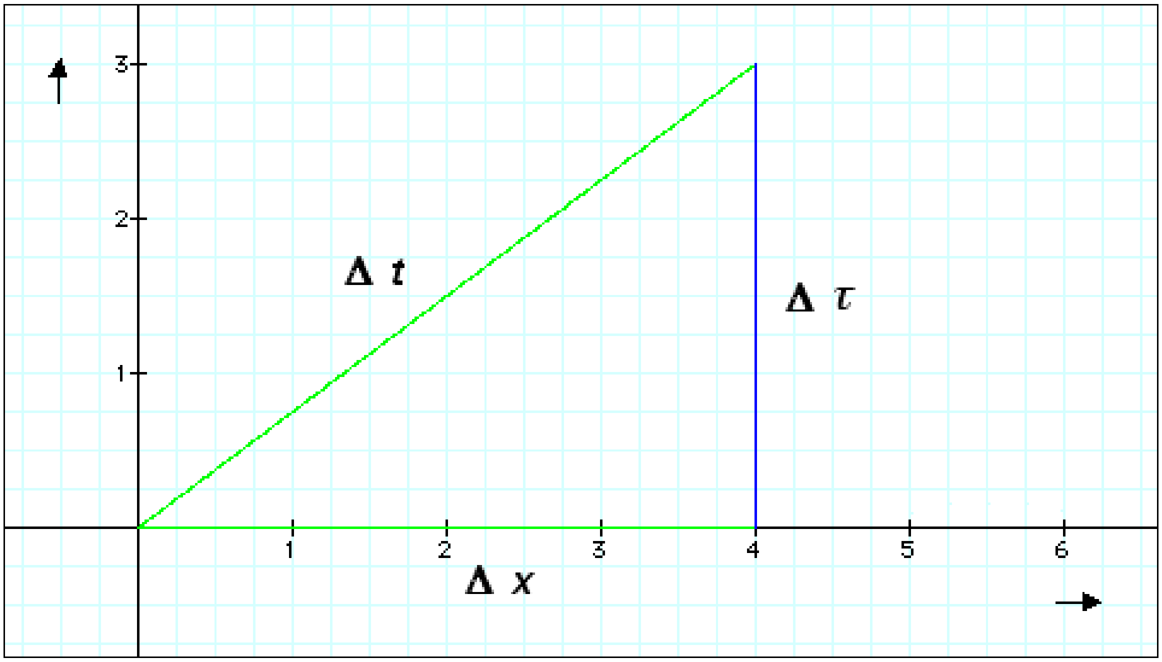
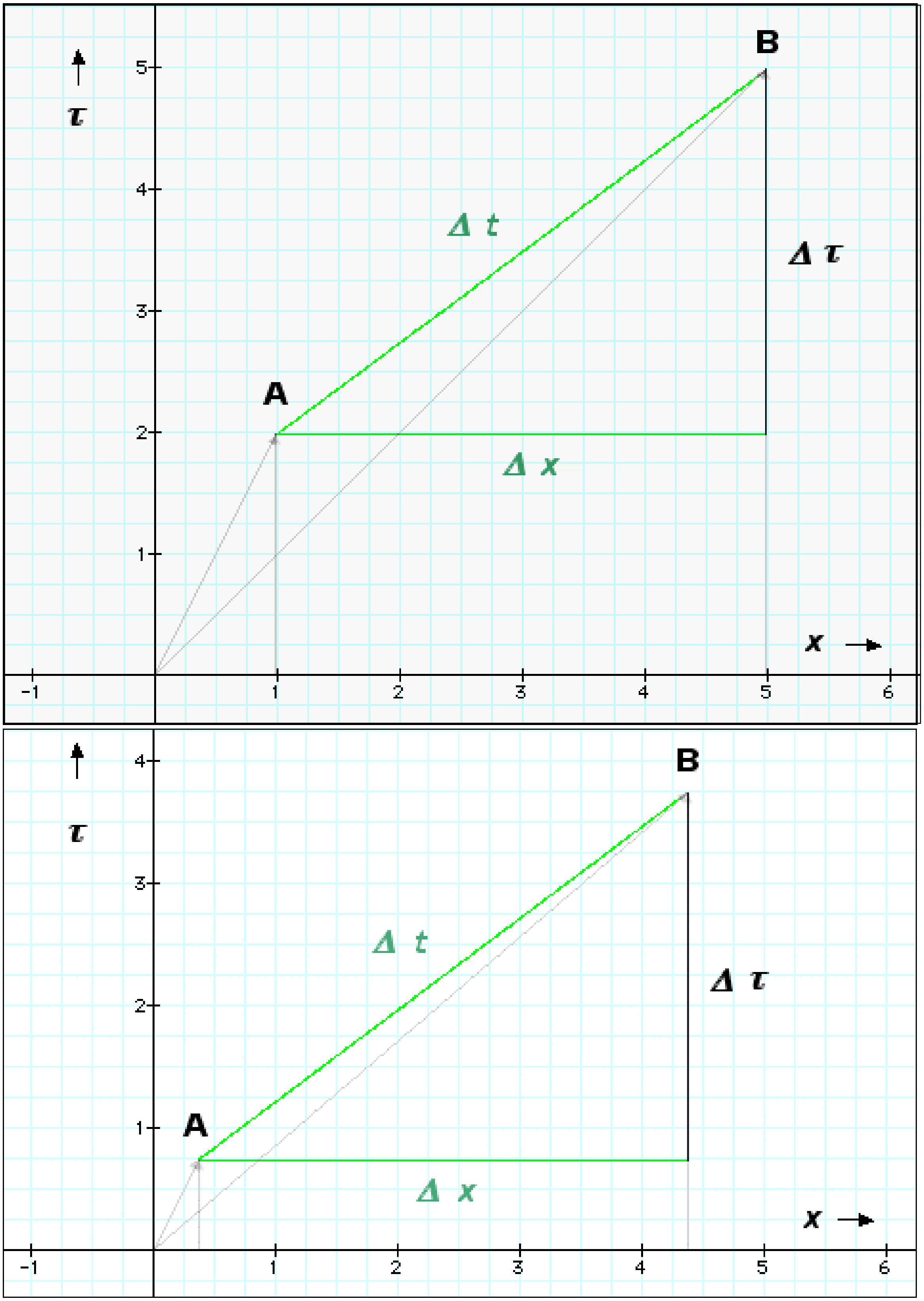


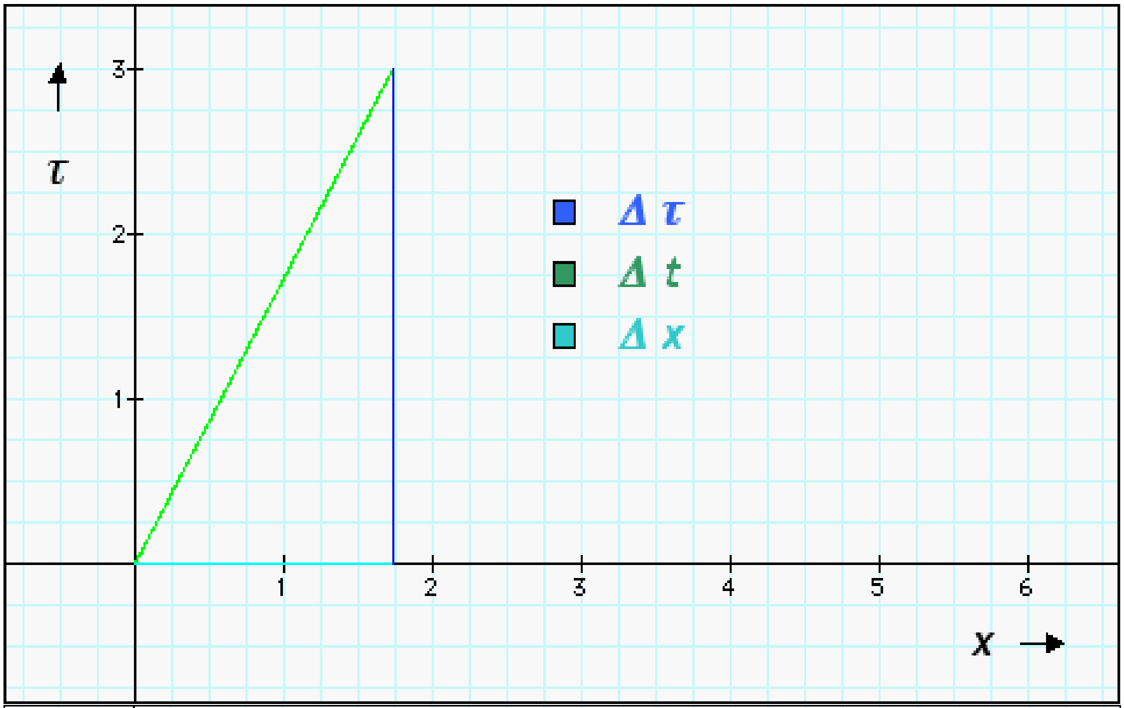
Figure 3. A diagram of the Minkowski spacetime invariant. The square of the invariant distance between two spacetime points is $(\Delta\tau)^2 = (\Delta t)^2 - (\Delta r)^2$. Actually, only the component Δx is shown here. The y and z components of Δr are suppressed. (See Stephens, 2004.)

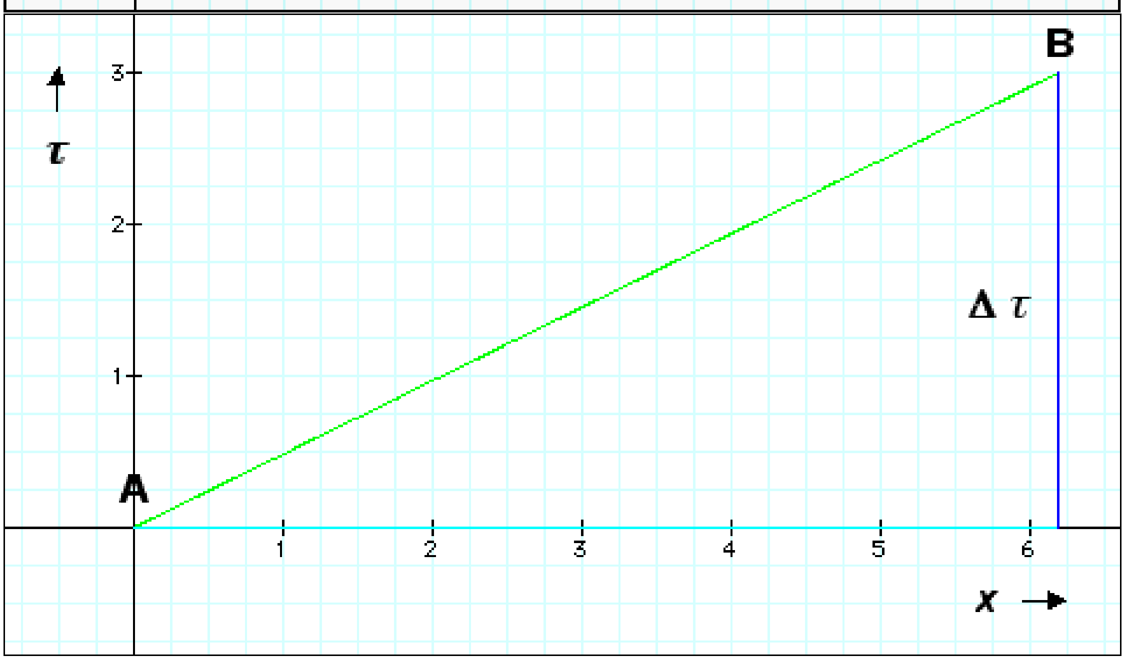
distinctions we can make are between the following three groups: translations involving one or more space axes, those involving just the time axis, or those involving the time axis plus at least one space axis. In normal conditions of relatively flat space and relatively short periods of time, all of these translations are equivalent to a simple change in position of the observer. (See Fig. 4). This is not true of rotations.

Rotations between any two or three space axes are described by the Euclidean metric (p. 23). With only one time axis, there is no possibility of a rotation through a purely time direction. (The case of multiple time axes will be discussed in Section 4, below). But we can have rotations between space and time axes. In fact, the symmetries of these rotations are described by the Einstein-Lorenz transformation equations.



Figures 4a-b. Translations. Changing the relative position of the coordinate system does not change the spacetime distance between points A and B. (Double click the top image for animation.)





Figures 5a-b. Rotations. Each position of the graph shows what we would measure for the space and time components for the distance from A to B from within a coordinate system moving at some constant speed relative to the two points A and B. In the bottom graph, we are moving past the points at 0.90 the speed of light and we measure a lot of space and a lot of time between the two points. The top graph initially depicts a velocity of 0.50 light-speed (but that will change if you double-click the QuickTime version).

Changing our speed is equivalent to rotating our coordinate system between the space and time axes. (This is a hyperbolic rotation.) Rotating a coordinate system changes the space and time components, but it does not change the *spacetime* distance between the two points, which is $\Delta\tau$, represented here by the vertical blue line.

The physical meaning of a rotation between space and time axes is the change of perspective of an observer traveling first at one speed and then another, relative to the event being observed. But we know from above that our invariant quantity in this spacetime is $d\tau^2 = dt^2 - dr^2$. So all observers, no matter what their speed relative to an event, will agree on their measurements for $d\tau$, the spacetime interval. However, they may disagree on the values they assign for the space components and the time components of the event. (See Fig. 5. Stephens, 2004 has a more complete visual exploration of the concept.)

The question we want to ask is whether any new symmetries (or possible ways to break symmetries) would be introduced by the inclusion in the space of additional time axes. We want to ask this because string theory postulates additional space dimensions, but most versions do not postulate additional time dimensions. What would be the differences in symmetry if some of those extra dimensions were time-like?

Let us look at the following metric:

$$\begin{matrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{matrix} .$$

We can flip all the signs and come up with an equivalent space, so it really makes no difference whether we assign the minuses to the space dimensions or to the time dimensions. This implies that, if we define our additional time dimensions according to the time of the Minkowski metric, their relationship to each other is Euclidean, just as is the relationship between the space dimensions. We would get the same results if we examined a spacetime with just two directions each of space and time, or, indeed, with any other combination of space and time, as long as all the dimensions were space-like or time-like in the sense of special relativity. Therefore, there are only two distinct kinds of rotations in any of these spaces: rotations through two dimensions of like sign (Euclidean) or rotations through two dimensions of opposite sign (hyperbolic). We get both of these as long as we include at least one time dimension in our space. We therefore conclude that adding additional time axes in this way offers no possibilities for new kinds of translations or rotations, although the numbers of each kind will be greater.

What does this mean? If we define additional time directions so that all additional directions consist of arrows pointing in the direction of increasing entropy (as will be discussed further in Section III-B), then only the gradient vector of that 3-D time will be physically important. Therefore, there will be only one physically significant space-time rotation, between the time gradient vector and d*r*. (See Fig. 5.) Only rotations between the time axes will have the potential for offering anything new. We discuss this briefly in the context of time loops, Section 4, below.

3. Inversions

There is at least one other possibility at which we can look, and that is the symmetry operation of inversion. Mathematical inversions are usually considered physically impossible with rigid bodies, but this hypothetical operation can point to physically possible operations on those same bodies in a higher dimensional space. An inversion through one dimension can be created by obtaining a one-dimensional slice of a 180 degree rotation through a 2-D space, a two-dimensional slice of a 180 degree rotation through a 3-D space, and so on. (See Appendix B.) We have a one-dimensional inversion in our 4-D spacetime equations, that of the time-advanced and time-retarded wave equations, and this inversion can be seen as a four-dimensional spacetime slice of a 180-degree rotation through a 5-D space. Feynman refers to this. (See Section III-D-3, p. 59.) So these equations can be interpreted in at least three ways: as a mathematical "trick," as representing a standing wave (where the imaginary part does not have a physical impact unless there is interference), or as describing quantities that exist in a 5-D spacetime. We suggest that a next step in this analysis could be to ask if the fifth dimension in this case could best be thought of as space-like or as time-like, and what would be the implications of each supposition.

4. Time shear

A shear is a deformation where parallel layers of a body slide over each other. The interesting thing about shear is that it is a tangential effect—if the source of the force is moving forward, then the shear force acts sideways. If we are to be aware of any effects from mass/energy in spaces parallel to ours, whether those spaces are the result of extra time-like or extra space-like dimensions, then the effects would have to come from some kind of shear forces operating between the 3-D spaces.

There are shear effects predicted in Einstein's theory of gravity. The term T^{ij} in the Einstein stress/energy tensor (described in Appendix C) has indices from two different space directions, and describes shear in space. In the Randall-Sundrum (1999) version of string theory, there are extra space dimensions and gravity can, in fact, act between them. However, according to Randall-Sundrum, gravity is the only force that can do this. To account for either the statistical results or the interference that we observe in quantum phenomena, we would have to have tangential effects from electromagnetic forces, and this is not predicted in any of the current string theories.

If there were more than one time dimension, it would also be possible to have time shear terms in Einstein's tensor. (This is discussed in more detail in Appendix C). This tensor describes the gravitational force only, but it could still be instructive to take a look at the implications of the shear terms, especially since some of those implications have already been investigated. This was done, for instance, in the context of the Gödel (1949) universe, which has two time coordinates. The problem discovered in this case is that the theory implies the existence of time loops, areas in space where objects are forced to cycle over and over in time without being able to break free. Actually, time loops *per se* are not the problem. Theorists think each electron-positron pair is an electron caught in a time loop—the electron the particle going forward, and the positron the same particle going backward. What we see when we look at the situation is this: a vacuum, then an electron and a positron appearing from nowhere, then the two particles recombining, then nothing. This kind of time loop not only is allowed by our current theories, it gives the best (or at least the most widely accepted) explanation for what we observe. But Gödel's theory allows also for the kinds of time loops that involve paradox. (You could go back and shoot your father before you were conceived, etc.) The challenge with this kind of issue is to exclude paradox without also excluding free will; we do not want to be forced to posit a past that cannot be altered by a visitor from the future. It is possible to modify the theory so that the past can be added to but not changed (Lockwood, 1996a, 1996b; Deutsch, 1996), but this modification produces a theory that excludes the possibility of time loops all together. This leaves no mechanism by which electron-positron and all other virtual pairs {endnote 8} could exist. Interest in this scenario has largely died out; although Vafa and Morrison (1996) have proposed an additional compactified (extremely tiny) time dimension in their version of string theory.

Even though the shear terms in the Einstein tensor deal only with gravitational waves, transverse shear waves exist elsewhere in nature (Feynman, 1963). This suggests that a next step in the inquiry could be to investigate shear in the context of the transverse waves of electromagnetism, considered within a spacetime with multiple time-like directions.

5. Conclusions from symmetry considerations

In order for a model that uses additional time dimensions to be a useful replacement for current models, it needs to be able to do something that our current models cannot, or to do the same thing in a simpler fashion. Our comparison of symmetry characteristics of space with the equivalent characteristics of time has shown that if we assume one preferred direction in time, say in the direction of increasing entropy (discussed further in the next section), this limits the possibilities for the existence of symmetry characteristics that differ from those of models with one time dimension. It might, however, be profitable to look more closely at higher order rotational symmetries using more powerful mathematical tools such as geometric algebra (Hestenes, 2003; Doran and Lasenby, 2003).

A look at the stress-energy tensor has suggested that there could be physical effects between mass/energy in adjacent time streams. However, this requires careful consideration of the meaning of loops in time and leads to the consideration of the nature of cause and effect in general. We will examine this more in Section III-E, below.

B. Concerning our Perceptions of Time

Now we turn from the essentially mathematical problem of how to define extra dimensions in time to the examination of a biological issue. As unrelated as the two approaches may seem, this new approach can shed more light on some of the issues we just examined above and will help prepare us for ideas to be introduced below.

Given the apparent fact that our cognitive apparatus constrains our perceptions, it should be interesting to investigate what effect, if any, this has had on the development of our scientific theories. Our apparatus has clearly evolved to construct a 3-dimensional image out of the bits of information it deems important enough to bring to our awareness {endnote 9}. We perceive this image as changing from moment to moment in a sequence of moments that we experience one after the other. A question can be posed regarding how much of this image is due to the nature of our perceptual apparatus. How does this apparatus constrain our modes of thinking?

If our macroscopically-developed spatial instinct has, as Stuckey (1996) suggests, limited our ability to think about well-verified data that nonetheless cannot be given a consistent description in 3-dimensional space, would not the same hold true for our ability to think about phenomena that cannot be described in terms of a 1-dimensional time? Bohr (1928) reminds us that the very idea of observation belongs to the causal space-time way of description. When we speak of an observation, we normally refer to our experience of electromagnetic waves (light waves) that extend from a point in the past to our perceptual apparatus in the present. We absorb photons and gain information from them about events that happened in our environment a moment before. We normally do not use the term "observation" to refer to our experience of electromagnetic waves that extend from us out into the environment, for they begin with us in the present and are absorbed by our environment in the future, even if that "future" is only a millisecond later.

Both Feynman (1961) and Cramer (1986) have given interesting and quite plausible answers to Feynman's question about why we do not perceive the backward wave from shaking electrons. (See Section III-D-3, p. 59.) But the question remains an interesting one. The fact is that we apparently receive useful information from the past but not from the future. Despite Aharonov et al. (1964) and Dowe's (1997) demonstrations that an event can collapse the wave function that comes after it—destroying our knowledge about the past state of the system but giving us information about its future—we experience our free will as operating only into the future. We cannot change past events, but we can select future ones. This, one suspects, is what is really at the root of scientists' reluctance to entertain seriously ideas about time as anything other than a one-dimensional arrow. Any other construct would imply a profound change in how our free will functions. The ability to go back and forth in time would mean that we could change or displace events in the past, leading to all kinds of paradoxes well-explored in the literature of science fiction. Being able to receive information from the future would imply that the future is immutable to the influence of a free will, an option that most scientists (though not all) would reject. Therefore, it is not only perceptual questions that must be addressed, but philosophical

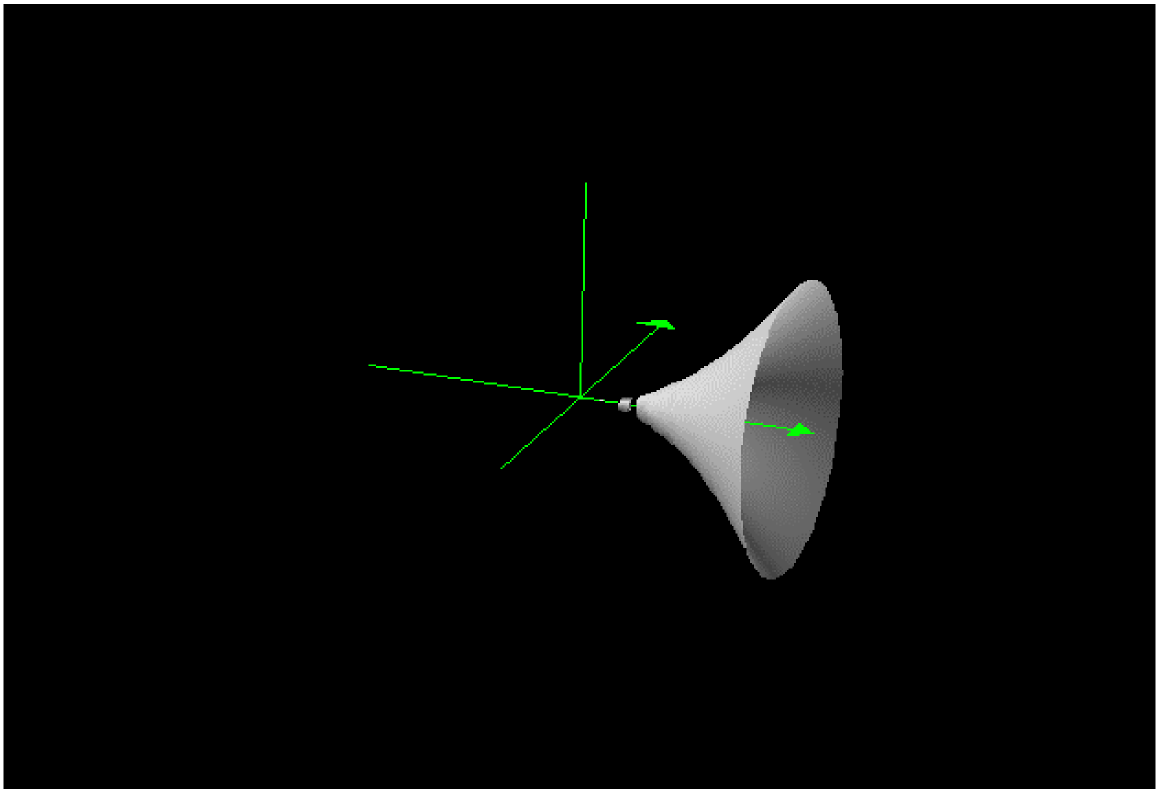


Figure 6. Increase of universe with time. Only 2 dimensions of space are shown.

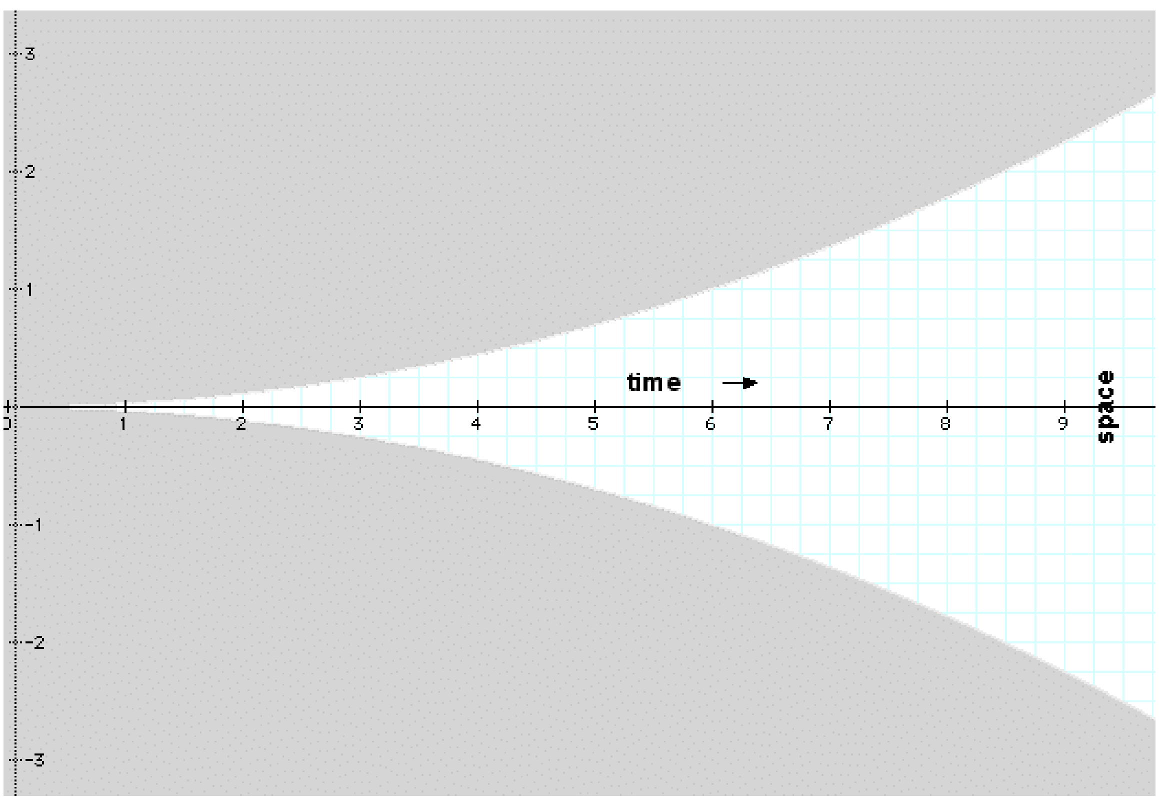


Figure 7. A slice through the image in Fig. 6. As space has increased, so has entropy.

ones as well. Setting aside the philosophy until Section III-E, we turn the free-will argument here to a different purpose; that is, to suggest an evolutionary mechanism that could help account for our perception of time.

Let us start from a new perspective. If we imagine the whole universe existing at once, at one end we see the big bang. (See Figs. 6 and 7, previous page. We are imagining this in Euclidean fashion, because it is easier and because it will make the point.) As we look in the other direction, as far as we can see, it does not have an end—it just keeps on expanding into the distance. Looking back in the first direction, we realize that if there is anything before the big bang, we cannot see it.

Now let us imagine another universe, similar to ours, but where time has multiple dimensions. For convenience, let us imagine it has two time dimensions. (Figs. 8 a-c show possibilities.) We are multi-dimensional beings, and we can look around this large spacetime in any direction. Again, as we look, we see an asymmetry in the distribution of matter. In this instance, we are not necessarily assuming that the additional time dimension is an arrow moving in the direction of expanding space and increasing entropy, as is our normal time. In fact, we take the general case where the new time dimension may not be an arrow and where we can travel in any direction relative to the change in entropy (Fig. 8c). We will see where this leads us. At one area in this spacetime, all the space and all the matter/energy is concentrated into a single point of space (but not necessarily a single point in the times). As we travel through spacetime in any time direction away from this area, we see that the matter/energy becomes spread out, more dilute, and less organized. Eventually, we reach areas with very large space, completely random distributions of energy, and no organized matter at all.

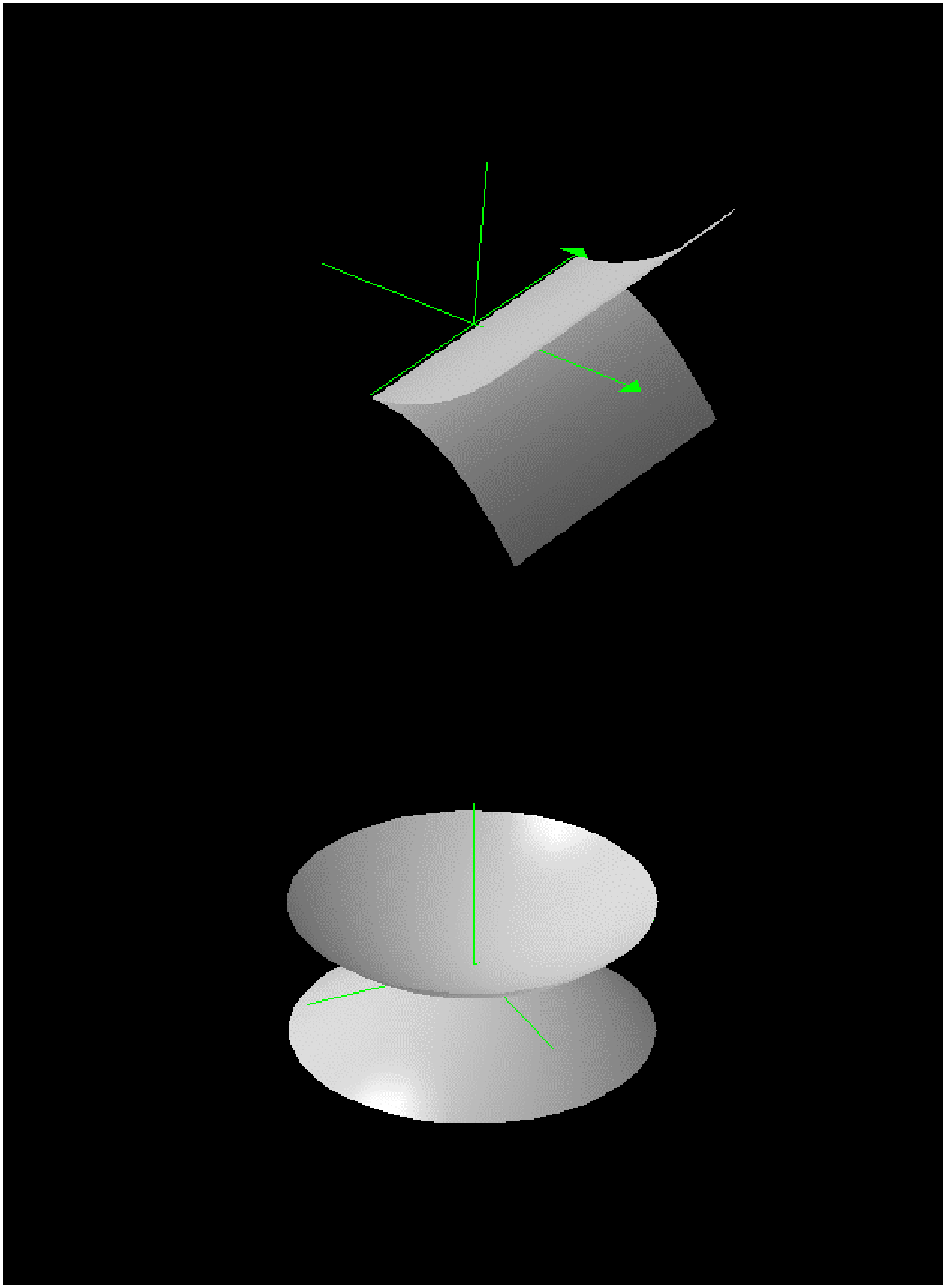


Figure 8a. Rather than adding back a second space dimension, we have added another time dimension. In this scenario, there would be almost identical parallel universes. But this is not the only way the universe could expand through multiple times.
Figure 8b. The universe could also expand from a single starting point in spacetime.

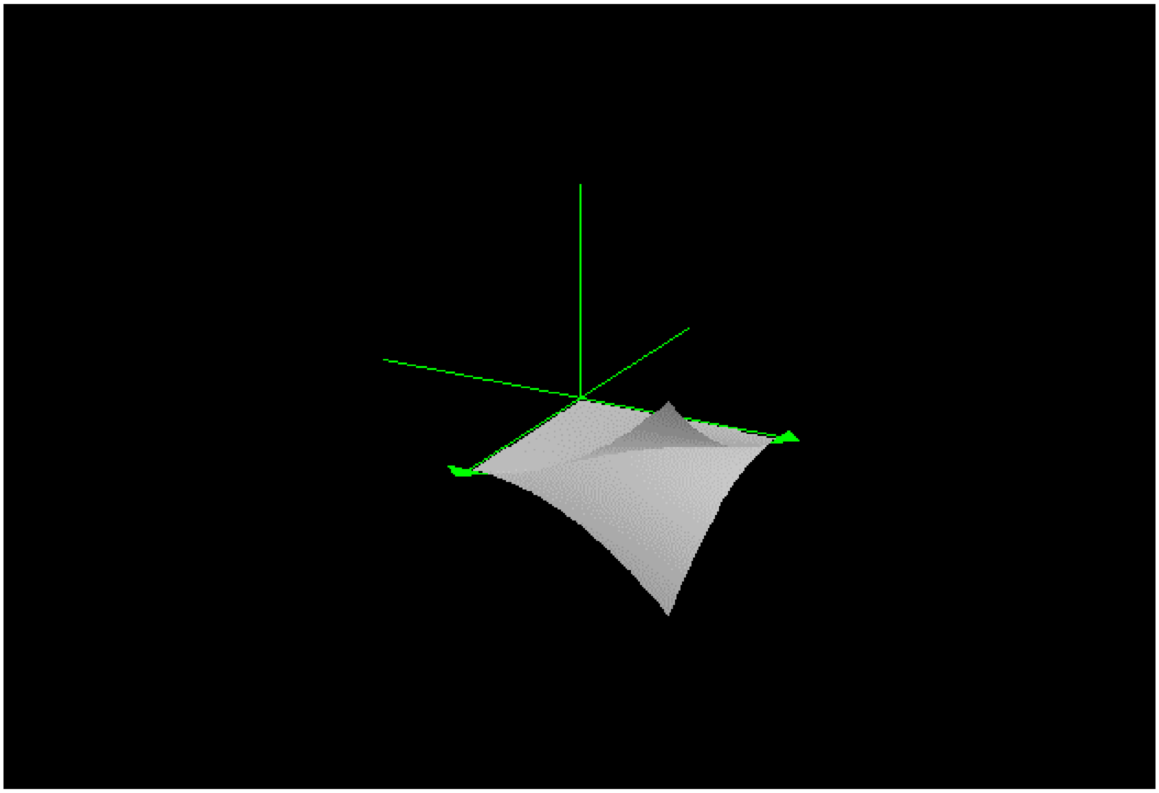


Figure 8c. Yet another way the universe could expand. In this case, although there are two times, there is a preferred direction in time. (Note that in all of the figures 8a-8c we can find the cross-section shown in Fig. 7.)

Imagine now that we are once again 3-dimensional creatures, but that we exist within the large spacetime of Figure 8c. We occupy a certain section of this spacetime, and we will develop our experience of this section by traveling through it in some time direction at an involuntary and constant speed (as we do in our own 3 + 1 spacetime). But there are different directions we could go in this two-dimensional time. We could go in a direction where order is increasing, where it is staying approximately constant, or where it is decreasing. In what direction would it be most advantageous for us, as organisms, to travel?

Before we answer that, given that we are in a place where our previous notions of space and time do not necessarily hold, we need to specify what we mean by *travel.*

We normally use that term to mean changing our position in space with respect to time. But that presupposes that we are already traveling through time, and that we are now simply deciding whether or not to change our position in space as we do so. If past and future did not have their current fixed meanings, what would constitute an experience of travel through time? To answer this, it will help to look more closely at what constitutes our present experience of travel through time.

Let us imagine that we are back in our usual 1-dimensional time and looking before and behind ourselves (Fig. 9). What precisely are the differences in what we see as we peer into the two time directions currently available to us? In one direction, we have definite knowledge of which events actually occurred at each point in time. It is relevant to note that we have this definite knowledge at the expense of free will

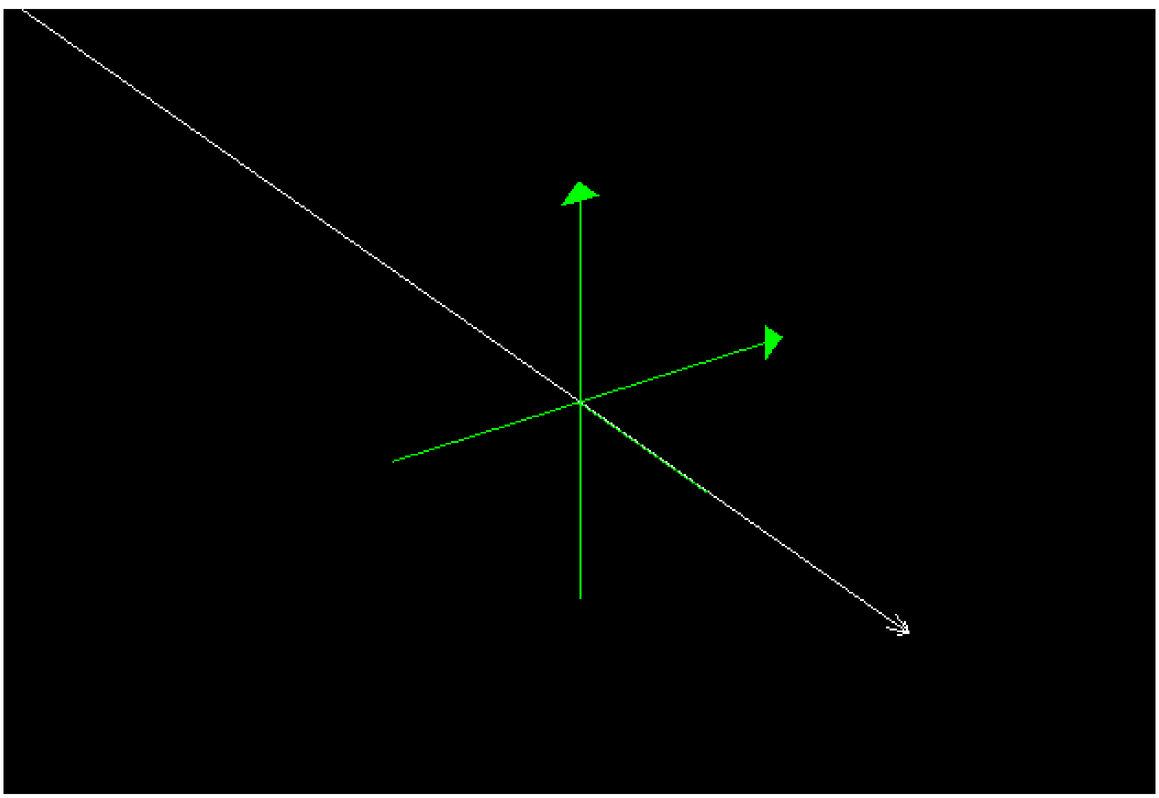


Figure 9. This is our single time direction. We can only look forward or behind.

concerning those events. In the other direction, we experience ourselves as having the free will to choose between many possible events. We do not have an infinite choice; our free will is actually bound by the wave equation, which tells us what is physically possible. (We cannot be here one second and jump to Alpha Centauri the next, for instance.) But the possibilities in that equation are huge, so we actually have a lot of choice in how we can interact with our physical surroundings.

When we consider a spacetime with many time dimensions and imagine viewing our trajectories through it “all at once,” the direction we imagine ourselves traveling along those trajectories becomes arbitrary. But we could define “travel through time” as the experience of having one direction where we have knowledge of a specific set of events, and another direction where we are only aware of many possibilities, which we are free to choose among. When we make a choice, it immediately becomes one of the fixed events of which we have definite knowledge. Our perspective at each point of that spacetime is this: the region where we observe a set of fixed events of which we have definite knowledge we label the “past,” the region where we observe multiple possibilities viable for each point we label the “future,” and the point where we experience ourselves in the process of making a choice we label the “present.”

Now what are the implications for the different possible directions of travel in our large spacetime? If we travel toward regions of greater order, at each step we have fewer possible configurations of matter/energy to choose among than we did “before.” But if we travel toward regions of greater randomness, we will have a greater number of possible configurations to choose among. It might be advantageous, therefore, for any organisms in such a spacetime (at least any organisms that are capable of making a choice based on some awareness of moments other than the moment being experienced) to "evolve" {endnote 9} in such a way that they could exercise whatever freewill they experience in the direction of widest choice. That direction would be the direction of the gradient vector of entropy in this spacetime, the direction of travel toward most steeply increasing numbers of configurations of matter. In that case, these organisms could all agree on what was "future" and what was "past," and might actually have no advantage in experiencing other degrees of freedom in time. Therefore, we propose that, even in such a large spacetime, we might evolve together to experience our movement through time in the single direction of most steeply increasing entropy (Fig. 8c).

C. Reinterpreting the Meaning of the Ensemble

When we arrange for a photon to be emitted and then try to predict where it will be absorbed later, we find that we can only give a statistical prediction. These statistical predictions are very, very accurate, in that we can predict, out of a thousand photons, almost exactly how many will be absorbed where. Or, out of a thousand experiments, how often we would detect a photon at a given location, at a given length of time after emission. But any individual photon could go in absolutely any direction. The only thing we know about an individual photon is how fast it goes, and that is at the speed of light.

To obtain our statistical predictions, we square the (normalized) wave function that describes the physical set-up. (Since the wave function contains imaginary quantities, we multiply it by its complex conjugate.) This gives us a (usually infinite) series of real numbers, each one between 0 and 1, each one the probability that the quantum could later be detected at some specific point in space and in time. In many

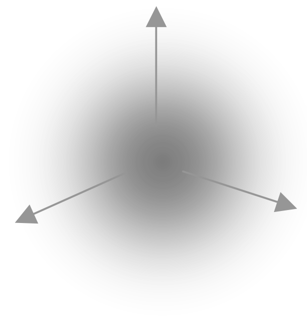


Figure 10. The electron cloud: a representation of the probability of finding an electron at different points in space. This shows a ground state orbital about a nucleus. If we actually arrange to detect the electron, we will find it at only one specific spot, but in some ways it acts as though it is smeared out through the entire cloud.

ways, the photon acts as though its energy were distributed over every one of those positions, but when we actually locate the photon, we always find all of its energy at one point. (We investigate this more in Section III-D.) This is also true of electrons; an electron's orbital is merely a picture of the probability distribution of the electron. We can give extremely precise laws for the behavior of the distribution, but virtually no laws for the behavior of the individual particle, except that we know how it will interact with a detector when we actually find it. Therefore, the distribution is often spoken of as an entity with its own physical validity, the *ensemble* of all the possible locations of the particle. (See Fig. 10. Also, there are elegant animated renditions of orbitals available online {endnote 10}.) Since a single quantum can only be located at one point at a time, the image of the entire ensemble cannot exist in 3 + 1 space. The space in which it does exist—often called the *probability* space—is a hypothetical mathematical space, but in some ways the ensemble acts more "real" than the single quantum that inhabits our usual 3 + 1 space.

For a variety of reasons, it has not worked to think of probability space as an actual physical space of large extra space dimensions. For one thing, the conservation of mass/energy would be violated, especially if the particles could move freely within the large space. The task chosen in this section is to look briefly at the possibility that the ensemble could be given a physical representation with the use of extra time-like dimensions. What implications can we derive from the conservation laws that apply to a change in time? (See Section III-A, above.) Would such a model agree with what we observe? We want to know if it would give us anything new or more elegant than the currently accepted model, the fictitious probability space.

1. Conservation of energy

As stated above, we cannot encounter a single object at multiple points in space at the same point in time. However, we can encounter it at multiple points in time at the same position in space. This suggests that the law of conservation of mass would not be violated if all of our probable copies of the quantum had physical existence in adjacent slices of an additional time-like dimension.

2. Entropy

In order for the ensemble that exists at our normal time *t* to be represented in an additional large time-like dimension, we would have to postulate that the amount of entropy was constant as we moved in this new direction. If entropy increased along this direction the way it does in our normal time, the meaning of the ensemble would be profoundly changed. (The number of possible states in a system is related to the entropy. The ensemble is a representation of the possible states in which we could find the quantum at a later point. If different points of the ensemble were considered to exist in regions with different amounts of entropy, we would have to rethink exactly what it is we mean by "the ensemble.")

We could instead imagine the extra dimension as being associated with a particular particle, so that the entropy remains constant, but the wavefunction is defined in terms of the possible points of *detection* of the particle; that is, the possible interactions the quantum can have with all other quanta. Another tack is to formulate the model in terms of the interactions themselves, rather than in terms of the particles (as has been done by Cramer, 1986 and Finkelstein, 1996). We will go into this more in Section III-D-3 (p. 61). The point introduced here is this: to use a large extra dimension to represent the ensemble, the dimension would probably need to be time-like with respect to the conservation of mass but space-like with respect to conservation of entropy. This raises the issue of how to define such a dimension (and we do not attempt to do so in the present work).

3. How do 3 + 1 spacetimes map into a large 3 + >1 spacetime?

There are actually many ways we could slice a large spacetime into sections the size of our normal 3 + 1 spacetime. Let us begin this visualization by imagining an infinitely long line that represents our normal time (Fig. 11a). Imagine that each point on this line represents a different moment of our 3-D space. One vertical blue line represents our entire 3-D world at 5:00 PM today, and another blue line our world at 5:01 PM, etc.

Now imagine another time line intersecting the first (Fig. 11b). This is a second time dimension. By each point of our original time line we can now have multiple blue lines, representing 3-dimensional spaces that exist in time-lines parallel to our own. In fact, we can have an infinite number of such parallel times. If so, we can imagine

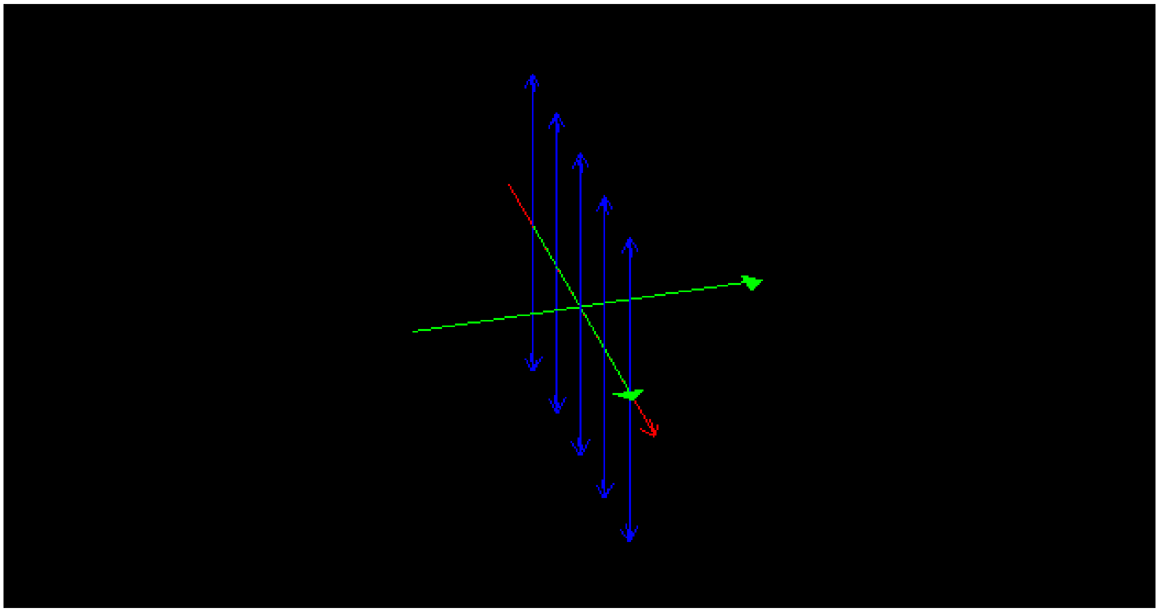


Figure 11a. Our time is flowing in the direction of the red arrow (coming out of the page). We have shrunk our space down to one dimension. The vertical blue arrows each represent that one space dimension at a different point in time.

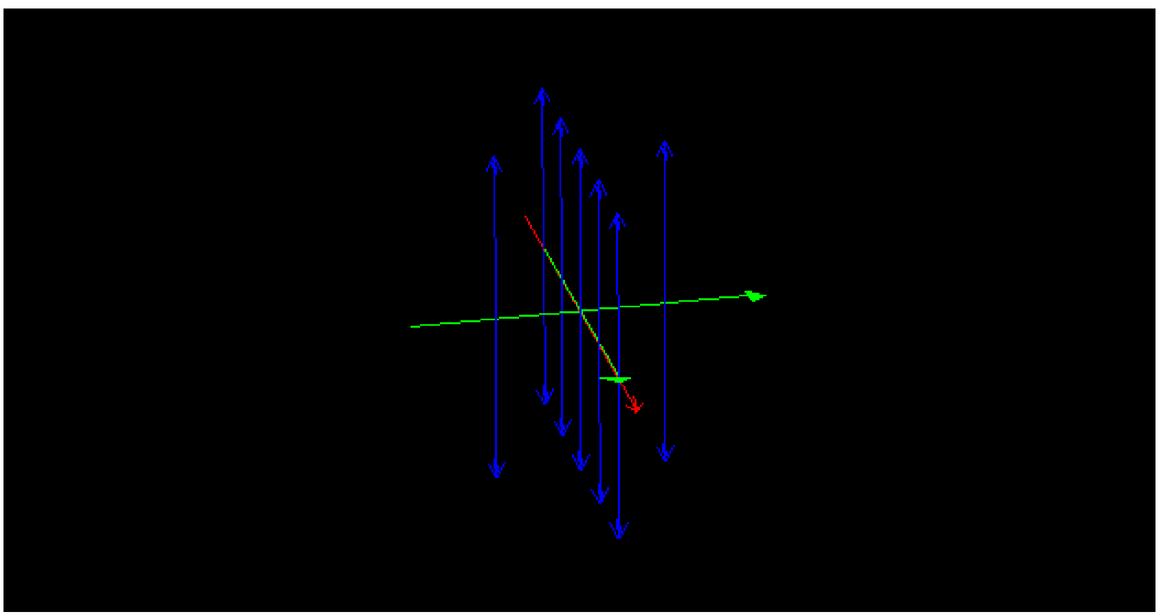


Figure 11b. Our normal time is still flowing in the direction of the red arrow. If we had another direction in time, we could have more than one 3-D moment at 5:00 PM.

traveling forward in a parallel time—but we can also imagine traveling sideways, from one timeline to another (Fig. 12). This is a model used frequently in science fiction, but the question is, can it represent our ensemble? The answer is no, it is not large

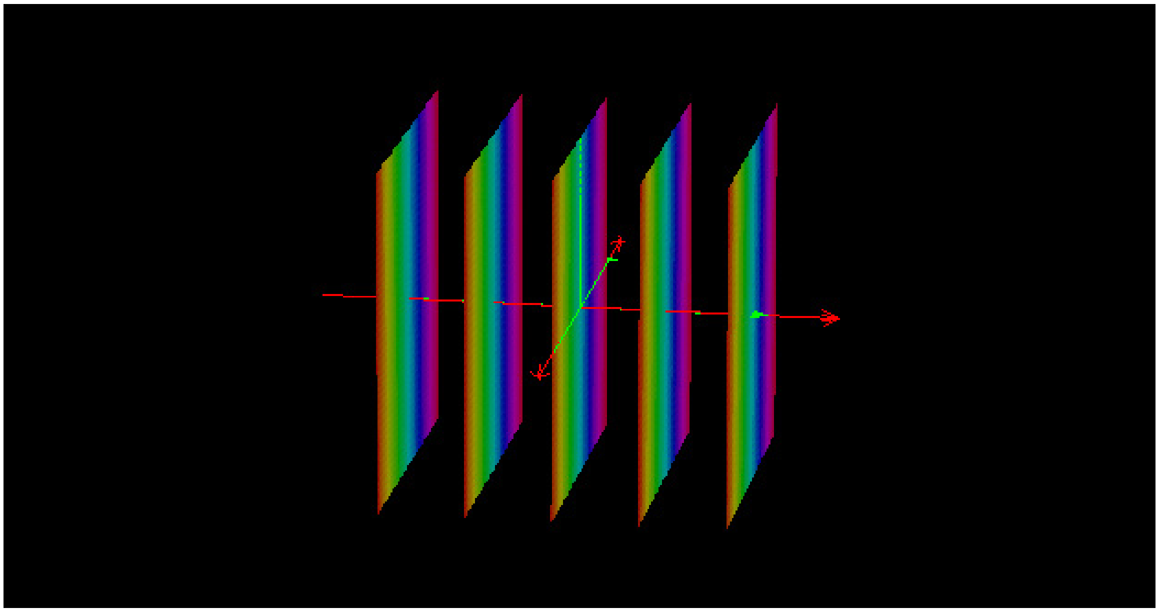


Figure 12. Each plane exists at a different moment in our time. The multiple 3-D spaces at 5:00 PM might not be separate, but could flow into each other, as one color flows into the next—as one moment flows into the next in our normal time direction.

enough. The ensemble represents a single moment of time, and would have to be contained in one of the planes in Figure 12. We cannot use a plane to represent all of the points of an ensemble in any simple way. (If we use more complicated ways to map a higher order of infinities onto a lower order, we lose a lot of the advantage of this model. There are far simpler models around.)

So let us next imagine a 3-dimensional "space" that is actually composed of three time directions (Fig. 13). In this 3-D time, we can imagine the ensemble existing just as it does in our 3-D probability space. The next step is to imagine that each point of this 3-D time intersects with a different 3-D space (Fig. 14). Now we have room for the ensemble as it exists in Figure 10, but we must remember that this image is just a snapshot of the ensemble at one point in our normal time. So we have to add back in our normal time (Fig. 15). Now we have four time-like dimensions and three space-like. Is this large enough to represent our ensemble? The answer again is no.

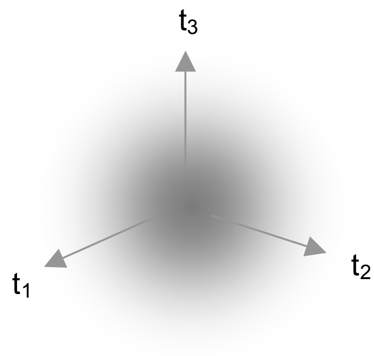


Figure 13. We can imagine the probability cloud existing within three *time* directions. Perhaps that way we can explain why we only detect one quantum of the ensemble at a time.

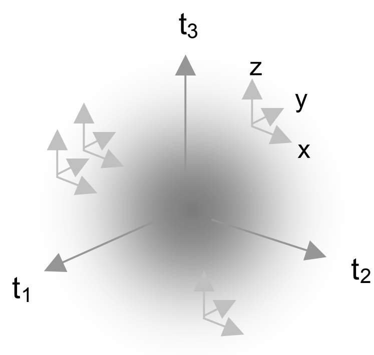


Figure 14. But to do this, we will have to imagine a separate 3-D space intersecting with each point of this 3-D time.

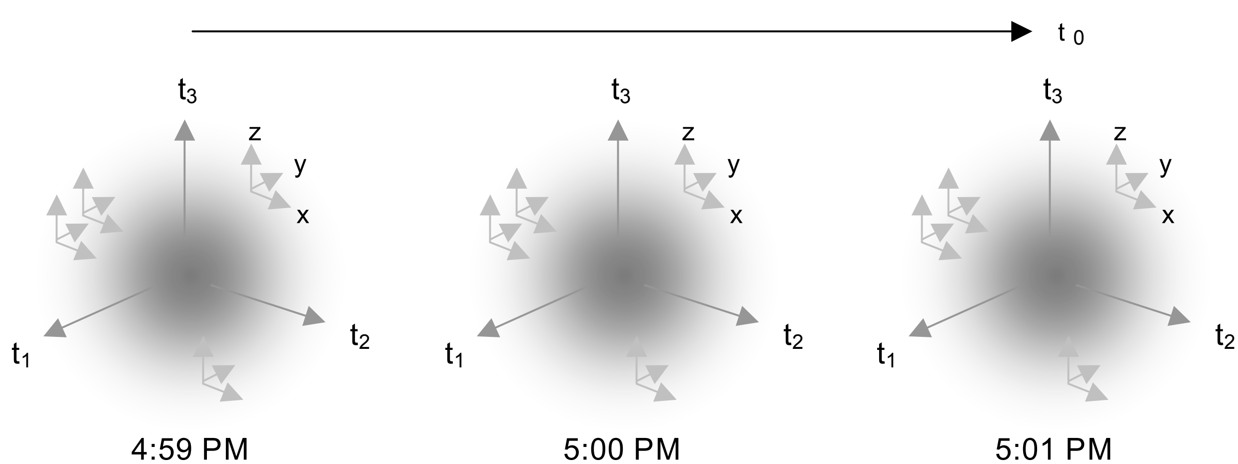


Figure 15. Fig. 14 is just a snapshot of one point in our normal time. Here, we include multiple points in our time. Our normal arrow of time is represented at the top.

The problem is that we need a way to represent the difference in probability of finding the particle at each point. What we find at each point of this 3-time world is either a portion of a particle or many copies of the particle (depending on how we normalize the probability space; see Fig. 16). There is no way to construct it so that we find one and only one particle at each point of the 3-D time—each point of which is associated with a separate 3-D space—which is what we must do if we want our model to match what we actually observe in our physical world. We know that at each moment

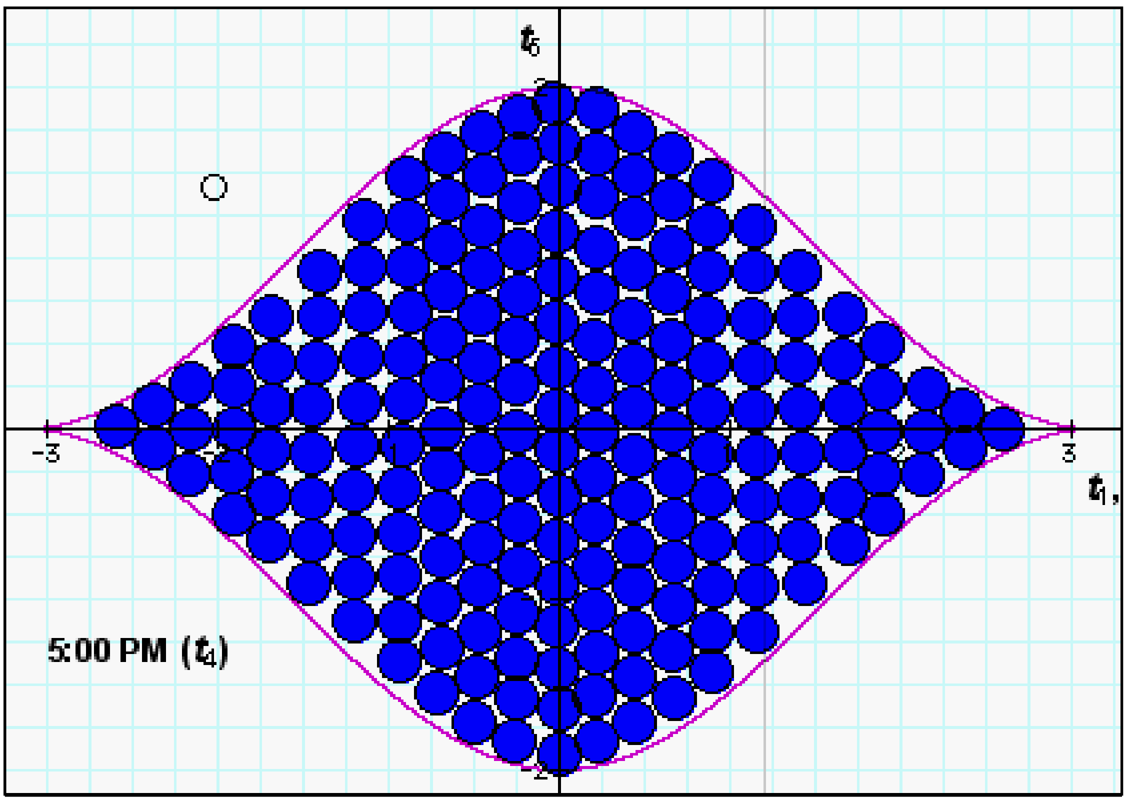


Figure 16. A different kind of snapshot of the electron cloud, showing the varying density of the cloud. The horizontal line represents any one of the three time directions shown in Fig. 13. The number of blue discs intersected by the faint gray line (which will animate) represents the probability of finding the electron at the equivalent point in space. Actually, the shape should not pinch off completely at either end but should get thinner and thinner; there is some small probability of finding the electron anywhere.

We have imagined this density as spreading into a fifth time dimension so that we can try to create a spacetime that has only one quantum (one blue disc) at each point of the spacetime. But if we do that, we end up with spaces that have no quantum. The small empty circle in the upper left quadrant indicates such a space.

in time within our own 3-D space, we always have the ability to detect the particle *somewhere*, although we may not be able to predict where that somewhere will be—and we know that when we do detect it, we always detect one whole particle. We must imagine a fifth time-like direction, so that at regions of high probability, we have many parallel universes, each of which contains a copy of the particle at the same point in space.

To recapitulate, we now have three time-like dimensions forming a 3-D time with one point corresponding to each point in our 3-D space; we have another time-like dimension that allows for a “thickness” to accommodate the probability level at each point—the number of members of the ensemble that exist at that point in space; and we have yet another time dimension that corresponds to our usual conception of time. Now each 3-D space contains at most one quantum, but in regions of low probability, there are many spaces that do not contain a copy (as in Figure 16). This is a problem, because we still do not have a 1-to-1 correspondence between quanta and spaces, as we should if we want to maintain conservation of mass for the interaction.

There is another way we can go. We can imagine that the number of spaces that intersect each point of the 3-D time is proportional to the probability of detecting the particle there. (Compare Figs. 17a and b.) This actually gives us what we want. Starting from the emission of a quantum, we can travel to any number of 3-D spaces, and each space has exactly one copy of the particle. We still have to use five time dimensions, but mass is conserved and the probability distribution has been replaced by a mathematically identical distribution of mass/energy. However, we have not solved the problem of entropy discussed in the previous section. Most of our new “time”

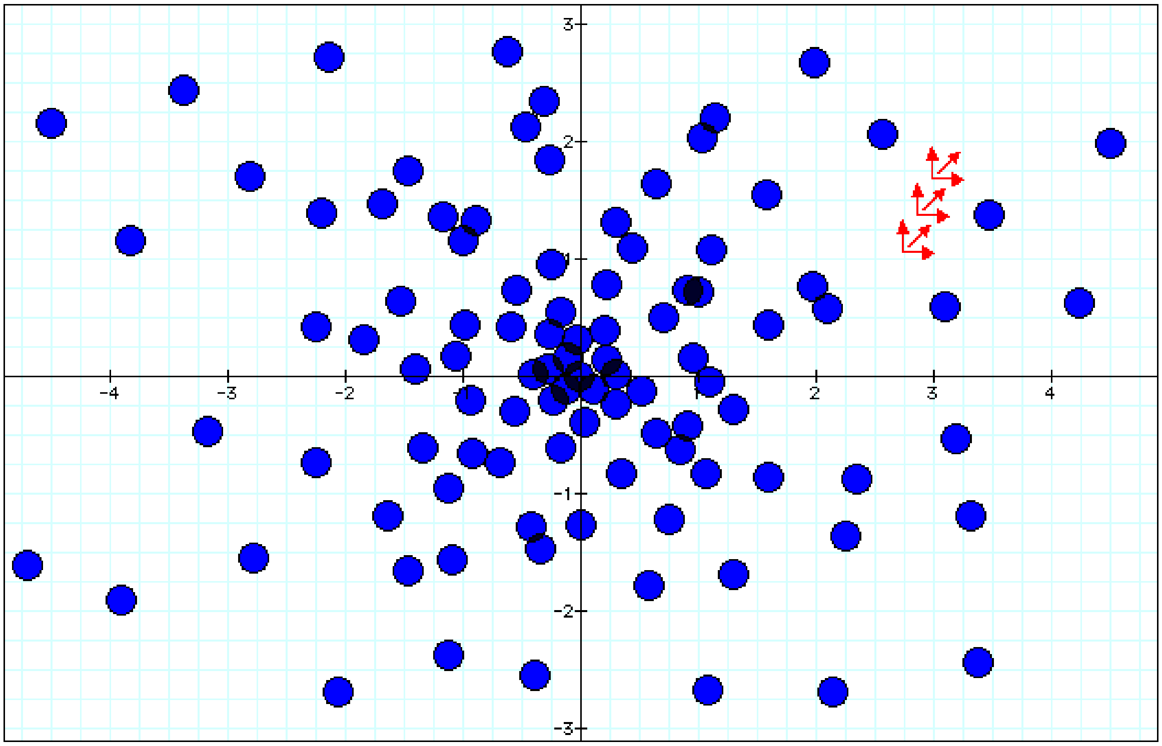


Figure 17a. Cross-section of Fig. 14. We show only a few members of the ensemble so that we can represent the density without using another dimension. 3-D spaces branch off at each point. But in the center, each 3-D space will intersect many quanta.

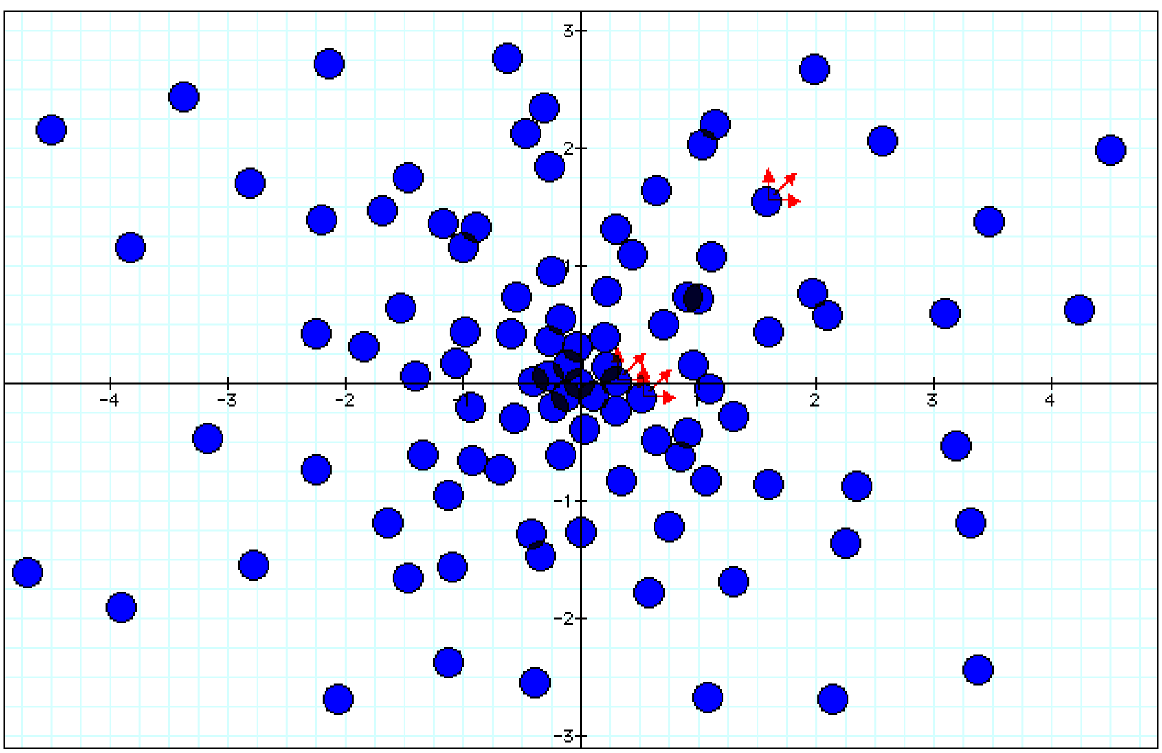


Figure 17b. Alternatively, we can posit one 3-D space branching off per quantum. In that case, there will be many 3-D spaces branching off from each point near the center.

dimensions still have a mixture of time-like and space-like conservation properties.

4. Results for the ensemble exercise

Our attempt to account for the statistical ensemble by postulating a real and unambiguous existence for each member of the ensemble within additional time-like dimensions has run into several complications. We have not succeeded in simplifying the picture, and therefore it is hard to justify the choice of this model over statistical ones. Also, in terms of conservation properties, the extra dimensions we have produced seem to have a mixture of space-like and time-like characteristics. But the really important drawback of this large time model is that even with all of these extra dimensions, it gives us no easy way to account for interference. This is the focus of the next section.

D. Reinterpreting the Meaning of the Quantum *i*

1. The Schrödinger equation

a. The two slit experiment. In Part I, we discussed the fact that the equations that successfully describe the results we see both in the laboratory and in nature require complex quantities. What motivates this description is the existence of interference phenomena. The puzzling nature of these phenomena is probably best grasped visually {endnote 11}. In the classic two-slit experiment, we have a set-up with light being emitted one photon at a time (the source) toward a photosensitive screen (the sink). An opaque panel with two slits through it is placed between the source and the sink. If only one slit is open, the light captured on the screen will form a diffuse pattern that is bright on the part of the screen directly opposite the slit and that gradually fades out to either side of the bright area (Fig. 18). This pattern can be understood from within the contexts of both the particle and the wave models of light. With the wave model, the pattern is a simple diffraction pattern; the wave diffracts and spreads as it passes through the slit. Using the particle model, we can see that the bright center area is formed from photons that pass directly through the slit, and the fainter side areas from photons that hit the sides of the slit and deflect through various angles en route to the screen.

The strangeness begins when we open the second slit (Fig. 19). Some areas of the screen become brighter, as we would expect, but some areas that were bright with one slit open are now dark. With the wave model this is easy to explain. The waves

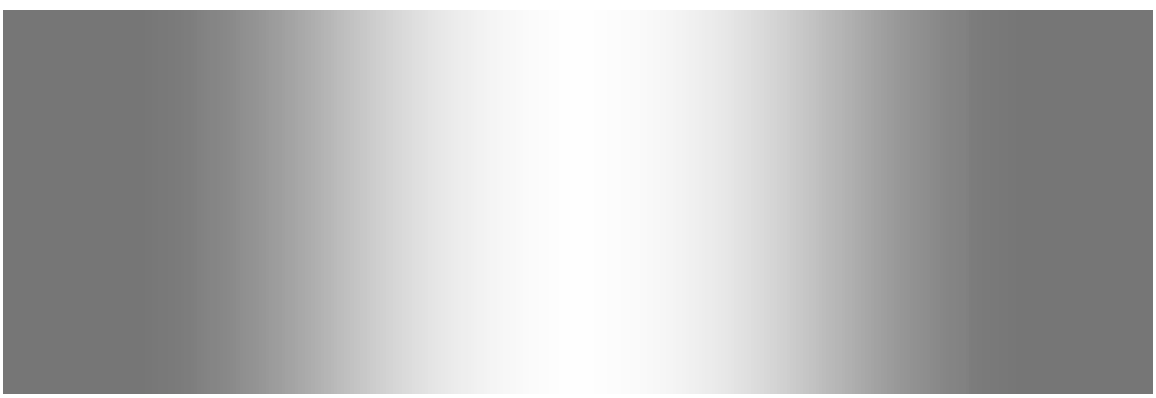


Figure 18. The pattern of light through a single slit.

Figure 19. The pattern of light through two slits. At certain angles, the light from the two slits interferes and the light waves cancel each other. At the middle of the dark ridges are areas where no photons can hit the screen.

through the two slits interfere and we get a pattern similar to the pattern we would obtain with water waves passing through a similar set-up. With the particle model, the explanation is more difficult. But before we try to give one, it must be noted that this is not the end of the weirdness.

If we turn down our light source until it sends only one photon at a time to the screen, say one every second, at first it will seem as though the photons are hitting at random points. But when enough photons have hit, we can see that they are again forming an interference pattern. The puzzle is, when a single photon has passed alone through the slit, how does it know where it *cannot* hit? How does it know whether or not the second slit is open? If the second slit is open, we know that there are some areas where that photon, in no circumstance, will ever hit. But if the second slit is closed, the photon can hit the screen anywhere, although it is more likely to be absorbed somewhere near the bright mid-line. It seems as though, in some odd way, the photon is interfering with itself. (For more on this, see Appendix D.)

b. Proposing additional time-like dimensions. The question we want to pose here is whether we can use additional time-like dimensions to form a theory of how a photon (or electron or proton) can interfere with itself. Let us look at the complex portion of the wave equation and speculate about where additional time axes might enter in. (All of the following ways have no doubt been tried, but that should not stop us from fresh speculation, if for no other reason than to see what does not work and why.)

The wave function can be written

$$e^{-i\omega t}\,\psi(x)$$

where the portion of the wave that depends on time, the complex portion, is

$$e^{-i\omega t}.$$

One way to proceed might simply be to break the time into components:

$$e^{-i\omega(\Delta t + \Delta u + \Delta v)} = e^{-i\omega t} e^{-i\omega u} e^{-i\omega v}.$$

But the only thing here that could have physical significance is the vector sum of the times, so the new components would not affect our representation of interference phenomena. We can try another tack:

$$e^{-i\omega t} = \cos \omega t - i \sin \omega t . \qquad \text{eq. (1)}$$

This gets us into the actual mathematics of the interference phenomena. (For a visual review, see Appendix D.)

Equation (1) has a real cosine term and an imaginary sine term, as shown in Figs. 20 and 21. In the interactive Graphing Calculator versions of the figures, the wave can be viewed from different angles. (See Note to the Reader, p. 3, concerning these versions.) Seen from head on, it appears much like Fig. 1; that is, as a circle rotating in the complex plane. The real direction (think of it extending vertically, rather than horizontally as it was in Fig. 1) represents one dimension of our 3-dimensional space. The real component of this wave has a physical interpretation—it is one component of the polarization vector. This means that if a test charge were put into the path of this wave, it would vibrate up and down in the one space dimension that we can see.

The imaginary direction (think of it here horizontally) is thought of as extending at

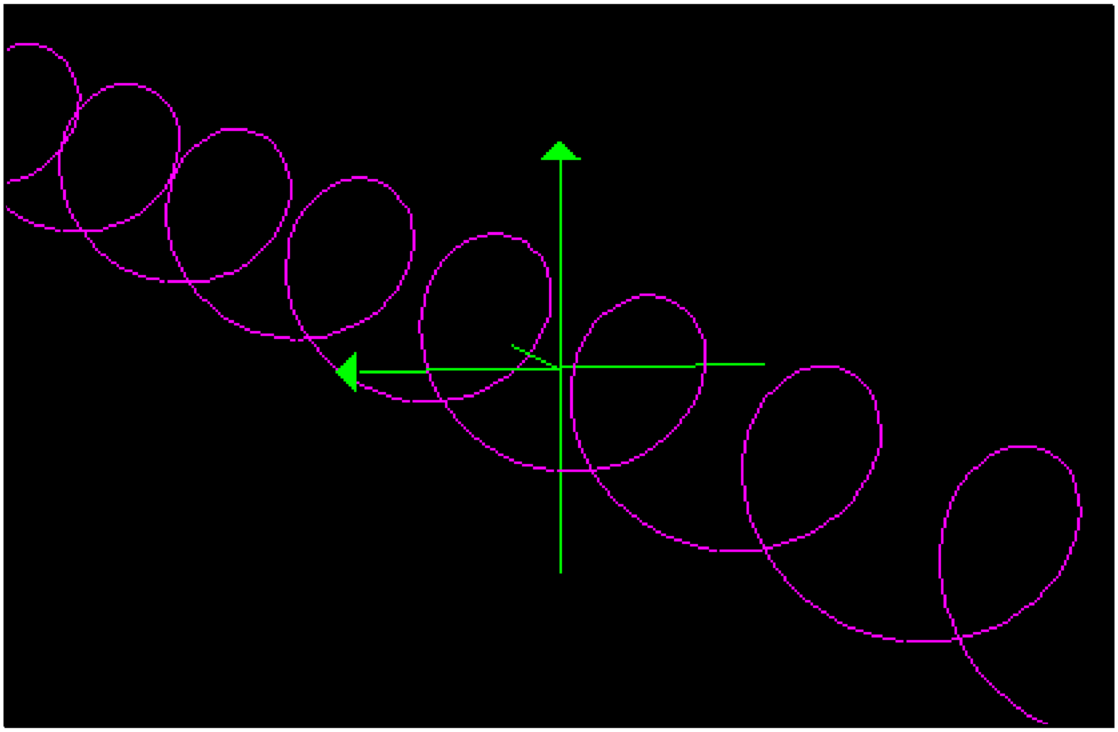


Figure 20. The $e^{-i\omega t}$ part of the wave function, which travels forward in time. (An animation.)

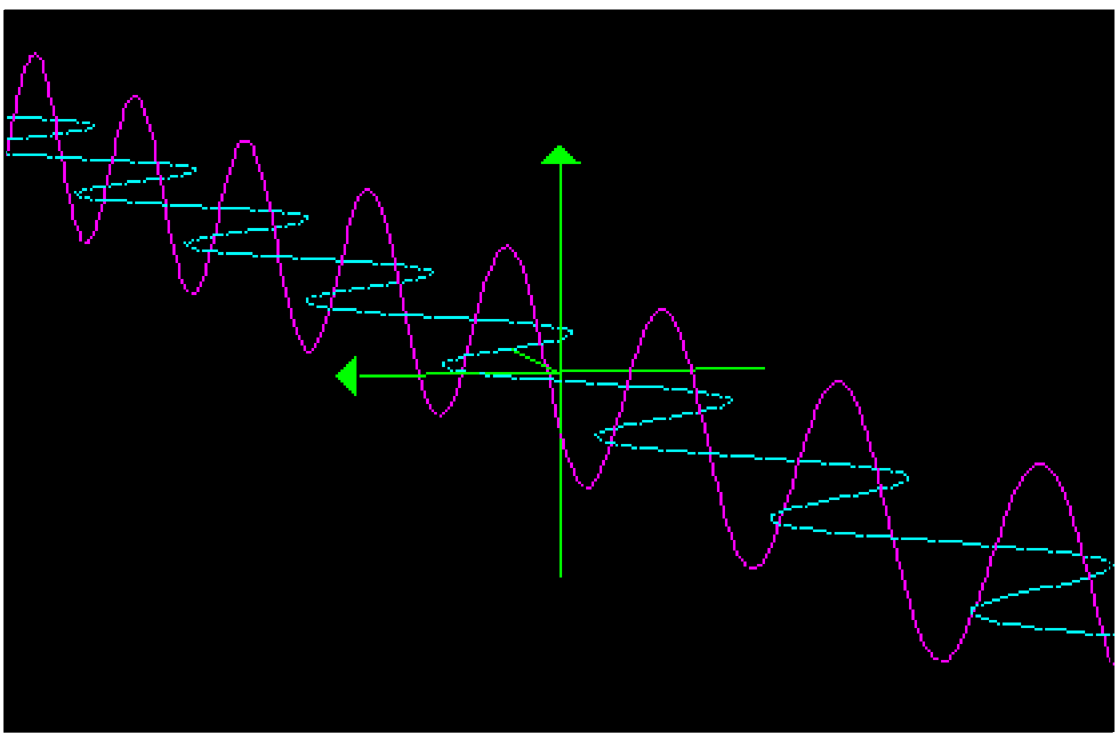


Figure 21. The same wave, split into sine and cosine components. In the original graphing calculator files, it can be seen that Fig. 20 was actually produced by adding the two waves of Fig. 21. (An animation.)

right angles to our three space directions and our time direction. The component of the wave that extends in this direction represents quantities that have to do with interference. Since they exist in an imaginary direction, they are usually thought of as non-physical quantities. But one way the imaginary component of this wave could be interpreted is as extending along an additional time axis, so that the rotation of the wave is through a small extra time-like dimension. We next ask whether we could observe effects from such a dimension.

We know that we can obtain an extremely accurate probability distribution (Fig. 16, p. 48) by multiplying the wave function by its complex conjugate. This washes out the time portion of the wave:

$$e^{-i\omega t} e^{+i\omega t} = e^{-i\omega t + i\omega t} = e^{0} = 1 ,$$

and so the probability is determined solely by the spatial portion

$$\mathrm{P(x)} = [\, e^{-i\omega t}\, \psi(x)\,]^{2} = [\, \psi(x)\,]^{2} .$$

Therefore, even if the imaginary portion were in a physical, time-like direction, nothing happening in that direction could affect our probabilities of finding the photon. If there is interference, we compute things differently and the complex portion of the combined wave squares to something other than one. But, although this affects the photon density at each point on the screen, the proportional accumulation in this case does not change with time. In fact, we can express the final distribution resulting from interference in terms of distances rather than in terms of time, and the time t disappears from our final expression for the ensemble. All of this means that the new dimension might be adequate to express interference, but it does not contribute toward the dimensions needed to show the ensemble. So, at this point, we would need the five additional time directions we discussed in Section III-C-3 plus this new direction for interference. And all of this deals just with the electromagnetic field! Once we dealt with nuclear forces, we would need whole new sets of dimensions. We really have not simplified anything.

For more ideas, we can look at two explanations proposed by others. The first was proposed by Feynman. The second, Cramer's, is a recent spin-off from a Wheeler-Feynman idea.

2. Sum Over Histories and multiple timelines

When there is more than one possible path a quantum can take to reach a given point, the result is interference. Whether there is interference or not, we can obtain the probability that a photon will arrive at a particular point by squaring the amplitude of the Schrödinger wave function at that point. As mentioned in Section II-B of this paper, Feynman (1963, 1964, 1965; see also Feynman and Hibbs, 1965) showed that if one derives mathematical expressions for every path a quantum can take from point A to point B and then adds every possible path, the result is a real amplitude—the real square root of the probability that the photon leaving point A will arrive at point B. The probability is the same as would be obtained by the Schrödinger method, but we have obtained it from its real square root rather than its complex square root {endnote 12}. This is because all of the imaginary components from the separate paths cancel each other out.

Feynman's interpretation is similar to the ensemble approach of Schrödinger

(1926), but Feynman (1965) demonstrated that his mathematical formulation is much easier to compute with than is the Schrödinger formulation. Also, note that we have moved from analyzing *positions* to analyzing *paths.*

If we use this sum-over-histories formulation for our model, we can speculate that the quantum could *in fact* take every path, if it did so in adjacent times. Therefore, our time parameter would be the summation index used in Feynman's formulation. This hints at the possibility of using one extra time dimension where above we used two. There are a couple of factors that complicate things, though. One is that this formulation only deals with a single point in the ensemble. We would still need all of the additional dimensions to deal with the ensemble that we needed above. The other factor is that included in the summation are not only paths that are adjacent to each other, but also loops, U-turns, and zigzags. So we would not have anything as neat as the nice, small, cyclical plane perpendicular to the path of the quantum that we had in the last section. A time dimension resulting from Feynman's summation would not be very easy to visualize.

One interesting ramification of this approach is that it results in a quantized time. This model of particle paths is probably much closer to what happens in our physical world, where everything is in motion, where detectors are always a source or a sink, and where position has been shown to have only relative meaning. To follow this investigation further would require delving into the path integrals used in this analysis, which we do not do here. (For the author's commentary on an exercise in particle paths using Feynman's checkerboard, see Appendix E.)

3. Transactional approach

Feynman (1961) explained that quantum electrodynamics takes the real classical (relativistic) electromagnetic wave and splits it into advanced and retarded parts, $e^{+i\omega t}$ and $e^{-i\omega t}$ (Figs. 22 and 23), retaining only the retarded part for computations involving the absorption of a quantum and the advanced part for computations involving an emission. Actually, until Wheeler suggested using the advanced wave (Fig. 22), it was normally dropped from consideration altogether; it can be seen as describing a wave traveling backwards in time and was therefore regarded as unphysical (Feynman, 1965). The fact that $e^{-i\omega t}$ had to be multiplied by its complex conjugate—which is $e^{+i\omega t}$—in order to give the probability that a quantum would be absorbed (and $e^{+i\omega t}$ by $e^{-i\omega t}$ to give the probability of an emission) was regarded more or less as a mathematical trick that happened to give the right answer. But nothing within the theory itself explains why the advanced part should be discarded (in the case of absorption, that is), so Wheeler and Feynman explored the idea that the physical reality behind the mathematical description is the combination of an advanced and a retarded wave.

Cramer (1986), using their results along with the work of Dirac (1930) and others, has proposed a transactional approach to quantum mechanics. The retarded and advanced waves $e^{-i\omega t}$ and $e^{+i\omega t}$ are both viewed as having physical existence. They combine to produce a "handshake," a standing wave that is the real wave (Figs. 24 and 25). It has been suggested (Snyder, 2000a, 2000b) that this proposal is a step toward a time-free version of quantum mechanics where no time dimensions would be needed at all, but Cramer does not take this view. In fact, he has gone to some lengths to restore the arrow of time {endnote 13}. His theory is, however, a non-local theory, because he

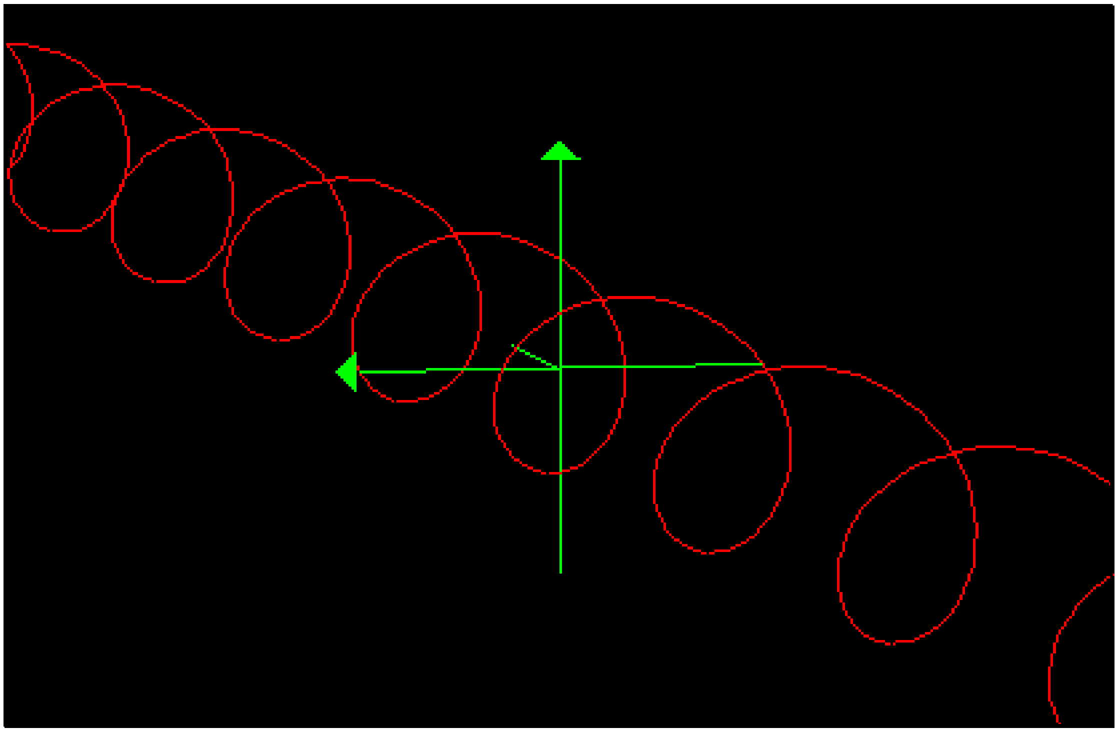


Figure 22. The advanced wave $e^{+i\omega t}$, which travels backward in time. (An animation.)

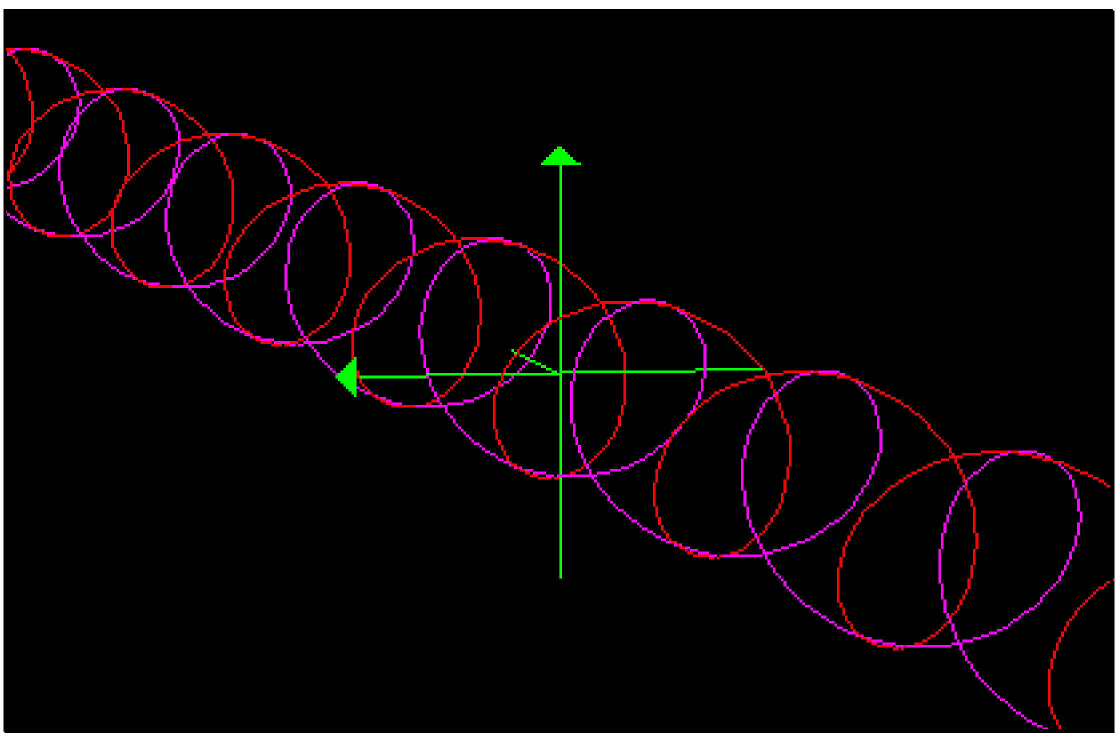


Figure 23. The two waves $e^{-i\omega t}$ and $e^{+i\omega t}$, one traveling forward and one backward in time. (But note, while the animated version will play forward and then backward when you click it, you cannot make one wave go forward and the other backward at the same time. This is not trivial: it illustrates that *we* cannot travel in more than one direction through time simultaneously. We will view both waves from the perspective of the direction in which *we* are moving through time.)

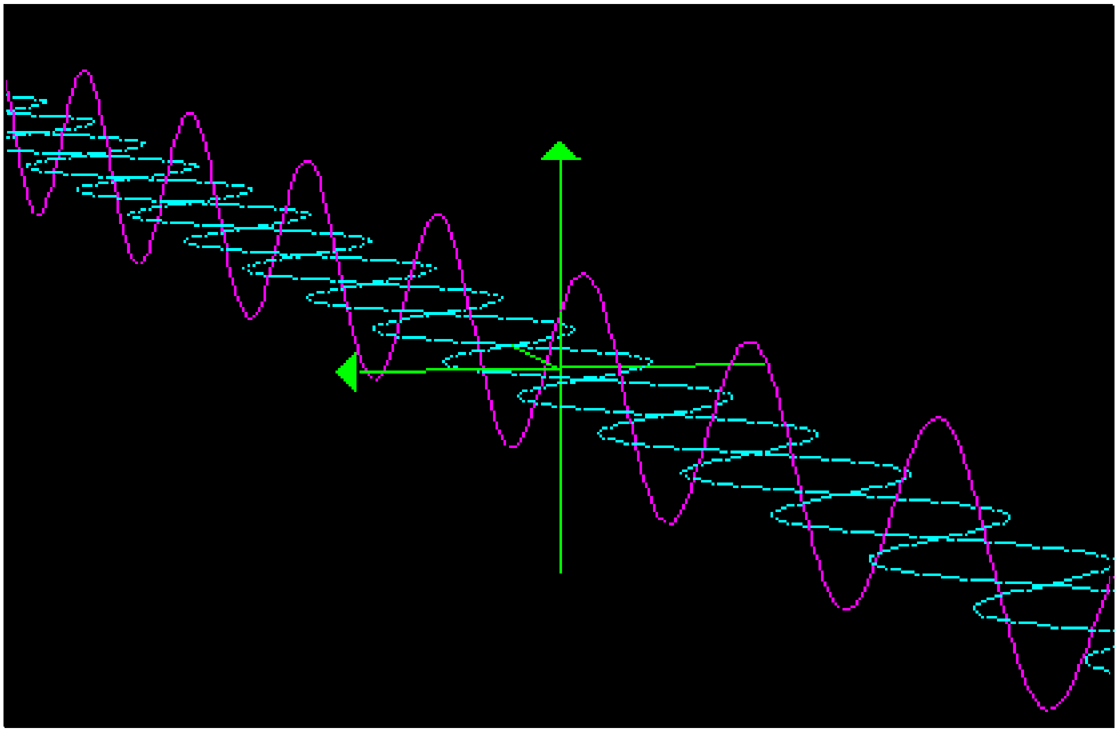


Figure 24. The forward and backward waves, as in Fig. 23, but this time split into their sine and cosine components. The second copy of the vertical, cosine wave is overlaid on the first and cannot be distinguished. (An animation.)

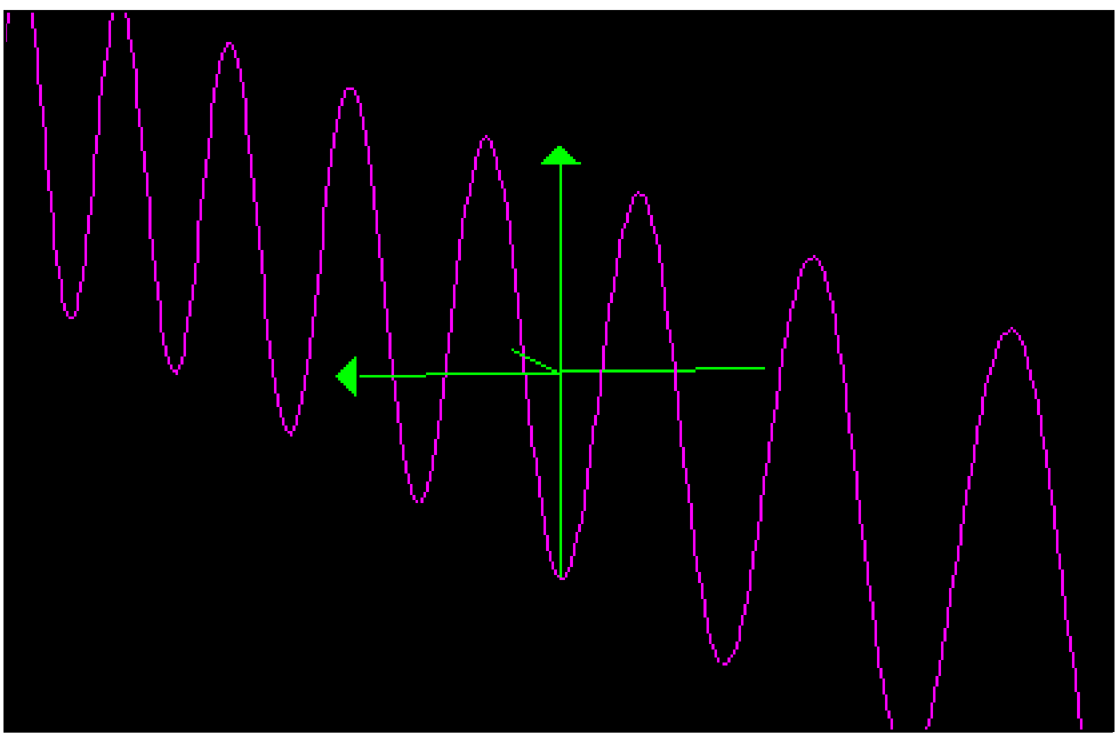


Figure 25. The two waves from Fig. 23 (or, equivalently, from Fig. 24) have been added. The imaginary components have cancelled and we are left with a real wave. This wave can be viewed from either a forward or backward in time perspective. (An animation.)

argues that cause and effect happen through combined effects from a past point and a future point separated by space as well as time. The advanced and retarded waves combine to form the real wave, which is the time-independent probability distribution function. This, then, is why we must multiply the original time-dependent wave function by its complex conjugate: the probability at a point is determined by the interaction of the forward in time wave and its backward in time counterpart.

This is equivalent to taking the cyclical extra dimension suggested in Section III-D-1 (p. 53) and allowing it to cancel out, leaving only one time dimension per member of the ensemble. This remaining time dimension is our usual time dimension, although some effects travel from the future into the past for a limited distance.

Cramer believes his theory gives a mechanism for the collapse issue (see section II-B-2, p. 13) and that it attributes a physical meaning to the imaginary quantities. In his 1986 paper, he noted an explanation for the fact that only one transaction forms: the boundary conditions generally allow for only one transaction. But he admitted that his theory does not solve the question of *which* transaction; that is, it does not explain how one destination for the quantum is picked out over all the other destinations that meet the boundary conditions.

It has been noted (Snyder, 2000) that apparently there have been no references made to Cramer's idea since 1996. (Presumably, Snyder was referring to Lockwood, 1996a, 1996b; and Deutsch, 1996.) It would be interesting to discover whether problems have been identified with Cramer's approach. Very recently, this question has gained in significance. After writing of the present paper began, the prestigious *Physical Review Letters* published experimental evidence for the existence of time-reversed effects from electromagnetic waves (Lerosey et al., May, 2004). Among other things, this evidence has important implications for the incorporation into theory of additional time dimensions, because it modifies the concept of time as an arrow—and that concept, as we have seen, complicates attempts to give unambiguous definition to additional time dimensions. Time reversal has been proposed only at the quantum level; at least partly, no doubt, because the existence of reversal on the macroscopic level would introduce paradox. Additional reversible time dimensions could eliminate at least some of the paradoxes. But whether or not we are entertaining the idea of additional time dimensions, in light of the new experimental evidence, a fresh look at Cramer's approach seems to be warranted.

E. Philosophical Considerations: Implications for Free Will

Considering a one-dimensional time that is an arrow, with increasing entropy and quantum choice lying ahead of us, it seems apparent that we can exert our freewill in the forward direction only (and within the constraints posed by quantum boundary conditions and other physical laws). But each physical interpretation of quantum theory gives rise to different philosophical implications concerning the structure of physical existence and the functioning of our free will within it.

The statistical interpretation of the wave function leads to a picture of fundamental randomness underlying our universe, with very dependable averages on which a stable macroscopic physical world is constructed. This interpretation leaves open the question of the nature of objective reality and focuses exclusively on what it is that we can perceive. Not answered is how one event is picked out of the untold number of possible quantum events available at each instant. Theorists have reacted to this situation in a variety of ways: with dissatisfaction (Einstein), with a refusal to speculate (Bohr), and even with a belief that consciousness is required to collapse the wave function and produce an event (Wigner and Wheeler, see {endnote 14}).

An alternative to the statistical interpretation is Everett's idea that the universe splits into multiverses at each quantum event (1957). But in Everett's multiverse, there is no going back and no going sideways; different future universes can never communicate with each other. Questions concerning free will could be somewhat irrelevant in this picture. Then there are the backwards and forwards in time models that deal with the possibility of events going both directions in a one-dimensional time. If this could happen on a macroscopic level, we would either have the existence of paradox or we would have to renounce our belief in any freewill that could allow us to act in a way that could change the past.

In a universe with multiple time directions, there is the possibility of multiple present moments, of multiple pasts that can interact with each other, and of multiple futures that can do the same. Quantum theory and relativity both give restrictions on how such interactions could happen, but if the elementary quanta of which we are composed turned out to exist within a larger-dimensional space, then it stands to reason that in some sense so would we. We perceive ourselves as moving forward and exerting freewill into the future, but in a universe where multiple time dimensions do not "flow" (as in the philosophical writings of Roberts, 1977-79; Lockwood, 1996a, 1996b; and Deutsch, 1996), this could be related more to the development of our perceptual apparatus in relation to entropy than to the actual nature of our freewill. In a universe where all of reality was happening "at once," then our freewill, happening solely in the present moment, could affect the past as well as the future. A possible mechanism for this is depicted in one of Feynman's formulations of quantum mechanics, wherein each event relies on the wave function that describes all possible points at which the quantum may be absorbed at any time in the future (1965; discussion in Cramer, 1986, sect. 3.1). In this scenario, the entire future universe has a part in determining the present moment. The interesting thing is that this formulation is entirely consistent with quantum mechanics, if somewhat unwieldy to use. The philosophical and freewill implications for such a universe have been explored in Roberts, *ibid.*, and Lockwood.

In a universe with enough degrees of freedom (enough dimensions), it is possible that every event that could happen, would happen. This is a familiar speculation, introduced many times by theorists and non-theorists alike. It is similar to the many worlds idea, with two important differences. In a multi-dimensional time with no rigid arrow of causality, we could have an infinite number of pasts branching backwards from each present. To imagine what this might be like to experience, we could use the analogy of our experience of the future: our memory of the past would be quite precise for the immediate past and would grow more fuzzy with increasing distance into the past. (To anyone over the age of fifty, this might sound familiar.) The fact that we can read precise contemporary descriptions of past events and view photos that fix past moments is a complicating factor in this scenario—which, however, is dealt with fairly effectively in the Roberts' work. The function of freewill in such a scenario becomes rather interesting and complicated. Either we experience everything or we choose which present moment to experience—or we have some combination of these. If we can choose which alternate present moment to experience, what happens to people who experience other alternate presents? What happens if they run into some version of me that I am not choosing to experience? One possible answer is suggested by the uncollapsed wave equation. Perhaps we would experience everything but could choose how much “weight” to give each version of the experience. Rather than a spike, personal experience could be a sharply peaked function. (This raises the interesting possibility that I could share a moment with someone else who is experiencing our shared moment somewhere other than on his own personal peak of experience.)

The central point in all of this is that it is possible that such a universe could be experienced in a way that is not all that different from our normal daily experiences—close enough, in fact, that the differences could be accounted for by the interpretations that we give to experience. These interpretations are influenced by our cultural and private beliefs about the nature of time and of freewill. At the very least, we can conclude that the existence of additional time or space dimensions does not necessarily imply a loss of freewill, and could even imply a greater flexibility in that regard.

IV. CONCLUSIONS, SUMMARY, SUGGESTIONS FOR FUTURE RESEARCH

A. Summary of Results

We have examined a number of different interpretations, almost all of which spring from the same mathematics. We have not, however, attempted to reconcile the differences. Rather, we have looked at the physical ideas suggested by each to see what new ideas might emerge. In his Nobel lecture, Feynman made the observation that even though different theories may be equivalent in their mathematics and in all of their predictions, if they use different physical ideas—such as the idea of collapse of the wave function, the idea of a handshake, or the idea that every possible path is taken—they are not *psychologically* equivalent (1965). When the theorist tries to move from the base of current theory into the unknown, it is her physical view that suggests new modifications to the mathematics, and therefore, new hypotheses. Feynman maintained that a good theoretical physicist will have available a wide range of mathematical expressions and interpretations for the same theory. So we have left the base wide, hoping this will provide more possibility for springboards into the unknown.

We have taken a first tiny jump into the unknown by proposing some naïve large-time models, some of which have led to dimensions that have a strange mix of space-like and time-like attributes. None of these models appears to offer any easy physical explanations for the quantum wave function, but their exploration has, perhaps, helped clarify why theorists have tended to choose the statistical interpretation of the wave function over proposals for additional time dimensions.

The short answer to the thesis question of whether we can rule out the possibility that there is more than one time dimension is "Probably not." There is probably no way to rule out the existence of extra dimensions, time-like or space-like. The beginning assumption was that the thesis question is a falsifiable one, but we conclude that it may in fact not be either falsifiable or provable. If there does exist more than one degree of freedom for matter in a time-like direction, the effects are not necessarily observable in our 3 + 1 space (as, for example, in Everett's many-worlds interpretation, 1957). If such effects are observable (or have already been observed), there is not necessarily any way to prove beyond doubt that extra time-like dimensions are the source of them. The results of Lerosey et al., (2004) provide hope for provability in some of these areas. However, as we have seen, sets of observed phenomena often stimulate a wide range of interpretations, and we cannot always obtain proof of the validity of one interpretation over another. Often, the best we can do with experimental results is to use them to put boundaries on the kinds of interpretations that we can accept. For instance, we might be able to prove that *if* there were any extra time-like dimensions, they would have to be in the form of an arrow; or, conversely, that they could not be; or that they could only contain exotic matter; etcetera. Or we could conclude that the idea of large dimensions is simply not a useful one and that science is better served with models based on the degrees of freedom exhibited by individual interactions.

It might well be asked whether, if a matter turns out to be neither falsifiable nor provable, there is any room in science for speculation about it. The traditional answer to this question has been negative. It is certainly important to the integrity of science to be clear about the nature of provability of any question that we pose. But as Feynman (1965), and later Cramer (1986), has pointed out, though they may be un-provable, the mental models that we entertain to explain a given physical situation will shape the questions that we pose, the experiments that we conduct, and the directions in which we give our theories a chance to grow. In areas where we ask too few questions or do not entertain a variety of explanations for what we think we know, our science is likely to grow stagnant. Therefore, a valid approach to theoretical research must be to look for questions that have not been asked and to ask them. There are many more questions to ask than we have time or resources to answer; we cannot sit back and ask only those that are driven by experiment. Rather, we must ask the questions that occur to us, knowing that we cannot anticipate what they may inspire. Our sciences, and the possibilities that they bring to the human experience, can grow only in the areas that are given attention.

B. Suggestions for Future Research

1. Experiment

In spite of what was just said above, one thing a theorist can do is to scan experimental results to see if they have implications for the theoretical questions at hand. There is often a chance, if not an expectation, that a particular idea can be ruled in or out. Even though there may be many formulations that can account for the same selection of known phenomena, these formulations do not necessarily make the same predictions for phenomena that have not yet been observed (Feynman, 1965). It is important for the theorist to examine her formulation for any differences, no matter how small, between its physical predictions and the predictions of other existing formulations. Concerning multiple-time scenarios, what could be looked for is some difference between interference-thorough-time phenomena and interference-of-probabilities phenomena. If a predicted difference could be found in theory, this would indicate areas of experiment to watch closely.

2. Theory

The recent backwards-in-time experimental results of Lerosey et al. (2004) suggest that a useful direction for future inquiry will be to investigate further the possibility that time does not “flow,” with a more thorough study of the theoretical results of Cramer. Also suggested is a revisit to the philosophical works of Lockwood and Deutsch. It will be interesting to observe the feedback from the scientific community to Lerosey et al., especially to watch for any alternative explanations for their results. With regard to a flowing time, one may ask what the (apparently now proven) ability to do time-reversed focusing of electromagnetic waves would imply for signals traveling in some of the more complicated spacetime paths suggested by general relativity, such as Thorne’s (1988) large time loops, where a local future intersects a global past.

A tangential line of investigation could be to look within such fields as neurophysiology and neuropsychology for information about the nature of the constraints that our perceptual apparatus places on our perceptions, and to ask what implication this may have for our perception of a time that flows.

As an alternative to string theorists’ search for large-dimensional spaces, it seems profitable to focus instead on models that do not rely on one single large spacetime as a container. In addition to Cramer’s approach, there is also Finkelstein’s action physics (1996), where dimensionality is viewed as springing from the interactions themselves. This model allows us to view our 3 + 1 spacetime strictly as the space of interaction between our perceptual apparatus and the rest of physical reality. This approach respects Bohr’s dictum always to begin with what it is that we can know, but it goes a little further, by focusing on defining the boundaries of that which we can know, and then spending its time at those boundaries. Examining Cramer's interpretation from the perspective of Finkelstein's work—and vice versa—could prove interesting.

A study of Feynman's path integrals (Feynman and Hibbs, 1965) may prove helpful because they suggest a different way of quantizing additional dimensions. Another mathematical tool is Geometric Algebra {endnote 15}, the recently rediscovered branch of mathematics that inspired Figs. 3-5 and the diagrams of Stephens (2004). Viewing the topics through the lens of GA may suggest more easily visualizable models for large-dimensional interactions. In general, the use of new mathematical tools on previous formulations has the potential to suggest whole new areas of theoretical inquiry {endnote 16}.

To summarize, Cramer's interpretation has been given new experimental support. Applying new, largely forgotten, or under-utilized mathematical tools to the subjects of transactional and interaction models could provide more useful and more easily visualized models as alternatives to the endless configurations of additional space dimensions often used to explain our experimental results. The fledgling researcher into the nature of time finds no shortage of questions, having, perhaps, not enough expertise to rule most of them out.

APPENDIX A: Difficulty in Defining a Conservation Law Through Space

Our experience of matter is that it is unevenly distributed from point to point in space. It might seem at first that we could simply compare this with the distribution from point to point in time. But when we formulate our conservation laws by slicing up spacetime, we have to keep track of how we are slicing up things. Are we comparing a 1-dimensional point in spacetime to another point, as we generally are when we are analyzing a distribution through space? Or are we comparing a 3-dimensional slice of spacetime to another slice, as we normally are when we analyze a distribution through time? Or do we want to compare 2-dimensional slices? Considered from point to point, the distribution of matter is not consistent, whether we travel in a space-like or a time-like direction. So we cannot define a conservation law this way. From slice to slice, the distribution is constant (we believe) if we are moving in a time-like direction and if the 3-dimensional slices span the entire universe. But if we try to imagine moving from slice to slice in a space-like direction, we will find that the slices are difficult to define. (This is the relativistic constraint that we mentioned at the beginning of this section.) Imagine a slice through the Earth's equator, one millimeter thick, and extending through all of space—and through all of time (including both before and after the Earth's existence). Even if we could define such a slice, how would we evaluate the mass/energy that exists in it? We do not—and probably cannot—have a conservation law for mass/energy unless we do one of three things: (1) consider slices that cover all of space, (2) define the law locally, or (3) (a new option) consider a 1-dimensional time as dual to a 3-dimensional space.

The third option, suggested by the work of Rowlands, et al, (2001) might allow us to construct the conservation law by comparing the sum of the mass/energy that exists in all of space at each increment of time with the sum of the mass/energy that has and will exist through all of time at each increment of space. The problem is that we run into the relative nature of simultaneity when trying to define "each increment of time" and the relative nature of position when trying to define "each increment of space." This may be one of those many instances in physics when the very concept of large dimensions does not work well. The idea of spacetime as a pattern of relationships works much better, and that is the idea we employ in our concrete example in Section III-A-1, p. 19.

(Back to Conservation Laws, page 19.)

APPENDIX B: Inversions Equal Rotations

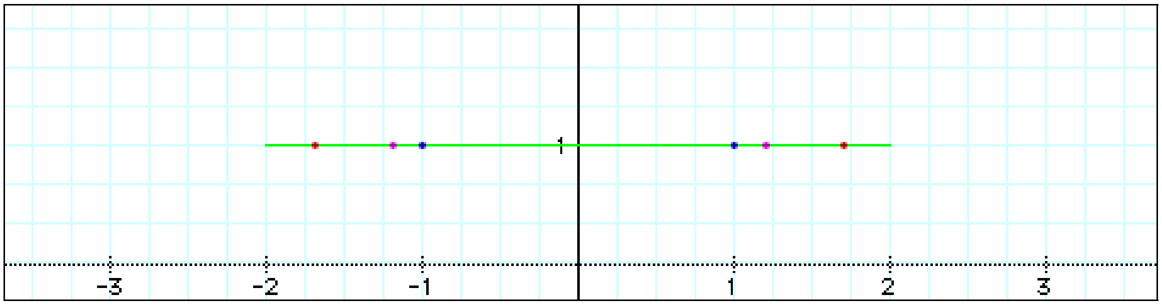


Figure B-1. The one-dimensional pattern of three dots on the left is inverted on the right.

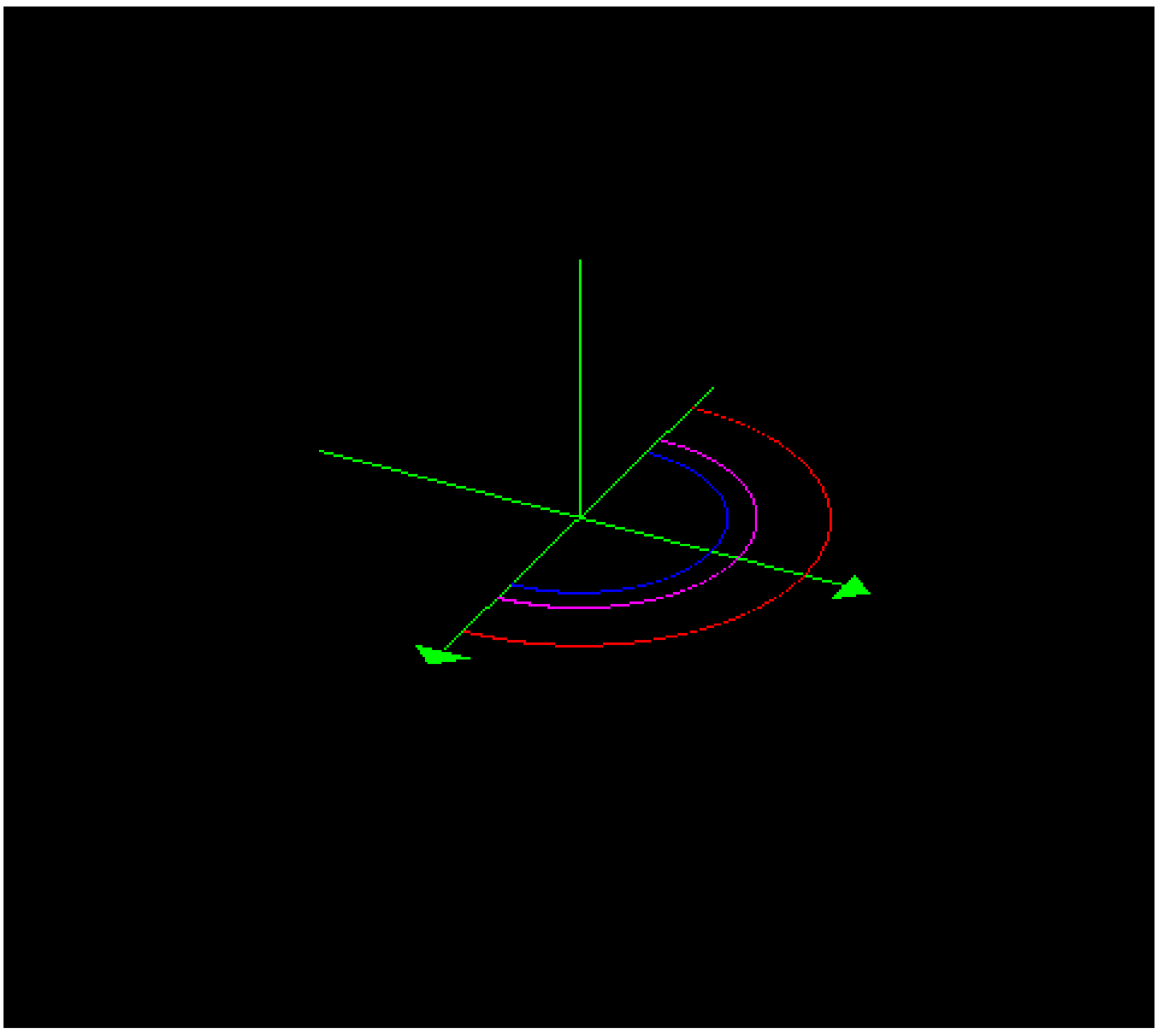


Figure B-2. The one-dimensional pattern from above has been rotated through a second dimension to produce the same result.

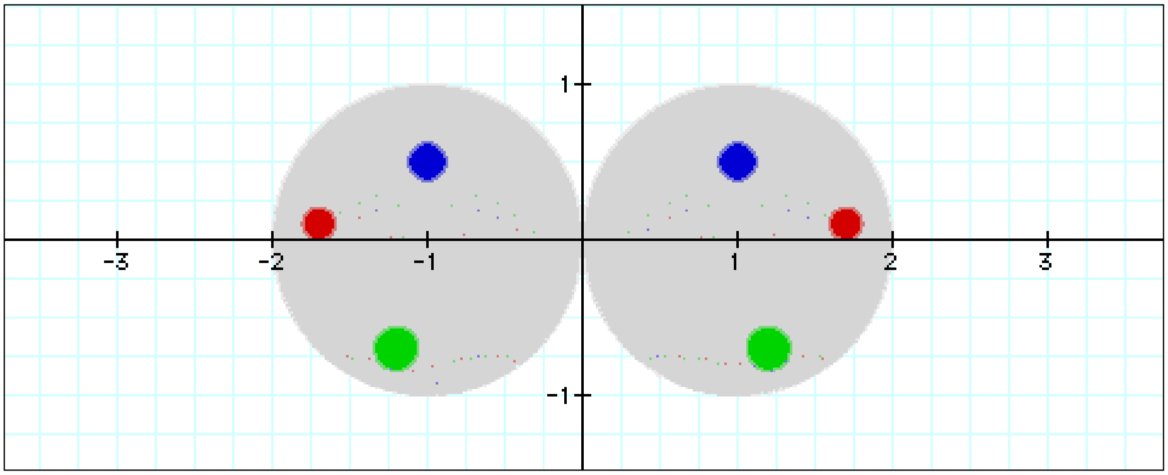


Figure B-3. The two-dimensional pattern on the left is inverted on the right.

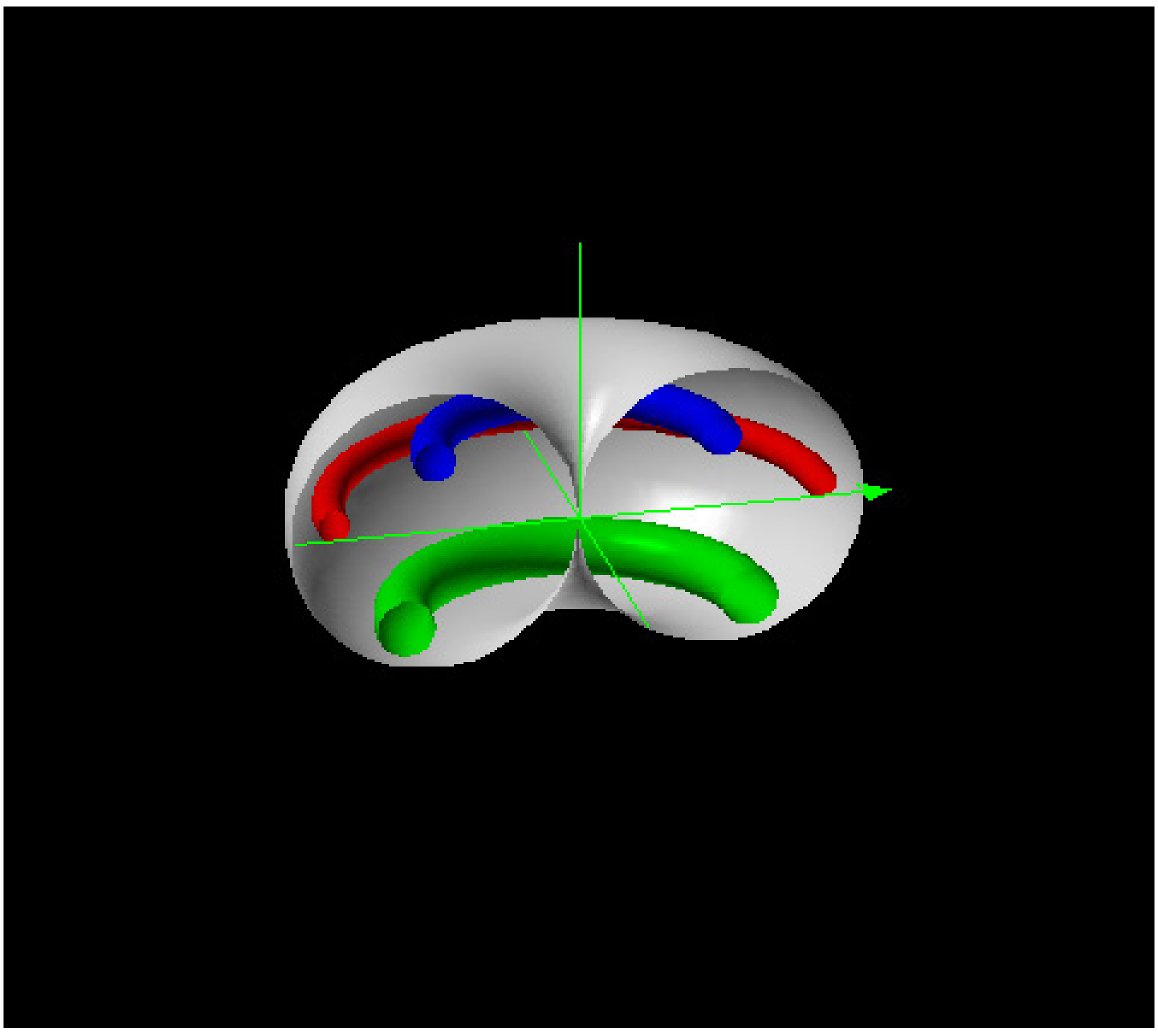


Figure B-4. The pattern from Fig. B-3 has been rotated through a third dimension.

(Back to Inversions, page 30.)

APPENDIX C: Einstein Tensor

(For the technical reader.) Here is a quick look at the terms in the tensor, which represent all possible directions of flow of the relativistic momentum. The terms take on the following meanings (Kenyon, 1990):

T^{ii} is the pressure in the x^i (space) direction from the x^i (space) component of the momentum density.

T^{i0} is the pressure in the x^0 (time) direction from the x^i (space) component of the momentum density.

T^{0i} is the pressure in the x^i (space) direction from the x^0 (time) component of the momentum density. This equals T^{i0}.

T^{00} is the pressure in the x^0 (time) direction from the x^0 (time) component of the momentum density. This is also the relativistic energy density.

Since we have more than one space direction, we also have terms with indices from multiple space directions:

T^{ij} is the pressure in the x^j (space) direction from the x^i (space) component of the momentum density.

This is a shear force, a gravitational force exerted at a direction orthogonal to the direction of the momentum of the mass that is exerting the force. It comes into play in general relativity when space is very sharply curved. Near a black hole, it can result in a loop, in which mass/energy affected by it begins to circle the singularity and cannot escape. In the Randall-Sundrum (1999) version of string theory, it comes into play in the context of extra space dimensions.

If we had more than one direction in time, then we would have terms with indices from more than one time direction. Instead of 0 for time, we could use, say, τ, μ, and ν as indices. We interpret the new mixed terms this way:

> $T^{\mu\nu}$ is the pressure in the x^{μ} (time) direction from the x^{ν} (time) component of the momentum density.

But what might this mean? By analogy with the above, it could imply a time loop, an area in space where objects are forced to cycle over and over in time without being able to break free (Gödel, 1949). But another implication is that mass/energy traveling in one time stream could exert gravitational pressure on mass/energy in an adjacent time stream.

Incidentally, nothing we have said would have any effect on the observed speed of light. The introduction of time shear terms would have no more effect on that than the introduction of space shear terms. To have shear, there must be a difference in the motion through adjacent slices of time, but this motion is, in this case, only one component of a motion through a larger dimensional time. The invariant of the motion could still easily equal *c.*

(Back to Time Sheer, p. 30.)

APPENDIX D: The Ensemble and Interference: A Visual Representation

Before we look at the interference of probability waves, let us look at how traditional waves interfere. The first three graphs represent features general to all waves. The wave could be thought of as a molecular wave, a disturbance in an electric field, or a probability wave.

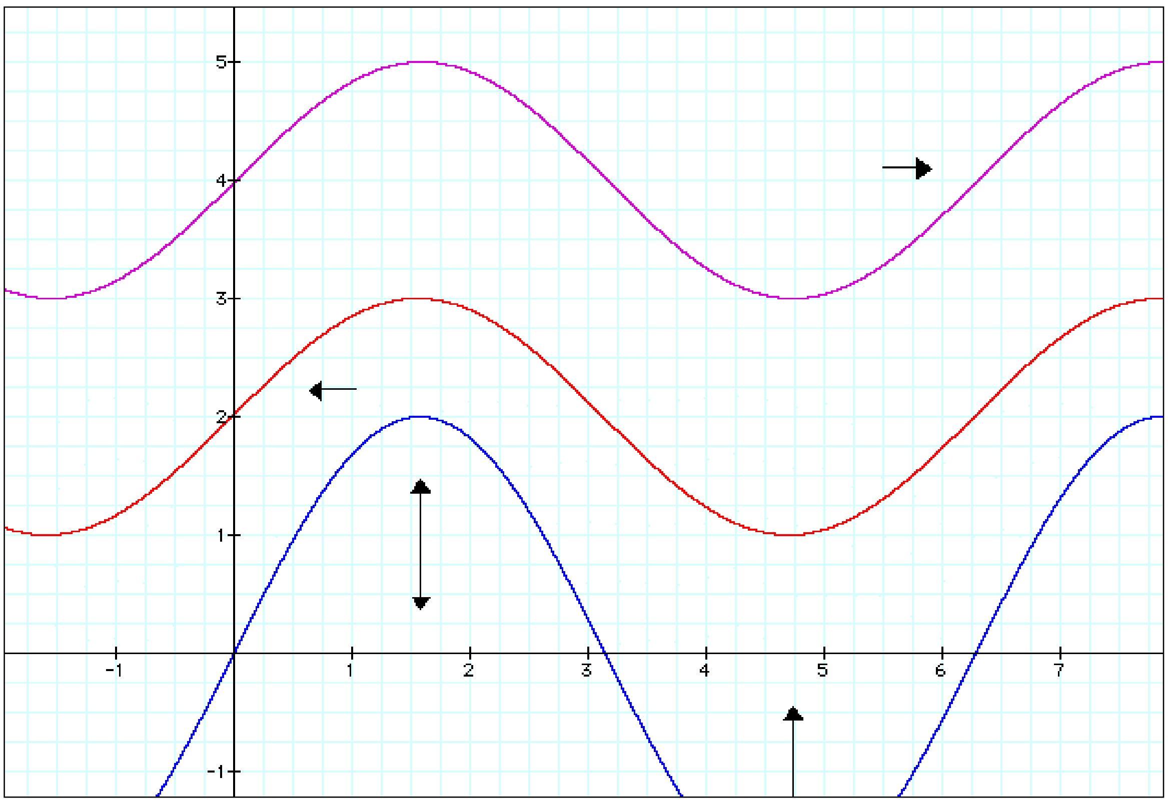


Figure D-1. The two waves at the top combine to produce the wave at the bottom.

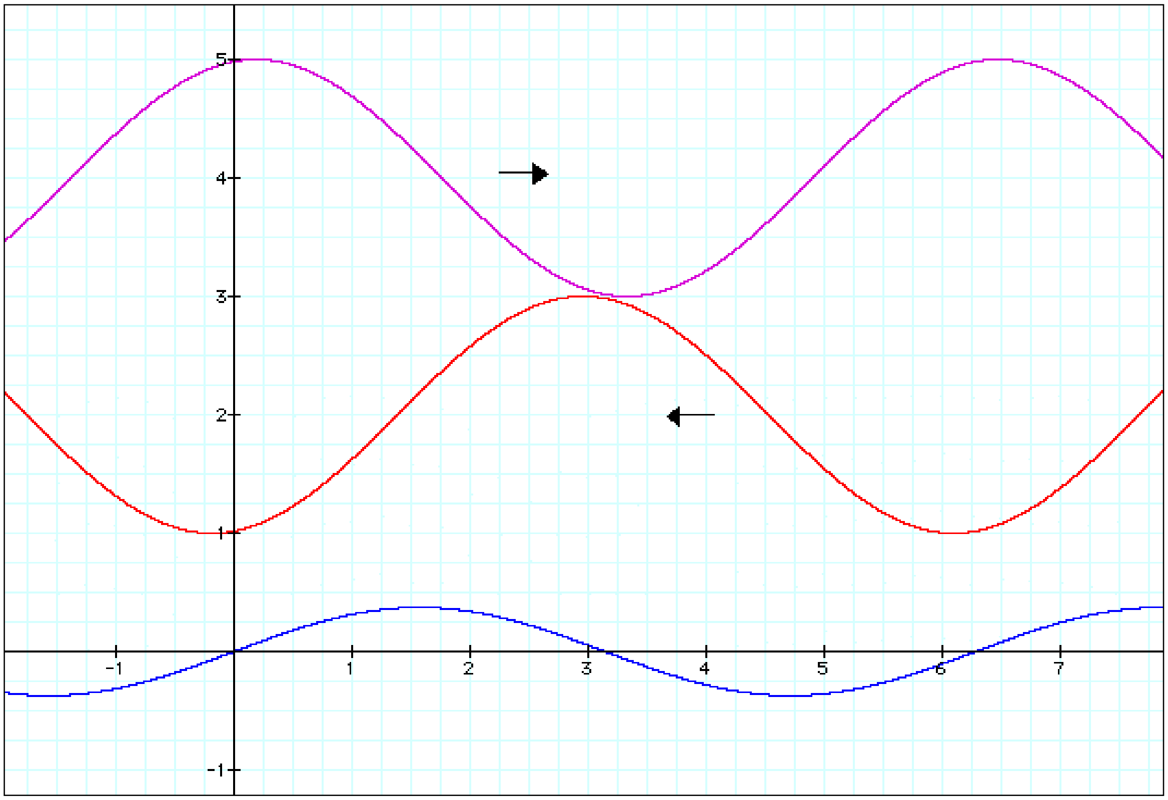


Figure D-2. The waves again combine, but this time they partially cancel.

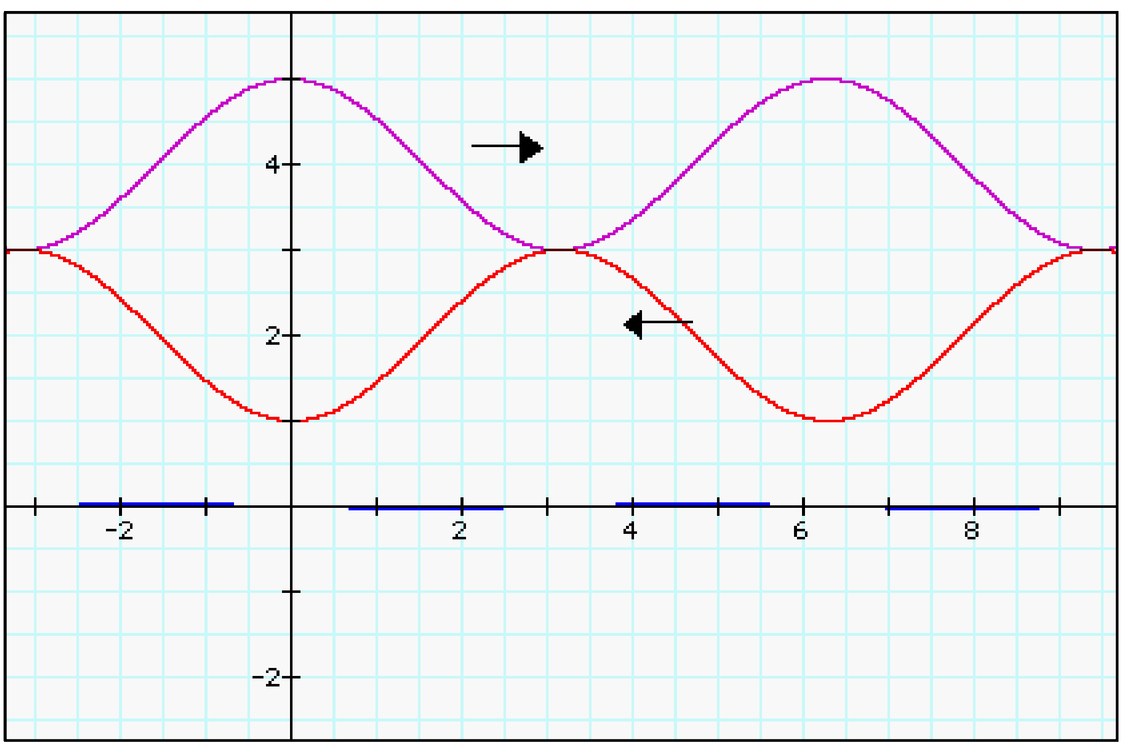


Figure D-3. Here, the waves cancel. (Double click for animation.)

Now let us look at a two-dimensional version of a water wave. In Fig. D-4, we can see that the number of molecules in a length π is the same in the absence or presence of a wave, although the wave changes the distribution.

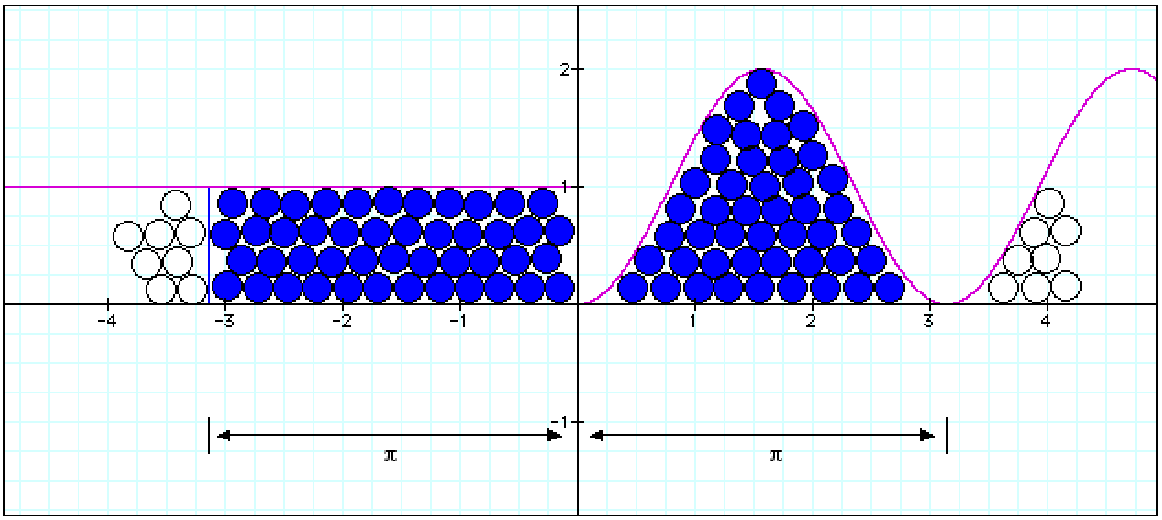


Figure D-4. On the left is a level surface, on the right a surface broken by waves. But the same number of molecules inhabits a space of width *x* whether or not the wave is present. Think of water waves. The original number of molecules is merely redistributed.

If we sent a large number of photons through the appropriate set-up, they would interfere to produce a pattern much like that of Fig. D-5. If we were to count the number of photons that hit the screen at each point across one of the bright bands, we would obtain a graph that looked a bit like our water wave.

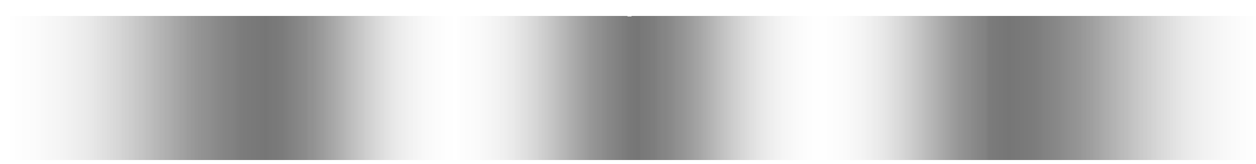


Figure D-5. Light striking a screen after it has passed through two slits.

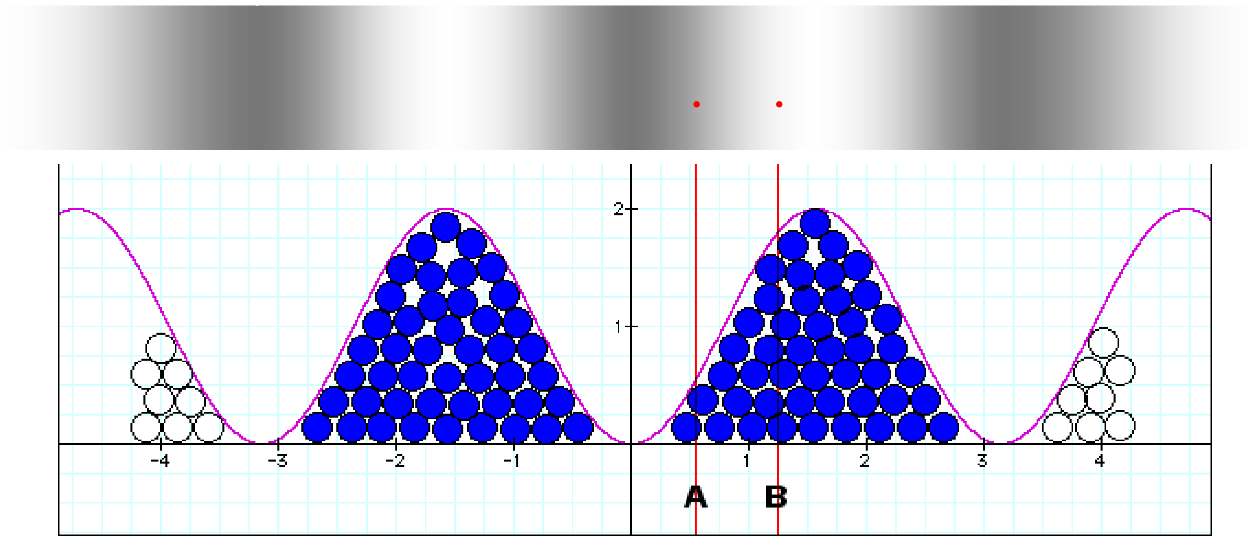


Figure D-6. The arrangement of discs represents the density of photons that have struck a screen. The (red) lines A and B correspond to the locations of the (red) dots on the screen just above. You can see that the screen is brighter in regions of higher probability; more photons have struck there.

For instance, we might detect two photons at Point A and seven photons at Point B. But if we turn down the light and send fewer photons through the set-up, we will have the same pattern—it will just have fewer dots. (Fig. D-7.)

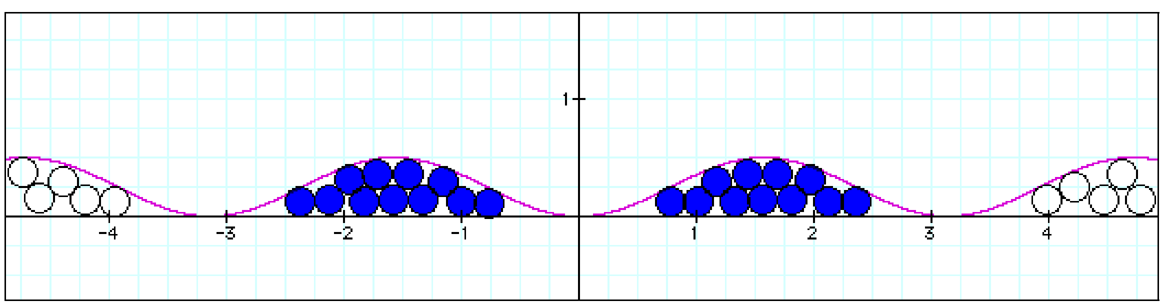
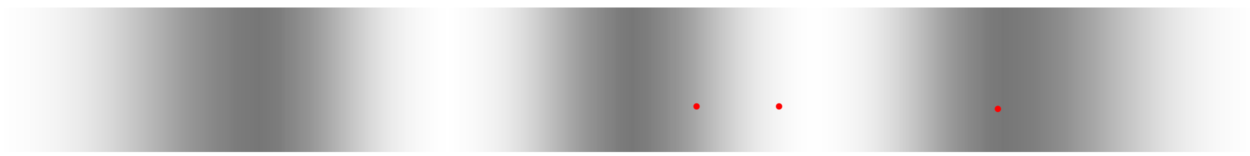


Figure D-7. I f we turn down the light source, we obtain the same pattern, even though it is more faint.

Now, if we turn down the light enough that we are sending only one photon at a time through the set-up, the photon could appear



here or here, but never here.

In fact, we can show experimentally that if we were to send single photons through a set-up that resulted in the distribution of Fig. D-6, out of forty-six times we sent a photon through, there would be a probability of 2 / 46 that any single photon would appear at Point A, a 7 / 46 probability that it would arrive at point B, and a 0 / 46 probability that it would strike Point C.

One way to imagine this is to picture many copies of the same photon, distributed as in our diagrams above, but with each copy existing in a different 3-D universe.

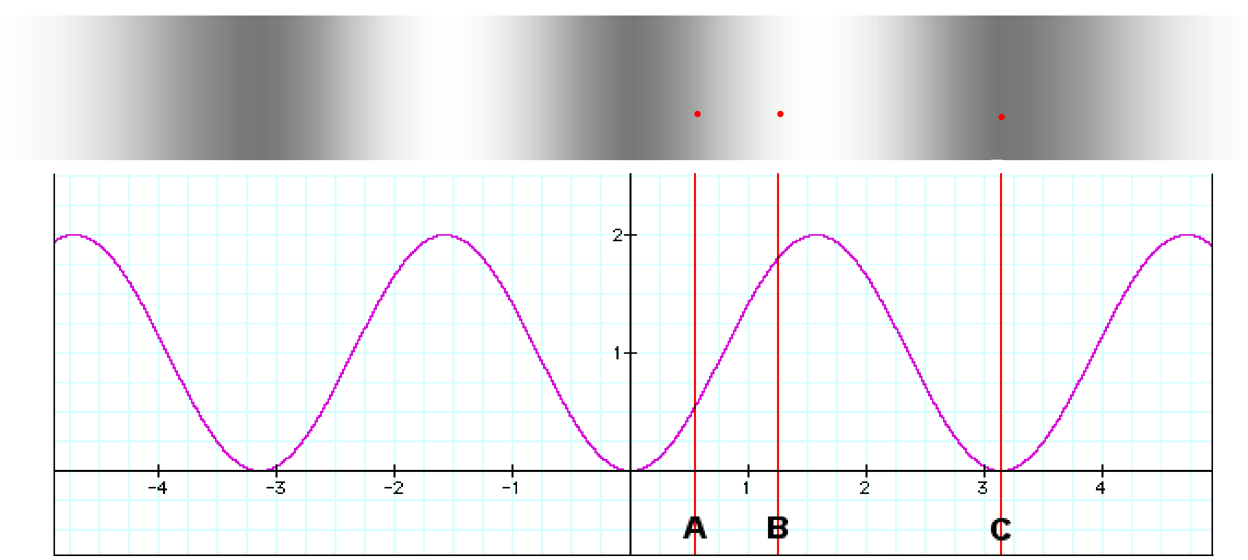


Figure D-8. If we just send one photon through the two-slit set-up at a time, it can hit anywhere except at points where the wave is zero. Using the probability distribution that is shown in Fig. D-5, we can say that the quantum has a 2 / 46 chance of striking at point A, a 7 / 46 chance of striking at Point B, and no chance at all of striking at point C.

Imagine all these universes arranged next to each other through some new dimension. The problem here is that if the new dimension is space-like, all of these universes together make up a 4-D space, and we would expect the quanta to be able to move around in it. We would see them arriving and disappearing rhythmically in our own 3-D space, just as extra molecules arrive with the crest of each water wave and leave again when there is a trough.

If, on the other hand, the new dimension is time-like, with the same conservation of mass/energy as our normal time, then only one copy of the quantum could exist in

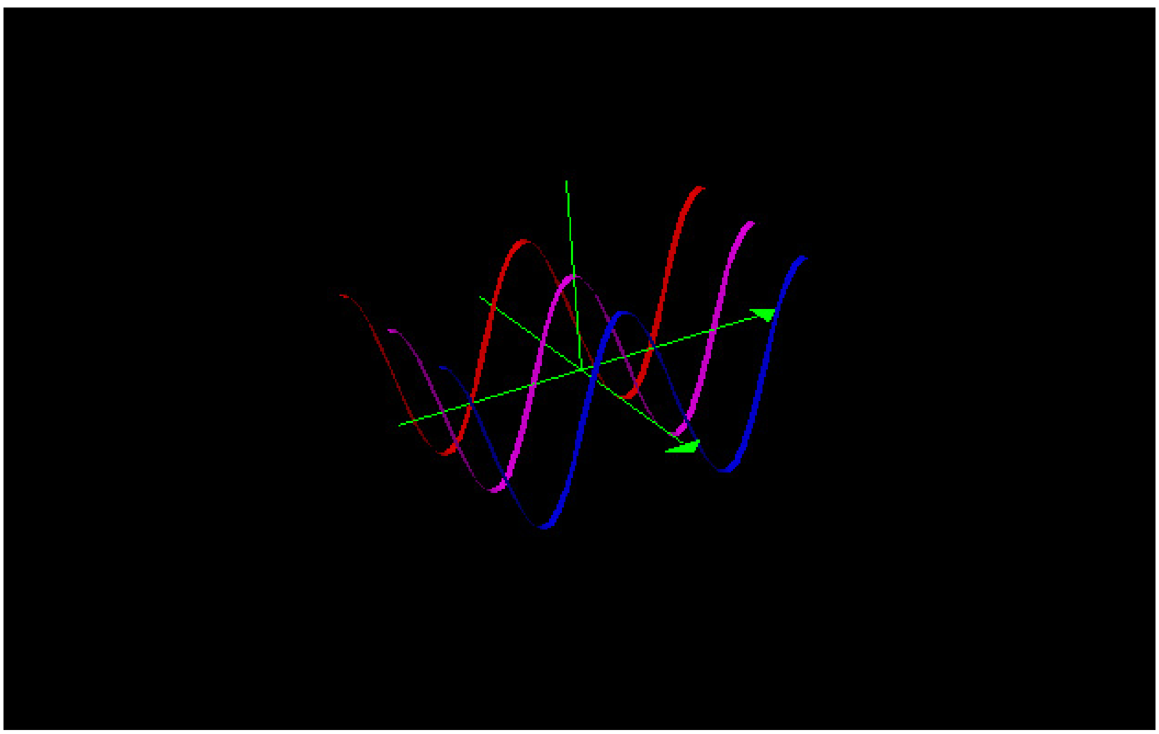


Figure D-9. A probability wave. The arrow that crosses all three waves represents one point in our space as it extends through time. The waves are actually snapshots of one probability wave at three different points in our time. The other arrow would normally be regarded as extending through a probability space, but there has been speculation about parallel universes in that direction. It could perhaps also be seen as an additional time dimension.

In this graph, our timeline extends along a peak of the wave. This means, for the point in our space represented here and for the photons represented by this wave, there is a high probability for the photons to strike that point.

each 3-D space. Also, just as a quantum is emitted at one point in our normal time and absorbed at another, before and after which it does not exist, we would expect that there would be one continuous segment of this new time direction in which the quantum has its existence. This is equivalent to the fact that between your birth and your death, at every moment there exists one and only one copy of you within our normal spacetime. There are no gaps within your life span where you do not exist (but see {endnote 17}). Fig. D-9 is an attempt to depict such a space (compare with Figs. 12-15, pp. 46-7.) Shown is a probability wave for one point of our space, at three points of our normal time. (The probability distribution represented by this wave has not changed during the time shown, although that will not generally be true of distributions.) Our universe follows the time-line of the arrow that crosses all three copies of the wave. We can see that on our time-line, the wave is at a peak. That means that, in our universe, the quantum represented by this wave would have been found in the region just to the right of Point B in Fig. D-8.

The other arrow, the one intersecting the second point of our normal time, is extending in the new time direction. There are parallel copies of our universe all along this new time direction, in some of which the photon will be found in peaks and in some of which it will be found off peaks, but in the universes whose time-lines are in the troughs, the photon will not be found. This does not work. In addition, in the universes at the peaks, several copies of the quantum will be found, not one. We could add another new dimension to account for multiple copies, but that would not help with the universes whose timelines are in the troughs.

In order to create a large-dimensional space where each 3-D space has one and only one quantum, there is at least one more option we can consider. We can suggest that in these two new time directions (actually a new time plane, extending horizontally in Fig. D-9), the 3-D spaces intersect the time plane only at the locations of the copies of the quantum, as in Fig. 17b (p.50). This model does not have a unified set of large dimensions within which everything exists. The idea is more akin to the action physics of Finkelstein (1996) or the transactions of Cramer (1986), at which we look in Sections II-B-3 and III-D-3. They both employ Leibniz' idea of spacetime as a pattern of relationships {endnote 5} rather than as an external, independent stage within which events occur.

(Back to The Two Slit Experiment, p. 54.)

APPENDIX E: 1 + 1 Feynman Checkerboard

(For the technical reader.) An intriguing exercise was undertaken by Kauffman and Noyes (1996), who used the one-dimensional version of Dirac's equation to investigate the Feynman checkerboard. They constructed discrete solutions to the equation and then interpreted them as sums over lattice paths. Their point was that the three solutions can be recombined to give solutions that start right or start left, and that the coefficients in these solutions are $(-i)^c$ where c is the number of corners in the path. This agrees with Feynman's result. They then conclude that each appearance of i corresponds to an elementary rotation.

This result suggests that the use of a mathematics called geometric algebra {endnote 15}, {endnote 18} could give new insight. However, a closer look shows that the Kauffman and Noyes conclusions, though significant, may not be as strong as at first appeared.

When the mathematics in the Kauffman and Noyes paper is followed closely, we can see that if $N_c\,(XY)$ denotes the number of paths with c corners of type XY, then

$$\psi_0 = \Sigma_c\,(-1)^{(c-1)/2}\,N_c\,(LR) = \Sigma_c\,(-1)^{(c-1)/2}\,N_c\,(RL)$$

$$\psi_R = \Sigma_c\,(-1)^{c/2}\,N_c\,(RR)$$

$$\psi_L = \Sigma_c\,(-1)^{c/2}\,N_c\,(LL)\,,$$

rather than as given in their paper. To obtain their next results, we must have

$$\psi_1 \equiv -i\,\psi_0 + \psi_R = \Sigma_c\,(-i)^c\,N_c\,(\mathrm{R})$$

$$\psi_2 \equiv -i\,\psi_0 + \psi_L = \Sigma_c\,(-i)^c\,N_c\,(\mathrm{L})\,. \qquad \text{eq. (2)}$$

The final summations are as they have them, but we can show that these are a bit arbitrary. In fact, by defining ψ_1 and ψ_2 appropriately, we can obtain any exponent we wish. Setting

$$\psi_1 \equiv \psi_0 + i\,\psi_R = \Sigma_c\,(-i)^{c-1}\,N_c\,(\mathrm{R})$$

$$\psi_2 \equiv \psi_0 + i\,\psi_L = \Sigma_c\,(-i)^{c-1}\,N_c\,(\mathrm{L})$$

gives exponents of (c – 1), but also (c + 3, c + 7, . . .). Likewise,

$$\psi_1 \equiv i\,\psi_0 - \psi_R$$

$$\psi_2 \equiv i\,\psi_0 - \psi_L$$

gives exponents of ($c \pm 2$, $c \pm 6$, . . .), and multiplying Eq. (2) by – 1 gives (c –3, c + 1, . . .). However, all of our definitions differ only by multiples of – i , and what remains consistent with the original conclusion is that the addition of Δc corners results in an expression with an exponent that is also increased by Δc.

It could be instructive to explore the relationships between rotations and the appearance of i in the expressions for the paths through larger–dimensioned Feynman checkerboards. The rotational interpretation of Kaufman and Noyes suggests that geometric algebra would facilitate this analysis.

(Back to Sum Over Histories, p. 58.)

ENDNOTES

1. See, for example, Dossey (1982) and Hunt and Hait (1990). The present author has had personal exposure to non-western concepts of time held by acquaintances among the Oglala Lakota and Nakota on the Pine Ridge and Rosebud reservations and among the Dine on the Navajo Nation reservation.

2. Examples of complex quantities are $e^{i\omega t}$, $e^{-i\omega t}$, and $3 + 4i$. The imaginary i in complex quantities equals the square root of -1.

3. The complex conjugate of e^{-ix} is e^{+ix} and of $2i$ is $-2i$, etc. Any sign before a term containing i is reversed.

4. The comment was found in a posting (Arendt, 2000) on an online bulletin board at Cornell. While such casual postings can be suspect, this one was made by an assistant professor of physics at New Mexico Tech.

5. Gottfried von Leibniz was a contemporary of Sir Isaac Newton. A brief description of his ideas is given by Siegfried (2000).

6. This gets a little tricky, because time enters into our definition of energy. That is why our slices have to have a finite thickness in time. Strictly speaking, their thickness should be long in comparison to the energy wavelengths being measured. Since we are measuring across all wavelengths, the concept loses some of its rigor in this formulation. In other words, the uncertainly principle from quantum mechanics puts limits on the precision with which we can define this symmetry. We only know time within a certain mass/energy. Therefore, it is not time that puts a limit on what we can know of energy, but energy that puts a limit on what we can know of time.

7. The reasoning is parallel to that of the previous note.

8. Virtual pairs are temporary particles that always come in pairs.

9. We leave for the philosophers to discuss what evolution might mean in the context of a time that does not flow. See Lockwood (1996a, 1996b) and Roberts (1977-1979).

10. Three-dimensional animated representations of atomic orbitals can be found at http://www.daugerresearch.com/orbitals/.

11. A quite wonderful animated visual applet can be seen at http://www.colorado.edu/physics/2000/schroedinger/two-slit3.html . On a previous page at this site is a depiction of the double-slit experiment with photons, but this page shows the even more dramatic results with electrons. The interactive animation is toward the bottom of the page; try pulling the operator's knob to the far left.

12. All probabilities are between 0 and 1, and are therefore positive. Therefore, they all have both real and complex square roots. For instance, some of the square roots of 1 are 1, -1, $e^{i\theta}$, and $e^{-i\theta}$.

13. Cramer (1986) has said that the past determines the future because the wave emitted from point A, in the past, precipitates the transaction. He has also related a macroscopic dominance of the retarded wave to the arrow of time of thermodynamics (1983). Although his use of the word "precipitate" may have been an unsuccessful attempt to avoid circular logic (the past causes the future because the past caused the future), he has apparently made a valid point concerning nature's violation of time reversal invariance in the case of the decay of the K^0_L meson.

14. The idea that consciousness is required to collapse the wave function is explored in multiple articles by these authors, several of them reprinted in Wheeler and Zurek

(1983). See, for example, Wigner (1961) and Wheeler (1980).

15. An important part of research, including theoretic research, is the discovery and development of new tools. Geometric algebra is a mathematical tool that was rediscovered in the 1960s (see Hestenes, 2003) and has gradually been gaining exposure since. It is beginning to be applied to areas of physics where the computational methods of current mathematics are especially difficult. As noted, it promises a greater ease in dealing with spacetimes of larger dimensionality. One of the strengths of geometric algebra is that it facilitates mathematical visualization. Because so much theoretical work is being done in areas that are extremely difficult to visualize, the formulation has been used to develop a number of software visualization tools. One choice for future research is to investigate the possibilities inherent in these new tools.

16. Mathematics is an area of theoretical research in her own right. Used as a tool in other disciplines, it can enable new modes of research; this is not surprising. But it is a tool that also has the power to suggest new theoretical ideas.

17. At least there are no gaps on a macroscopic time-scale. If time is quantized, we might not be able to make absolute statements about the nature of existence on a quantum scale, but that is another matter.

18. In geometric algebra, $i = \sqrt{-1}$ is given a geometric interpretation. However, there are some differences between the *i* of geometric algebra and the *i* of quantum theory. The interpretation of GA does not require the use of any extra dimensions, time- or space-like, and constitutes a distinct alternative to the approach taken in the present paper. See Doran and Lasenby (2003).

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